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FROM THE

MOUNT WILSON SOLAR OBSERVATORY

VOLUME VI

NOS. 106-123

WITH TWELVE PLATES

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A STUDY OF THE POLE EFFECT IN THE IRON ARC

By CHARLES E. ST. JOHN and HAROLD D. BABCOCK

I. INTRODUCTION

In the investigations made at this observatory upon the determinations of standards of wave-length and upon the relation between laboratory and solar spectra, it was early found that a detailed study of the iron spectrum would be required as a preliminary to further definite progress.[1]

The measurements by different observers of the wave-lengths of the iron lines that serve as standards in the international system show discrepancies which far exceed the limit of precision attainable with a grating spectrograph of high dispersion, and the interpretations of sun-arc displacements may be illusory and lead to diverse conclusions, if the presence and influence of lines of certain types are unrecognized. The divergent results were found to depend upon conditions in the arc, its length, the current-density, and the portion of the arc from which the light is taken, and to inhere in those types of lines that show more or less dissymmetry under varying pressure and line intensity and that appeared to have large or abnormal pressure displacements.[2]

The purposes in view in undertaking the investigation of these sensitive and unstable lines were: (1) to determine whether the variations in wave-length observed with different arc conditions are real or fictitious, that is, whether there are actual displacements of the maxima of the lines; (2) to determine whether there are general pressure differences in the arc sufficient to account for the displacements; (3) to examine some other conditions in the arc which may be effective; (4) to investigate a wide range of the spectrum for the identification and distribution of the questionable lines, which must necessarily play an important rôle in several fields of investigation; (5) to indicate working conditions which should be observed when lines of this character are to be used as standards of reference.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 75; *Astrophysical Journal*, **39**, 9–14, 1914.

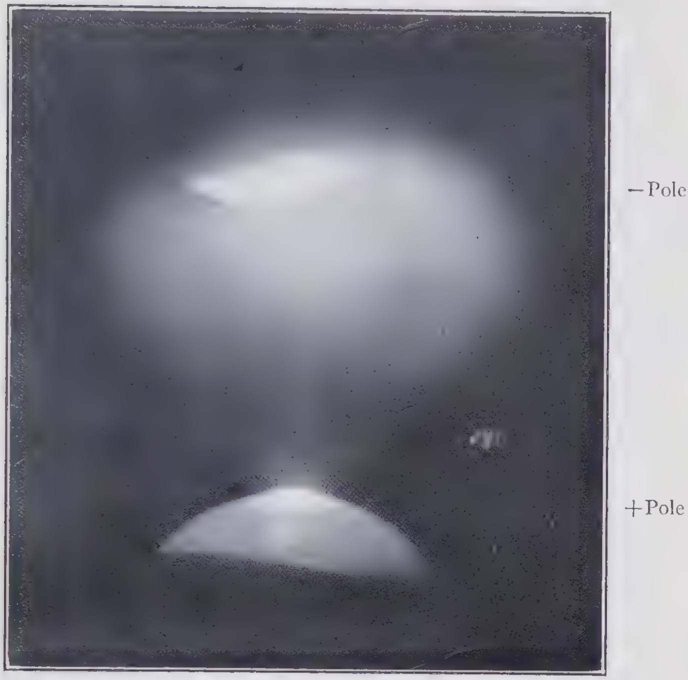
[2] *Mt. Wilson Contr.*, No. 61, pp. 2–5; *Astrophysical Journal*, **36**, 15–18, 1912; *Mt. Wilson Contr.*, No. 75, pp. 5–8; *Astrophysical Journal*, **39**, 9–12, 1914; *Mt. Wilson Contr.*, No. 93, pp. 33–36; *Astrophysical Journal*, **41**, 61–64, 1915; Goos, *Astrophysical Journal*, **38**, 141, 1913.

II. METHODS AND APPARATUS

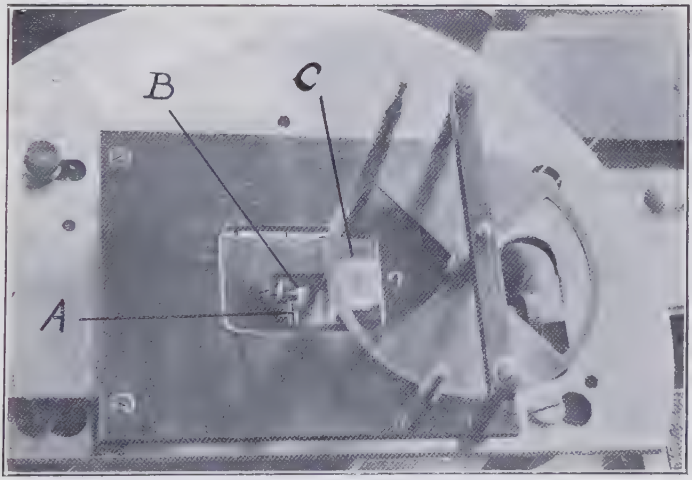
Our method has involved an accurate comparison of the spectrum from near the poles with that from the center of the arc. In the effort to carry such a comparison of wave-length to the third decimal place, difficulty was encountered in obtaining plates free from line displacements of small but determinable amounts due to instrumental causes or observing conditions. When the problem is such that crucial tests can be applied to the results, the difficulties involved in obtaining comparison spectra of consistent reliability are quite impressive. These were in the end eliminated by making the exposures rigorously simultaneous.

A greatly enlarged image of the arc was thrown upon the slit of a plane-grating Littrow spectrograph of 30 feet focus by means of an achromatic lens and a large totally reflecting prism above the slit. Plate I*a* shows an image of the arc as it actually appeared upon the slit. The light from the central portion passed directly to the slit, at *A*, Plate I*b*, while a system of two small totally reflecting prisms, *B* and *C*, permitted light to be taken from any desired point on the axis of the arc. The spectrum of the light from this selected portion of the arc fell between the two narrow spectra of the equatorial section. For measurements of high precision, it is of great advantage to have the lines of the two spectra of nearly the same intensity, and this is particularly important when the lines have a tendency to broaden unsymmetrically. The relative exposure times were controlled by a rotating sector of variable opening placed above the first prism in the path of the light coming from near the pole of the arc, as shown in Plate I*b*. The exposures began and ended at the same instant, one being continuous and the other rapidly intermittent. In this manner any effect due to slow changes of temperature, to flexures of apparatus, or to vibrations reaching

PLATE I



a. THE ARC AS PROJECTED UPON THE SLIT



b. SECTOR AND PRISMS IN POSITION

the instrument through the ground was effectively eliminated; no part of the instrument was touched during the exposures, so that no effects due to mechanical displacements were introduced. Great care was used to secure complete illumination of the grating from each light-source. A similar arrangement was used in comparing arcs with different current-densities and it is applicable to all comparisons where the highest precision is sought. It will be added to the reconstructed 60-foot tower telescope equipment for sun-arc comparisons.

Before the sector was employed the exposures were made of unequal length in order to equalize the intensities. They were simultaneous during the period covered by the shorter; but, though the apparatus is very stable, was not touched during the exposures, and has its optical parts in an underground chamber at practically constant temperature, there were minute displacements, positive on some plates and negative on others, differing even for successive exposures on the same plate. It is not a sufficient precaution to divide an exposure of a comparison spectrum with the thought of detecting an instrumental displacement when the quantities involved are of the order of a thousandth of an angstrom.

The Pfund[1] form of arc, 6 mm long, carrying a current of 6 amperes, has been found to be a very stable and reliable source. In obtaining standards of reference the slit of the spectrograph was placed normal to the axis and in the central plane of the enlarged image of the arc. Except where otherwise noted the Pfund arc has been thus employed for all wave-length comparisons given in this paper.

Some plates in the ultra-violet were taken with a 5-inch concave grating of 15 feet radius. On account of astigmatism the arrangement of prisms and sector described above could not be employed. The exposures were therefore not simultaneous, and the use of a shutter in front of the plate became necessary. Under these conditions minute displacements due to instrumental causes appeared upon the plates, for the correction of which a series of overlapping exposures was made to connect these photographs with those obtained by means of the plane grating.

---

[1] *Astrophysical Journal*, **27**, 296, 1908.

In a valuable paper Eder and Valenta[1] call attention to the precautions necessary to obtain reliable photographs of spectra. The care that we have given to this question may be seen from the following details. The 4-inch plane grating (ruled surface 63×72 mm, 42,386 lines) used in the Littrow spectrograph of 30 feet focus was ruled by Anderson on the reconstructed Rowland machine. It has been shown to yield over 90 per cent of its theoretical resolving power in the second order, and is particularly well suited to the purposes of this investigation, as it produces bright line spectra of a high degree of perfection. In the second order, which was used for this work, the diffraction pattern is distinct and symmetrical on the two sides of a good line, and when the spectrum is observed visually the diffraction fringes of the lines of pressure groups *a* and *b* remain sharp and distinct even near the poles of the arc. The focus for any particular region is read from an experimentally derived curve which is accurate to 0.5 mm; the focal length of the instrument is 9.16 m. The slit-widths were always very nearly four times the Rayleigh normal, and give 80 per cent of the resolving power available with an indefinitely narrow slit and 77 per cent of the intensity given by a very wide one, thus making an excellent compromise between opposing conditions. Tests were made to detect a possible variation of the gradation-curves of the lines with the slit-width, but no effect was found for the widths employed. Great care was taken to avoid overexposing the spectrum lines. For the microphotometric measurements with the Hartmann and Koch instruments no lines were used whose maximum blackness was such as to lie in the region of overexposure, as defined by the characteristic curve for the plate. On the plates used for filar micrometer measurements the exposures were timed to equalize the widths and intensities at pole and center. If the strongest lines were possibly overexposed, the weakest lines on the same plate were underexposed, but among the lines of intermediate strength some must have had correct exposures. Lines of all ranges of intensity, however, showed the effect.

For obtaining an independent check upon the magnitude of the pressure-effect some photographs were made by means of an interferometer of the Fabry and Perot type, recently constructed in our instrument shop. The plates, by Hilger, are of fused quartz, 40 mm in diameter and 5 mm thick. Their surfaces are of the highest quality and in performance they equal our glass plates furnished by Jobin. We have coated them with silver by the method of cathodic sputtering *in vacuo*, obtaining perfect uniformity in the films and a reflecting power so high that more than 60 images of the sun can be counted through them. The étalon used is made of invar and has a thickness of 10 mm. The rings are projected by means of an achromatic objective of 41 cm equivalent focal length upon the slit of a 13-foot Littrow spectrograph. The auxiliary dispersion is produced by a 4-inch plane grating having a remarkably bright first-order spectrum.

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[1] *Astrophysical Journal*, **19**, 251, 1904.

III. THE REALITY OF THE DISPLACEMENTS

As the lines exhibiting these variations in wave-length show more or less dissymmetry under pressure and are greatly widened at the pole, it has been thought by some that the displacements in question are only apparent and arise from the difficulties inherent in the measurement of lines of this character, and by others that they are due to increase of pressure or of vapor-density in the arc. The point is not only of theoretical interest because theories of the production of spectrum lines must be founded upon detailed study of the conditions of emission, but it is also of importance in view of the possible employment of these lines in solar and stellar investigations, for which their peculiar characteristics promise to be of value, provided their properties can be well established and the conditions determined under which they may be used.

To eliminate as far as possible the disturbing effects due to dissymmetry, we have made the net exposure times such as to give practically equal intensities to the lines from the two parts of the source. The lines λλ 5424 and 6400 observed in this way are shown in Plate II*a*, with a twenty-eight-fold enlargement. These are typical lines, showing displacements toward the violet and red, respectively. In order to make the shifts still more apparent in the reproduction, fine artificial lines have been ruled through the centers of the comparison spectra. For an extreme case, the relative exposures were such that the intensity and width of the lines in the spectrum from near the negative pole were less than for the same lines in the spectrum of the central section of the arc. In this case, lines that widen unsymmetrically to the red would not show at the pole a displacement toward greater wave-lengths having its origin in this quality of dissymmetry. An illustration is shown in Fig. 1, which is a diagram to scale from measurements of λ 6400 taken under these conditions. It is evident that if the width of the line, as observed near the pole, were increased by a longer exposure to equal that of the same line in the spectrum of the central section of the arc, the line would still show a distinct displacement to the red. When such lines are of moderate intensity upon the photographic plate, an experienced observer can make micrometer settings upon their maxima with surprising consistency.

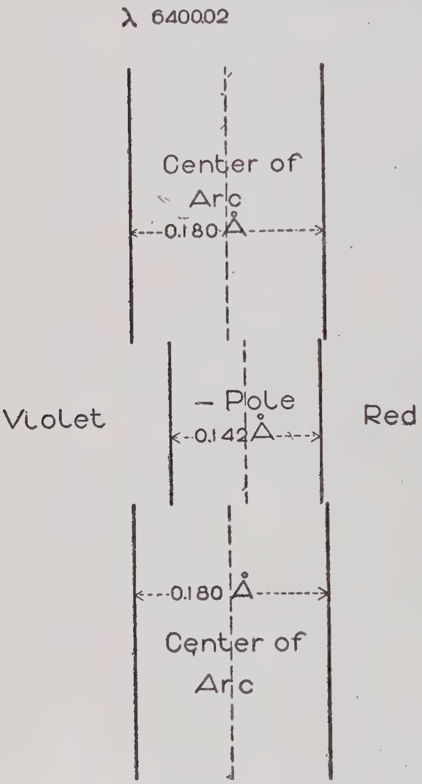
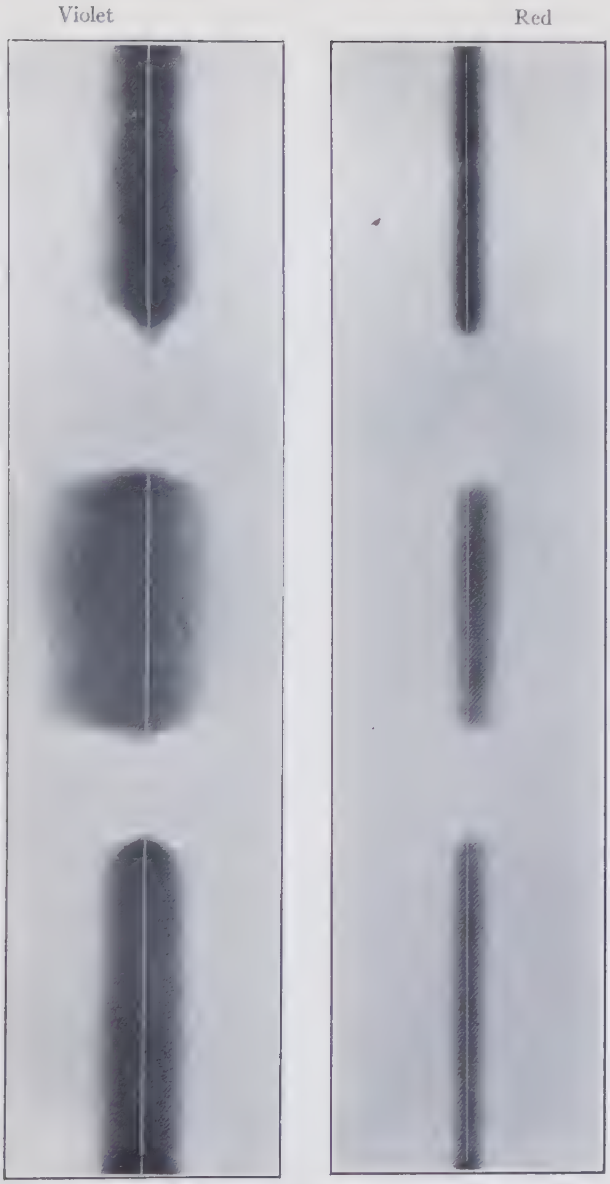


Fig. 1.—Persistence of displacement when width at pole is less than at center.

Dissymmetry is not evident in the case of every line that gives a displacement to the red at the pole of the arc as compared with its position at the center. For example, λ 5339 is apparently as good a line as λ5341, though the first belongs to group *d* and the second to group *a*. On one plate, for illustration, the widths of the line at the center and negative pole are 0.106 and 0.108 A, respectively, and the measured shift of the latter is 0.015 A to the red, which cannot be accounted for on the ground of unsymmetrical widening when the increase in width is only 0.002 A.

PLATE II



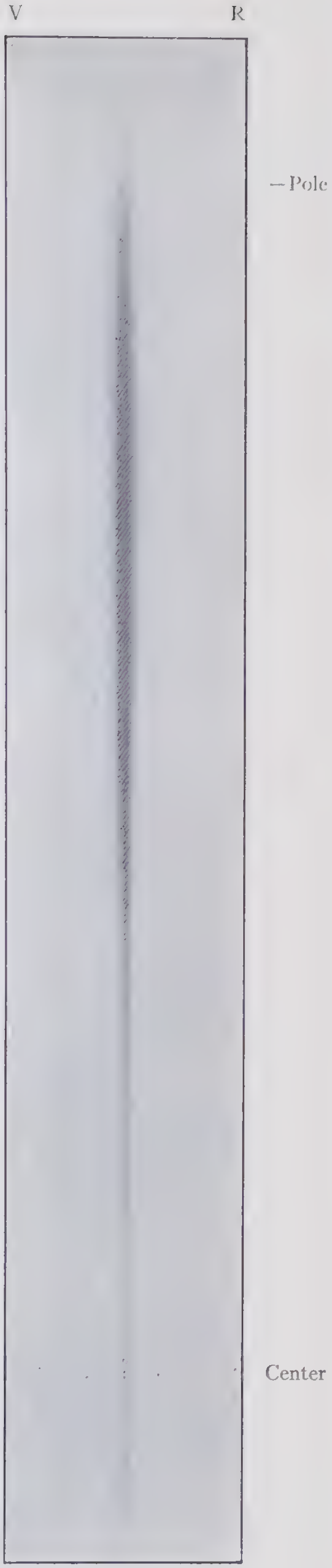
λ 5424

a

λ 6400

a. Displacements at pole toward violet for λ 5424, toward red for λ 6400. Artificial lines ruled through centers of comparison spectra. Enlarged 28 times.

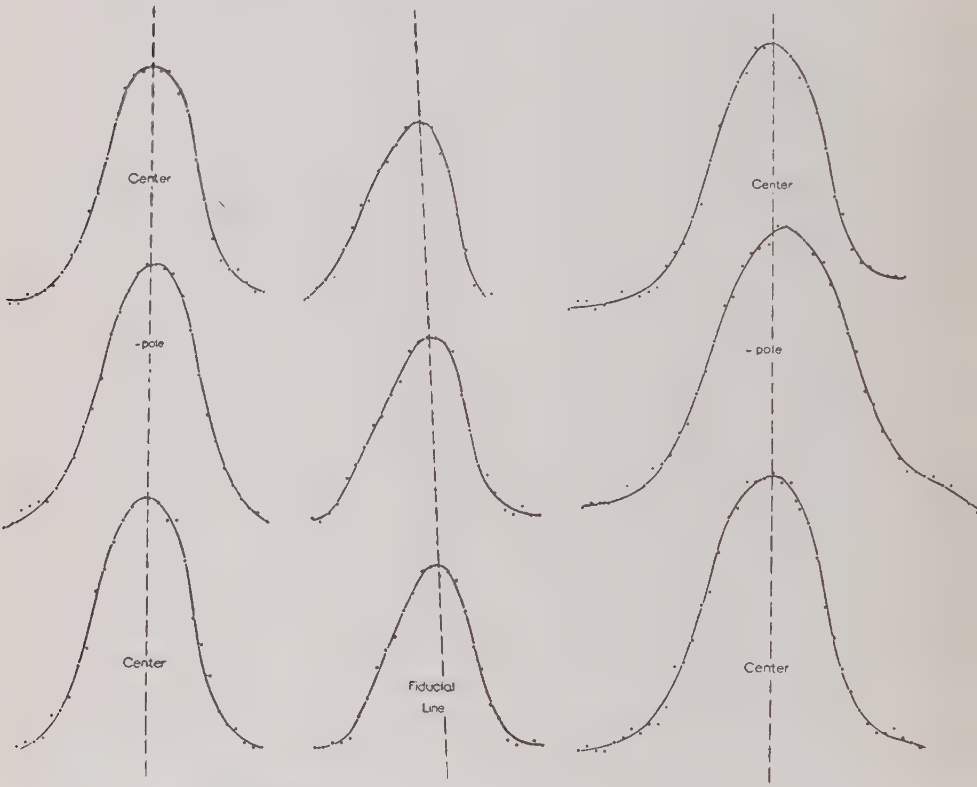
b. Iodine absorption line superposed upon iron line λ 5424, showing displacement of the latter at the negative pole. Enlarged 6.7 times.



b

In such cases the personal equation of the observer is involved, and one may question more or less the possibility of measuring lines of this character to the degree of precision suggested. In order to eliminate the personal element, measurements have been carried out with the Hartmann and Koch microphotometers. Fiducial lines were drawn upon the plate, generally one on each side and as nearly parallel as possible to the spectrum line under investigation. With the Hartmann microphotometer settings were made by 0.02 mm steps from one fiducial line to the other, across them and the spectrum lines along each of the three divisions of the spectrum. The density-curves were plotted to scale and the positions of the maxima determined relatively to the fiducial lines. Some lines of groups *a*, *d*, and *e* were measured in this manner, and in all cases the measurements showed displacements of the maxima of the *d* and *e* lines to the red and violet, respectively. Curves determined by the Hartmann microphotometer are shown in Fig. 2 for lines of groups *b* and *d*, and in Fig. 3 for lines of groups *a* and *e*. The results have been confirmed by the photographic records made with the Koch form of microphotometer. Both instruments show that the displacements of the maxima are unquestionable and yield values of the same order as those fonud by the usual method, but the dissymmetry is much less striking in the graphs than was expected from the visual examination of the lines. It would seem that the eye is very sensitive to contrast and overestimates the degree of asymmetry.

An entirely independent test for the reality of the displacements was made by superposing upon the spectrum of the iron arc the absorption spectrum of iodine vapor under low pressure. For this purpose the slit was placed parallel to the axis of the arc, the prisms and sector being removed, and a spherical glass bulb containing the iodine was inserted in the path of the light from the arc. A number of the iron lines in the green are found to have iodine lines superimposed more or less centrally upon them. Plate II*b*, illustrating this, shows the line λ 5424 enlarged 6.7 fold, the upper end corresponding to the negative pole of the arc, the lower end to a point near the center. On the original plate three iodine absorption lines can be distinguished, one of which is seen in the reproduction to lie clearly upon the iron line. At the center of the arc it is obviously well to the violet side of the maximum of the line, while at the pole it is plainly upon the red side. Although the other two iodine lines fall upon the violet wing of the strongest part of the line, cutting it off sharply and greatly reducing the width of this portion, the shift of the maximum of the iron line between pole and center is clearly apparent. It appears to us that the only possible interpretation of this observation is that the wave-length corresponding to the maximum intensity of the iron line is less at the pole than at the center of the arc. On the original

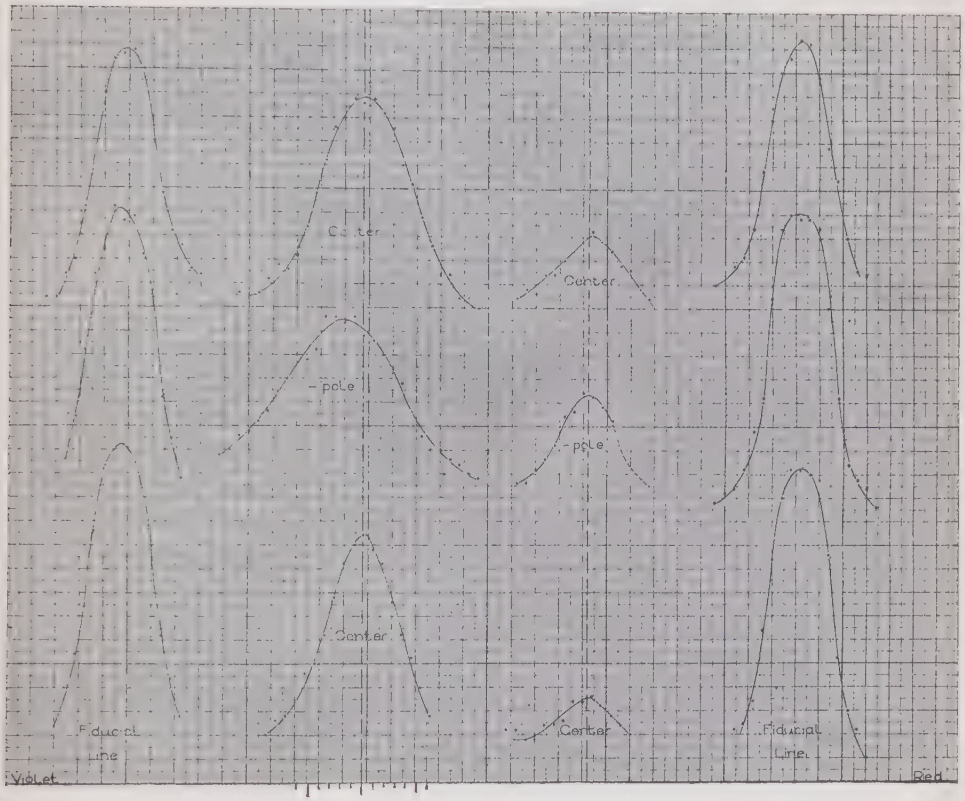


λ 6393 group *b* λ 6400 group *d*

Fig. 2.—Microphotometer curves showing displacement at pole for λ 6400, group *d*, but not for λ 6393, group *b*.

negative a smaller shift in the same direction is shown at the positive pole.

The question whether the unsymmetrical broadening of spectrum lines may be accompanied by displacements is an old one.[1]



λ 5364.8 group *e* λ 5365.4 group *a*

FIG. 3.—Microphotometer curves showing displacement at pole for λ 5364.8, group *e*, but not for λ 5365.4, group *a*.

Professor Kayser remarks that the position of a spectrum line is determined by the intensity maximum. In measuring these lines, the effort has been to set upon the maximum; a double cross-hair in the observing microscope, with the space adapted to the width of the particular line under examination, has proved a satisfactory device for the purpose.

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[1] Kayser, *Handbuch der Spectroscopie*, **2**, 297, 1902; *Astrophysical Journal*, **26**, 191, 1907; Exner and Haschek, *Die Spektren der Elemente bei normalem Druck*, **1**; *Sitzungsberichte der Wiener Akad.*, **116**, Abt. II*a*, 323, 1907; Eder and Valenta, *Astrophysical Journal*, **19**, 251, 1904.

There are three points, each of which appears to us to establish the reality of the displacements of the maxima of the line between pole and center: (1) the persistence of the displacements when the intensity and width of the line at the pole are less than for the same line produced in the central section of the arc, since the maximum is the effective portion of the line when exposure times are shortened; (2) the shift of the intensity maximum when photometrically determined; (3) the relative position of the maxima at pole and center with respect to the superimposed iodine lines.

IV. PRESSURE IN THE ARC

The question whether the pressure differs from point to point in the arc sufficiently to account for these displacements was investigated by examining the behavior of lines of known pressure-shift that remain symmetrical under wide ranges of pressure and density. It is a comparatively easy matter to make differential measurements of high-dispersion spectra of good lines to the third decimal place, but a difficult matter to obtain comparison spectra that are reliable to this degree of precision. It was only after developing a method of obtaining rigorously simultaneous exposures that we were able to convince ourselves that no general pressure differences occur in the arc. The data in Table I illustrate the relative behavior of the lines of groups *a*, *b*, *c5*, *d*, and *e*. At the bottom are appended the mean pressure displacements per atmosphere for the same groups of lines.

The displacements of the lines of groups *a* and *b* do not exceed the limit of error, but the pressure displacement per atmosphere, in the case of the *b* lines, is of a magnitude to show a change of pressure of one-tenth of an atmosphere, while to produce the displacements of the lines of groups *c5* and *d* an increase of pressure of one and a half to two atmospheres would be required. The behavior of the lines of group *e*, on the other hand, would indicate a great reduction of pressure. It appears therefore from the data that the displacements shown by the lines of groups *c5*, *d*, and *e* are not due to a *general* increase of pressure in the vapors near the pole of the arc.

TABLE I

NEGATIVE POLE *minus* CENTER OF THE ARC

GROUP *a*		GROUP *b*		GROUP *c5*		GROUP *d*		GROUP *e*	
λ	P−C	λ	P−C	λ	P−C	λ	P−C	λ	P−C
5328	0.0000 A	6136	+0.0007 A	4890	+0.014 A	5281	+0.019 A	5364	−0.028 A
5332	−0.0002	6137	−0.0004	4919	+0.016	5283	+0.018	5367	−0.025
5341	−0.0006	6213	+0.0003	4920	+0.013	5324	+0.015	5369	−0.020
5497	−0.0003	6219	+0.0010	4938	+0.018	5339	+0.019	5383	−0.025
5501	−0.0004	6230	+0.0004	4957	+0.014	5393	+0.017	5404	−0.025
5506	+0.0004	6252	0.0000	4957	+0.014	5653	+0.016	5410	−0.026
Mean.	−0.0002 A		+0.0003 A		+0.015 A		+0.018 A		−0.025 A
Displ. per atm.	+0.0036		+0.0094		+0.0094		+0.0092		+0.0017

V. OTHER CONDITIONS IN THE ARC

Density.—The absence of a general increase in pressure between the center and the negative pole led us at first to consider density as a possible cause of the pole effect. The concentration of luminosity is marked at the poles and is quantitatively indicated by the longer exposure upon the center of the arc required to equalize the intensities of the pole and center spectra. To test the possible effect of density upon the position of these sensitive lines, a pair of furnace plates was taken for us by Mr. King with 0.2 g and 2.0 g of iron, respectively. The iron deposit upon the walls of the graphite tubes showed that in both cases all of the iron was vaporized, so that the ratio of the vapor densities may be considered comparable to the quantities of metallic iron. Neighboring lines belonging to group *a*, the flame lines, were used as standards. Their wave-lengths are independent of the current-density and the part of the arc serving as a source, and King has found[1] that for lines of this type the pressure displacement does not depend upon the density of the furnace vapor. Our measurements are given in

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[1] *Mt. Wilson Contr.*, No. 60, pp. 21–27; *Astrophysical Journal*, **35**, 203–209, 1912.

Table II, and show no increase of wave-length with the increased density, though for the same lines the increase in passing from the center to the pole of the arc is 0.0195 A. Some lines of manganese present as an impurity in the iron poles of the arc show displacements at the negative pole of the same order as the neighboring lines of iron. It does not seem probable that the density of a trace of vapor would be very appreciable even at the negative pole.

TABLE II

LINES OF GROUP *d* UNDER VARYING DENSITY AND TEMPERATURE IN THE FURNACE

λ	Density $\lambda_{HD}-\lambda_{LD}$	Pole Effect	λ	Temperature $\lambda_{HT}-\lambda_{LT}$	Pole Effect
5266	0.000 A	+0.017 A	5232	0.000 A	+0.025 A
5324	−0.004	+0.015	5266	0.000	+0.017
5586	+0.001	+0.024	5324	−0.001	+0.015
5615	−0.003	+0.022			
Means	−0.0015	+0.0195		−0.0003	+0.019

Temperature.—The effect of a variation in temperature was investigated by comparing the wave-lengths of these sensitive lines at temperatures in the furnace as widely different as practicable (2100°–2600° C.). They are high-temperature lines, and hence appear strongly only at the highest furnace temperatures, but the three given in Table II are measurable at the lower temperature and are free from the lines of the carbon flutings appearing so abundantly at the highest temperature. With the neighboring lines of group *a* as standards, no increase in wave-length was found for a 25 per cent increase in temperature, though between the center and the negative pole of the arc the displacement is 0.019 A.

In vacuo.—In order to make a preliminary test of the possible effect of electrical conditions, a 6 mm arc of the same form as that used under normal pressure and actuated by a current of like intensity was operated in a vacuum chamber at pressures of 0.5 cm and 10 cm of mercury; comparisons were made between the negative pole and center with practical equality of intensities for the lines under examination. In Table III data are given showing the results of measurement for five groups of lines.

For the lines of groups *a* and *b* the measurements show the same results as at atmospheric pressure, that is, no pole effect in either case; but for these typical lines of groups *c*5, *d*, and *e* the pole effects disappear with pressures below 10 cm of mercury, though at atmospheric pressure they are +0.012, +0.017, and −0.027 A, respectively.

TABLE III

NEGATIVE POLE *minus* CENTER *in vacuo*

λ	Group *a*	λ	Group *b*	λ	Group *c*5	λ	Group *d*	λ	Group *e*
5270	+0.0010	4547	0.0000	4607	+0.0008	5266	−0.0006	5367	−0.0005
5328	−0.0005	4592	+0.0005	4611	+0.0008	5281	+0.0005	5383	−0.0005
5332	+0.0002	4602	+0.0002	4736	0.0000	5283	−0.0006	5410	0.0000
5341	0.0000	4786	0.0000	4859	−0.0005	5302	−0.0004	5415	0.0000
5397	−0.0003	4789	−0.0010	4872	−0.0002	5324	+0.0018	5424	0.0000
Means	+0.0001		−0.0001		+0.0002		+0.0001		−0.0001

Though the arc *in vacuo* was of the same type and length and carried the same current as the arc under normal pressure, its appearance was strikingly different; the luminosity in such a case is more evenly diffused over the poles, and does not issue from a point source. Though the polar region is brighter than the central zone, the concentration in the core is practically absent. The disappearance of the pole effect under these conditions indicates that the potential difference plays a minor rôle, if any, but a more definitive investigation is to be undertaken.

Relation to luminosity.—The observations of Fabry and Buisson[1] brought out a striking difference of luminosity in the positive and negative regions of the iron arc, a difference confirmed by the change in the relative exposure times required to equalize the intensities of pole and center spectra on our plates. The ratio of center to pole exposure is greater for the red than for the violet at the negative pole. There is, therefore, a greater apparent change in radiation conditions in passing from the center to the negative pole in red than in violet light, but the loss of light through scattering and absorption in the outer layers of the arc vapor is undoubtedly greater for shorter wave-lengths, and as the negative pole is surrounded by a much thicker envelope of cooler vapor than is the central portion of the arc, the apparent relative luminosity for these parts of the arc should be quite different for widely separated spectral regions. At the positive pole the greatest apparent change in intensity of radiation between center and pole is in violet light. If the pole effect is closely related to these conditions, one would expect the displacements for the longer wave-lengths to be greater at the negative than at the positive pole, and for the shorter wave-lengths the reverse relation to obtain. The observations are given in Table IV.

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[1] *Journal de physique* (4), **9**, 929, 1910.

TABLE IV

COMPARATIVE DISPLACEMENTS AT POSITIVE AND NEGATIVE POLES

Group	Ultra-Violet			Green		
	λ	Neg. Pole −Center	Pos. Pole −Center	λ	Neg. Pole −Center	Pos. Pole −Center
d	3322.50	+0.010	−0.001	5339.94	+0.019	+0.004
	3407.47	+0.006	0.000	5393.19	+0.017	+0.004
	3426.65	+0.004	+0.002	5569.63	+0.020	+0.003
	3667.28	+0.008	0.000	5602.96	+0.019	+0.007
Means		+0.007	0.000		+0.019	+0.004
e	3689.46	−0.011	−0.004	5367.46	−0.025	−0.005
	3694.00	−0.014	−0.004	5369.96	−0.020	−0.004
	3748.96	−0.014	−0.006	5410.90	−0.026	−0.004
	3754.50	−0.008	−0.006	5415.19	−0.030	−0.006
Means		−0.012	−0.005		−0.025	−0.005

For the region of λ 5400 the displacements are largest at the negative pole, but the reverse does not hold for the ultra-violet, though for the lines of group *e* the difference between the two poles is less for the shorter wave-lengths. The evidence against a relationship between luminosity and pole effect is strong in the case of the lines of group *d*, which appear to constitute a homogeneous group, since their pressure-shift varies as the cube of the wave-length; but the lines in the two sections of group *e* are not so connected and upon other grounds seem to be less intimately related. Under the reduced pressures used the pole effect disappears for the lines near λ 5400, but persists for the lines in the violet. The true pressure-shift for the lines in the green is very near zero, while for the violet region these lines show displacements to the violet under increase of pressure. A more complete consideration of this class of lines will appear in a later paper.

Local pressure.—In comparing the spectra of the core and the flame of the arc, Adams[1] found that the ratio of core to flame intensity is highest for the lines of groups *d* and *e*, and lowest for lines of group *a*. Not only are the lines under consideration relatively the strongest in the core when compared to the flame of the arc, but they also increase in intensity more rapidly on approaching the poles than any other group of lines, so that the energy density for them is high in the limited volume of vapor in which they originate. This raises the question of a local increase in pressure in the core of the arc near the pole, which might produce an effect on those lines of which the core is mainly the source, while in the case of lines of groups *a* and *b*, for which the core plays a less important rôle, the effect may escape observation. In general the measurements for such lines show a slight indication of displacement, but of a magnitude not considered to be within the range of precision of our measurements. A possible method of approaching the question of a local increase in pressure is to compare the pole effect with the pressure displacements for the same lines. We have measured the pressure-shift per atmosphere by comparing the center of the 6 mm arc under pressures of 0.5 cm and 10.0 cm with the center of a duplicate arc under normal pressure. Upon this point St. John and Ware say:

> Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine *in vacuo* and under normal pressure the behavior of an extended list of lines belonging to groups *d* and *e*.[2]

The precision which our preliminary measurements give justifies the opinion implied in the reference made. Taken in the central section of the 6 mm arc in both cases, the lines are of good quality, and the displacements may be determined with a surprising degree of accuracy, but the results show that the pressure displacements formerly attributed to these sensitive lines are greatly in error through pole effect.

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[1] *Mt. Wilson Contr.*, No. 40; *Astrophysical Journal*, **30**, 86, 1909.

[2] *Mt. Wilson Contr.*, No. 61, p. 21; *Astrophysical Journal*, **36**, 37, 1912.

TABLE V

PRESSURE-SHIFT, POLE EFFECT, AND WAVE-LENGTH

Group	No. of Lines	Mean λ	Δ per Atm.	Pole Effect	Pressure-Increase
d......	25	4085	+0.0048 A	+0.0099 A	2.1 atm.
d......	12	5528	+0.0089	+0.0206	2.3
d......	6	6350	+0.0160	+0.0185	1.2
c5	16	4766	+0.0093	+0.0119	1.3
e......	7	3755	−0.0035	−0.009	2.6
e......	8	5392	+0.0017	−0.026	?

The results for representative groups of lines are assembled in Table V. The increases of pressure on the assumption that the effect is due to pressure at the negative pole, given in the last column, show variations exceeding the limits of error.

The pressure displacements for the three sections of group *d*, shown in Table V, are related as the cube of the wave-length, confirming the conclusion of Gale and Adams.[1] The values calculated from the weighted equation

$$\Delta\lambda=\left(\frac{\lambda}{5000}\right)^3 0.00804$$

show residuals as in Table VI.

TABLE VI

AGREEMENT WITH CUBE LAW

MEAN λ	Δλ			WEIGHT
	Observed	Calculated	Obs. − Calc.	
4085......	0.0048 A	0.0043 A	+0.0005 A	25
5528......	0.0089	0.0108	−0.0019	12
6350......	0.0160	0.0164	−0.0004	6

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[1] *Mt. Wilson Contr.*, No. 58, pp. 22–26; *Astrophysical Journal*, **35**, 32–36, 1912.

On taking account of the weights, it will be seen that the small deviations of the observed points from the theoretical curve are properly distributed; but an examination of the pole effects in Table V shows that they do not follow the cube law even approximately. Since the pole effects are not so related to the wave-length, it is evident that pressure alone does not explain them.

VI. IDENTIFICATION AND DISTRIBUTION OF AFFECTED LINES

Between λ 2979 and λ 6678 we have examined 1570 lines; when the negative pole is compared with the center, we find 286 lines showing displacements to the red and 80 with displacements to the violet; that is, 23 per cent of the lines are affected. In Table VII the lines showing displacements to the red are listed, and in Table VIII those which are displaced to the violet. The first column identifies the lines by their wave-lengths in international units to the second decimal place. The present available determinations of wave-length show variations of such magnitude for the majority of the lines listed that it seems advisable for the present to omit the third place. The second column gives the intensity and character according to Burns[1] for the lines contained in his tables; the others are in general very weak lines. The displacements, negative pole minus center, are in the third column; when followed by I.S., the corresponding lines are international standards of the second order. No attempt has been made to separate the groups *c5* and *d*, nor to indicate any subgroups. This is best done by means of the pressure-shifts, which are at present under investigation.

The distribution is shown in Table IX. The regions are so selected that the number of affected lines per 100 A is fairly uniform in each. From the point of view of one who uses the iron lines as standards of wave-length, as reference lines in solar or stellar investigations, or as a basis for intensity comparisons, not only is the absolute distribution of interest, but also the proportion of affected lines in a given spectral region. The sensitive lines are numerous in sections of the ultra-violet; they form, however, a small percentage of the total number of lines there, but in the

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[1] *Lick Observatory Bulletin*, **8**, 27, 1913.

TABLE VII

Fe Lines, Groups *c5* and *d*

Negative Pole *minus* Center of Arc
Displacements to Longer Wave-Lengths

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
2991.65	4b	+0.006 A	3739.54	1h	+0.014 A
3012.46	2b	+0.006	3740.06	1	+0.019
3048.47	2H	+0.007	3787.60		+0.007
3093.89	2b	+0.006	3789.44	1	+0.015
3154.51	2b	+0.007	3811.01		+0.010
3188.59	4b	+0.004	3814.79		+0.012
3208.48	4	+0.004	3817.65		+0.030
3209.33	4b	+0.010	3830.87	1	+0.008
3211.69	4b	+0.004	3920.85	1b	+0.008
3322.50	4b	+0.010	3928.09	1h	+0.010
3407.47	7l	+0.006	3941.29	2b	+0.014
3410.90	1	+0.006	3947.00	2b	+0.004
3426.65	6	+0.004	3948.11	3b	+0.009
3438.51	3b	+0.004	3955.36	2b	+0.020
3445.78	2b	+0.007	3957.03	2b	+0.018
3459.74	1b	+0.004	3963.11	2b	+0.012
3474.44	2	+0.007	3965.44	1	+0.010
3518.68		+0.011	3976.62	2	+0.004
3522.27		+0.004	4018.28	2b	+0.004
3532.56		+0.012	4024.75	2	+0.013
3568.98	4	+0.010	4030.51	3b	+0.011
3583.67		+0.013	4058.23	2b	+0.006
3586.61		+0.010	4065.40	1	+0.006
3587.25	2H	+0.010	4072.52	1b	+0.008
3592.66		+0.010	4073.76		+0.008
3599.13		+0.012	4083.78	1	+0.008
3604.69		+0.007	4084.51	4	+0.007
3607.55		+0.008	4101.27	1b	+0.008
3612.08	4	+0.004	4104.13	2b	+0.008
3613.47		+0.007	4109.07	1	+0.004
3616.32		+0.009	4112.98	2b	+0.020
3620.47		+0.007	4118.90	1	+0.010
3635.20		+0.010	4125.63	1	+0.017
3636.49		+0.007	4133.87	2b	+0.009
3643.14		+0.009	4150.28	2b	+0.018
3644.81		+0.009	4153.92	4b	+0.012
3655.68		+0.016	4154.82	3	+0.004
3662.85		+0.014	4157.80	3b	+0.018
3664.56	2	+0.004	4158.81	2b	+0.014
3665.81		+0.006	4171.70	2	+0.007
3666.26		+0.011	4187.05	6	+0.008
3667.28	4h	+0.008	4187.81	6	+0.010
3667.99	2h	+0.004	4191.44	6	+0.010 I.S.
3676.88	1b	+0.006	4195.34	3b	+0.009
3688.48	1b	+0.005	4196.22	2	+0.011
3697.44	2h	+0.006	4198.31	6	+0.010
3703.70	1h	+0.005	4198.64	2	+0.009
3721.51	1	+0.005	4210.36	6	+0.006
3735.33	3	+0.005	4217.56	2b	+0.008

TABLE VII—*Continued*

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
4222.22	5	+0.006 A	4938.83	5	+0.018 A
4225.46	4b	+0.004	4944.34		+0.024
4227.45	7	+0.014	4946.40	2	+0.024
4233.16	1	+0.007	4950.12	1	+0.020
4233.61	6	+0.010 I.S.	4954.11		+0.016
4235.95	8	+0.009	4954.41		+0.029
4238.83	4b	+0.011	4957.31	5	+0.014
4247.44	5b	+0.011	4957.61	10	+0.014
4250.13	7	+0.010	4960.65		+0.020
4260.49	10	+0.012	4966.10	5	+0.017 I.S.
4264.21	2	+0.006	4969.94	2b	+0.045
4271.17	7	+0.010	4970.80		+0.026
4299.25	7	+0.009	4973.11	2	+0.018
4343.28	2	+0.012	4978.61	2	+0.018
4401.30	3	+0.004	4982.52	4b	+0.026
4407.71	2	+0.011	4983.27	3b	+0.020
4446.85	2b	+0.012	4985.27	3	+0.016
4462.01	3b	+0.020	4985.56	3	+0.019
4469.39	4b	+0.008	4988.97	2	+0.018
4531.64	2	+0.006	4991.29		+0.030
4581.52	2	+0.012	5001.88	5	+0.023
4598.13	2b	+0.012	5002.82		+0.022
4607.66	4	+0.014	5005.73	4	+0.032
4611.29	4b	+0.006	5006.13	5	+0.016
4613.23	3	+0.011	5007.31	2h	+0.030
4625.06	4	+0.012	5014.96	4	+0.022
4637.52	4	+0.012	5022.25	4	+0.024
4654.64	3b	+0.007	5027.14	2h	+0.014
4668.15	4	+0.013	5039.27	2b	+0.026
4707.29	5	+0.012 I.S.	5044.22		+0.018
4709.09	2	+0.011	5048.45	2	+0.021
4727.42	2	+0.024	5068.78	4	+0.030
4736.79	5	+0.012 I.S.	5073.67		+0.035
4754.05	5	+0.010 Mn	5076.28		+0.022
4783.44	4	+0.012 Mn	5090.79	3h	+0.010
4823.53	4	+0.011 Mn	5099.05		+0.020
4859.76	5	+0.014 I.S.	5125.14	2b	+0.020
4871.33	8	+0.010	5126.21		+0.020
4872.15	8	+0.014	5136.06		+0.028
4878.22	6	+0.012 I.S.	5137.39	3	+0.014
4890.77	7	+0.014	5139.27	6	+0.021
4891.50	9	+0.012	5139.48	8	+0.020
4903.32	5	+0.009 I.S.	5162.32	5b	+0.030
4915.60	1	+0.018	5165.43	2b	+0.010
4917.20		+0.012	5191.47	7	+0.019
4919.01	8	+0.016 I.S.	5192.36	8	+0.019 I.S.
4920.52	10	+0.013	5208.61	4	+0.020
4922.38		+0.014	5215.20	4	+0.020
4923.94		+0.046	5217.41	4	+0.016
4930.33	1	+0.015	5226.88	5	+0.023
4932.21		+0.021	5229.52		+0.019
4933.64		+0.045	5229.86		+0.015
4934.02	1h	+0.005	5232.96	8	+0.025 I.S.
4938.18	1	+0.018	5236.19	1	+0.020

TABLE VII—*Continued*

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
5263.32	5	+0.020 A	5655.51	2	+0.016 A
5266.04		+0.012	5658.84	4	+0.023 I.S.
5266.57	8	+0.017 I.S.	5662.53	3	+0.018
5273.18	3	+0.014	5701.47	4	+0.010
5281.80	5	+0.019	5705.48	1	+0.031
5283.64	7	+0.018	5709.40	3	+0.024
5302.32	5	+0.014 I.S.	5711.87	2	+0.010
5324.19	6	+0.015 I.S.	5712.15	2	+0.017
5339.94	3	+0.019	5715.11	1	+0.007
5353.38	2	+0.014	5717.85	3	+0.018
5389.46	2	+0.015	5731.78	3	+0.010
5391.49	1	+0.028	5753.14	3	+0.028
5393.19	4	+0.017	5763.02	4	+0.028
5466.42	3	+0.020	5775.10	3	+0.025
5472.72	1	+0.022	5782.15	1	+0.020
5473.91	3	+0.014	5791.04	2	+0.020
5476.58	4	+0.019	5809.25	2	+0.041
5480.87	2	+0.011	5859.61	3b	+0.025
5487.78	3	+0.036	5883.84	3	+0.024
5522.46	2	+0.013	5905.68	2	+0.007
5525.55	2	+0.010	5934.68	4	+0.028
5543.94	2	+0.018	5952.75	4	+0.021
5560.23	1	+0.025	5976.80	2	+0.029
5563.61	3	+0.021	5983.71	2h	+0.018
5567.40	2	+0.016	6003.04	3	+0.020
5569.63	5	+0.020 I.S.	6008.58	3	+0.024
5572.86	5	+0.025	6013.52	2	+0.019 Mn
5576.10	4	+0.021	6016.66	2	+0.020 Mn
5586.77	6	+0.024 I.S.	6021.82	2h	+0.018 Mn
5600.24	1	+0.018	6141.13		+0.016
5602.79	2	+0.016	6180.22	2	+0.011
5602.96	3	+0.019	6232.67	2	+0.017
5608.16		+0.021	6246.34	4	+0.020
5615.66	6	+0.022 I.S.	6301.52	5	+0.016
5618.65	1	+0.010	6302.51	3	+0.015
5624.56	5	+0.023	6336.84	4	+0.020
5633.97	2	+0.023	6400.02	5	+0.019
5638.28	3	+0.015	6408.04	4	+0.017
5641.46	2	+0.025	6411.67	5	+0.019
5655.18		+0.014	6419.99	5	+0.024

region λ 4900–λ 5050, for example, there are 34 affected lines. Burns gives for this region 24 Fe lines of intensity 2 or stronger; of these 20 are sensitive to the pole effect. From λ 5500 to λ 6000 they are practically the only lines in the iron spectrum. The lines giving displacements to the violet at the negative pole have a stronger gregarious tendency showing in the ultra-violet and to the red of λ 5364. When the ultimate groups of the iron lines are once determined from their reactions to various physical conditions, it may be possible to find definite series relationships even in so complex a system as the iron spectrum.

TABLE VIII

Fe Lines, Group *e*

Negative Pole *minus* Center of Arc

Displacements to Shorter Wave-Length

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
3157.88	4	−0.004 A	5074.75	2b	−0.022 A
3160.65	6	−0.004	5079.00		−0.013
3205.40	7b	−0.004	5096.99	3b	−0.016
3210.46	2	−0.006	5133.67	5b	−0.045
3244.19	1	−0.004	5153.20		−0.035
3251.24	5h	−0.004	5364.86	3h	−0.028
3516.41	3	−0.008	5367.46	3b	−0.025
3518.86		−0.006	5369.96	4b	−0.020
3529.82	4	−0.004	5383.37	5b	−0.025
3532.10		−0.004	5400.50	2b	−0.011
3533.00	4	−0.004	5404.13	3b	−0.025
3549.87	3	−0.010	5410.90	3b	−0.026
3582.58		−0.013	5415.19	4b	−0.030
3588.52		−0.006	5424.06	4b	−0.027
3594.63	5	−0.004	5432.96		−0.023
3604.28		−0.008	5445.04	2h	−0.020
3610.15	5h	−0.007	5462.96	2b	−0.014
3616.58	4h	−0.014	5463.27	4b	−0.016
3633.84	4h	−0.006	5543.18	2	−0.014
3634.68		−0.006	5554.88	3b	−0.019
3650.03	3h	−0.007	5565.78	3	−0.019
3682.21	1	−0.003	5594.66	2	−0.023
3689.90	1b	−0.011	5598.31	3	−0.025
3694.00	6	−0.014	5686.53	3	−0.028
3701.08	6	−0.009	5693.64	2	−0.007
3726.92	3h	−0.006	5705.99	2	−0.019
3744.09	2h	−0.011	5816.36	3	−0.026
3748.96	3h	−0.014	5862.35	4b	−0.018
3754.50	2b	−0.008	5914.16	6	−0.003
3773.69	2b	−0.012	5930.18	5	−0.020
3797.95	1	−0.012	5984.81	3	−0.022
3845.26	5	−0.007	5987.06	2	−0.016
3966.62	5b	−0.004	6007.96	2h	−0.016
4172.64		−0.017	6020.18	2l	−0.012
4200.92	2	−0.007	6024.06	4h	−0.015
4224.51	2b	−0.006	6042.08	2	−0.012
4433.22	2b	−0.006	6055.99	3h	−0.015
4960.93		−0.035	6078.48	3	−0.022
4967.89	2	−0.016	6102.18	3	−0.010
5065.02	3b	−0.029	6103.19	2h	−0.015

TABLE IX

Distribution of Affected Lines

Displaced to Red			Displaced to Violet		
Region	Total	Per 100 A	Region	Total	Per 100 A
3000–3400...	10	2	3100–3300...	6	3
3400–3900...	47	9	3300–3500...	0	0
3900–4300...	54	14	3500–3800...	25	8
4300–4600...	9	3	3800–4500...	6	1
4600–4900...	20	7	4500–4900...	0	0
4900–5300...	72	18	4900–5200...	8	3
5300–5800...	50	10	5200–5300...	0	0
5800–6100...	13	4	5300–5600...	18	6
6100–6420...	11	3	5600–6200...	17	3
	286			80	

VII. WORKING CONDITIONS IN THE IRON ARC

Aside from the theoretical interest in the changes of wave-length considered in this paper, reference may be made to the following practical considerations:

1. A number of these sensitive lines are included among the international standards of the second order adopted by the International Union for Co-operation in Solar Research.

2. There are regions of the iron spectrum in which few or no other lines are available for standards; for example, from λ 4900 to λ 5050 and from λ 5500 to λ 6000.

3. In various laboratories there are in progress redeterminations, based upon the iron standards, of the wave-lengths in international units of the lines of many elements. In these redeterminations the instrument most commonly used is the concave grating in the usual Rowland mounting, and in practice the slit of the spectrograph is parallel to the axis of the arc and includes the major part of its length. The astigmatism under these conditions introduces more or less pole effect, and to that degree vitiates results involving lines of the character under consideration. The practice of reversing the current in the arc in order to overcome the tendency to produce wedge-shaped lines when the slit and the axis of the arc are parallel, obscures, but does not eliminate, the pole effect.

Since the redeterminations aim at a precision of 0.002 to 0.003 A, it is necessary to take the pole effect into consideration.

4. Lines of the type considered are not limited to iron, but are present in the spectra of other elements, the detailed investigation of which is necessary before safe deductions can be made from their use in astrophysical investigations, or before their wave-lengths can be determined with the requisite precision.

5. The arc lines are often used as a basis for intensity comparisons, and for such purposes reliable results depend upon employing suitable arc arrangements.

TABLE X

Displacements, Pole Distance, and Current

	Neg. Pole and Center	1 mm from Neg. Pole and Center	Pos. Pole and Center	12–4 Amperes	7–5 Amperes
Group *a*.....	−0.0005 A	+0.0005 A	−0.0002 A	−0.0004 A	−0.0002 A
Group *d*.....	+0.021	+0.009	+0.003	+0.007	+0.001
Group *e*.....	−0.025	−0.014	−0.006	−0.012	−0.003

It is of importance then to determine the practical conditions under which these sensitive lines may be used and the limits of the precision obtainable. In furtherance of such a purpose we have made comparisons between the center of the arc and the positive pole, the negative pole, and a point 1 mm from the negative pole, using the Pfund arc 6 mm long, carrying a current of 6 amperes under a pressure of 110 volts; and also between the centers of arcs carrying 5 and 7 amperes, and between arcs carrying 4 and 12 amperes. The data are shown in Table X. For the *d* lines the current may vary between 5 and 7 amperes without introducing errors exceeding the desired precision, and it appears that the small changes in current obtaining in practice, when care is taken to hold it constant in the standard arc, are without measurable effect. A more insidious source of error is the introduction of the pole effect by using light from any part of the arc except the middle zone, as nearness to the negative pole is accompanied by easily measurable displacements. It is important to hold closely to the mid-point of the arc, and advisable to approach the positive rather than the negative pole if any considerable length of the arc is to be employed. In our experience the highest precision and the most uniform results are obtained by keeping the slit normal to the axis at its mid-point in a greatly enlarged image of the arc. The necessary conditions for high precision are difficult to realize in the classical mounting of the concave grating without some arrangement for rotating the image of the arc. The ultra-violet plates for this investigation were taken with a concave grating in a Rowland mounting arranged in a vertical plane and with the slit normal to the axis of the arc.

The effects arising from differences in arc conditions and types of spectrographs employed are, as has been mentioned, manifest in the determinations of the tertiary standards by different observers. They are also apparent in other lines of work. We wished to compare our results for the pressure displacement of these lines with those taken under the widest range of pressure, and turned to Duffield's interesting paper.[1] He worked with pressure differences of 3 to 100 atmospheres, and used the ordinary mounting of the concave grating with the slit parallel to the axis of the arc, the slit and the image of the arc being of the same height. His procedure involved the pole effect in somewhat varying degrees, as he changed both the length of the arc and the current-strength, and at high pressures the exposures were made by a series of flashes, a process that intensifies the effect of the polar influence. From his Tables I and II the lines showing pole effects are selected, and the pressure-shifts per atmosphere, deduced from the pressure differences used, are given in Table XI. Unreversed lines only are considered and the means of his two sets are taken. In the last column are shown the displacements for one atmosphere found by us for some of the same lines of group *d* in passing from a vacuum to atmospheric pressure. Duffield's results vary from 0.033 A per atmosphere, determined from the pressure difference of 3 atmospheres, to 0.006 A, deduced from the pressure difference of 80 to 100 atmospheres. These discrepancies are explicable as pole effect. At low pressures a larger proportion of its influence would appear as an increment to the pressure-shift, while with increasing pressures the pole effect would play a decreasing rôle. For the lines of group *a* no pole effect is shown by our measurements. There are in Duffield's tables two lines of this group. The pressure-shifts per atmosphere deduced from pressure differences of 10 and 80 atmospheres are 0.0022 A and 0.0015 A respectively, an agreement in striking contrast to the results for lines showing pole effect.

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[1] *Phil. Trans.*, A **208**, 111, 1908.

TABLE XI

PRESSURE-SHIFT PER ATMOSPHERE—DUFFIELD

λ	ATMOSPHERES						
	3	5	10	80	95	100	St. John and Babcock
4299	....	0.0128	....	0.0067	0.0060	0.0060	....
4236	....	0.0126	0.0147	0.0083	0.0082	0.0086	0.0046
4233	0.0330	0.0172	0.0163	0.0060	....	0.0064	0.0057
4227	0.0377	0.0240	0.0125	....	....	....	0.0090
4222	0.0287	0.0150	0.0151	....	....	....	....
4210	0.0300	0.0134	0.0090	0.0034	0.0040	0.0048	0.0019
4191	....	0.0162	0.0155	0.0057	0.00	0.0064	0.0027
4187	0.0407	....	....	....	....	....	0.0032
4187	0.0353	0.0152	0.0123	....	....	....	0.0038
Means	0.0326	0.0146	0.0135	0.0060	0.0061	0.0064	0.0044

The pole effect appears also in the interferometer determinations of λ atm.−λ *in vacuo* by Fabry and Buisson.[1] We have measured the values of λ atm.−λ *in vacuo* for the lines used by them, both with a grating spectrograph and with an interferometer. The respective results are given in the fourth and fifth columns of Table XII. The differences between the values of Fabry and Buisson and the means of our two determinations appear in the sixth column.

For the arc at atmospheric pressure Fabry and Buisson used a current of 3 amperes, while we used a current of 6 amperes. They say: "For the unsymmetrical lines the displacements would have been much greater if the intensity of the current had been stronger"; but the displacements observed by us are smaller than those found by them. A comparison of the differences in the sixth column with the pole effects in the seventh leaves no doubt that the large values obtained by them were mainly due to pole effect. As the maxima of the interference fringes correspond to the maxima of the emission lines, the measurements show actual displacements of the maxima under varying arc conditions, even when determined by the interferometer.

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[1] *Astrophysical Journal*, 31, 112, 1910.

TABLE XII

λ ATM. *minus* λ *in vacuo* FOR SENSITIVE LINES

Group	λ	Fabry and Buisson	St. John and Babcock		Fabry and Buisson *minus* St. John and Babcock	Pole Effect
			Grating	Interferometer		
d..	4187.05	+0.011	+0.004	+0.004	+0.007	+0.008
	4191.44	+0.010	+0.003	+0.005	+0.006	+0.010
	4227.45	+0.020	+0.006	+0.009	+0.012	+0.014
	4233.61	+0.012	+0.006	+0.006	+0.006	+0.010
	4235.95	+0.011	+0.006	+0.005	+0.005	+0.009
	4250.13	+0.013	+0.005	+0.007	+0.007	+0.010
	4859.76	+0.017	+0.008	+0.005	+0.011	+0.014
	4871.33	+0.010	+0.013	+0.008	0.000	+0.010
e..	5415.19	−0.015	+0.001	....	−0.016	−0.025
	5424.06	−0.017	+0.001	0.000	−0.017	−0.026

There is also evidence that in the furnace spectra studied by King[1] the positions of these sensitive lines are affected by the phenomenon under consideration; for when their pressure displacements are compared with those of the stable lines in the same spectral region the furnace displacements exceed by 0.007 A those of the stable lines given under the same conditions, while those obtained from our arc determinations exceed the mean given by the stable lines under like conditions by 0.0026 A. Moreover, as shown in Table XIII, a striking agreement appears between the differences, furnace minus arc, and the pole effects given in the fourth and fifth columns respectively.

So much emphasis has been placed upon the differences between various determinations of pressure-shift that it may be well to call attention to the fact that the lines showing marked discrepancies are those catalogued in our Tables VII and VIII, and that among the iron lines examined by us some 1200 are free from pole effect and will normally yield definite values for pressure-shift. For example, in Table XIV we compare our interferometer determinations with the pressure-shift per atmosphere deduced from the displacements for 8 atmospheres found by Gale and Adams for lines of group *a*. Those lines of group *b* which we have measured in this way also show differences smaller than the errors of observation.

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[1] *Mt. Wilson Contr.*, No. 53; *Astrophysical Journal*, **34**, 37, 1911.

TABLE XIII

DISPLACEMENTS PER ATMOSPHERE IN FURNACE AND ARC
FE LINES, GROUP *d*

λ	Furnace	Arc	F−A	Pole Effect
4187.05........	+0.012	+0.0038	+0.008	+0.008 A
4187.81........	+0.013	+0.0033	+0.010	+0.010
4191.44........	+0.013	+0.0027	+0.010	+0.010
4198.31........	+0.014	+0.0113	+0.002	+0.010
4210.36........	+0.013	+0.0019	+0.011	+0.006
Mean..........	+0.013	+0.0046	+0.008	+0.009 A

The agreement for lines of this type, shown in Table XIV, is in strong contrast to the consistent difference for lines of group *d*. For the lines given in Table XII, the mean pressure-shift per atmosphere deduced from the displacement for 8 atmospheres found by Gale and Adams is +0.011 A, while, from our measurements based upon the lines as produced in the central zones of 6 mm, 6 ampere arcs, its mean value is +0.006 A, the difference being referable to the pole effect introduced by the short arcs used by them.

TABLE XIV

DEFINITE PRESSURE-SHIFT FOR STABLE LINES
GROUP *a*

	Gale and Adams	St. John and Babcock
4376................	+0.0022	+0.0020
5371................	+0.0036	+0.0031
5397................	+0.0036	+0.0029
5406................	+0.0034	+0.0045
5429................	+0.0036	+0.0033
5447................	+0.0039	+0.0030
5455................	+0.0036	+0.0035
	+0.0034	+0.0032

A remarkable agreement appears between the pressure displacements found for this group of sensitive lines by Fabry and Buisson (arc and interferometer), Gale and Adams (arc and grating), and King (furnace and grating), namely, 0.013, 0.011, and 0.013 A, respectively, while our measurements give only 0.006 A. It seems probable that some common factor was effective in producing these larger values, and a comparison of the excesses with the pole effect points to it as the operating cause.

It is a matter to be considered, whether the differences in wave-length between the center of the arc *in vacuo* and in air, as we have used it, are wholly due to pressure, or are still measurably influenced by pole effect. One way of approaching the question is to compare our results with those given by Duffield's data. As our value for the pressure-shift is 0.0044 A, and that deduced from the pressure differences of 100 atmospheres used by him is 0.006 A, it appears that under the conditions of our arrangement we are approaching closely, if we have not actually reached, the true pressure-shifts for lines subject to pole effect. This is further indicated by their varying as the cube of the wave-length, a relation shown by Gale and Adams to hold for the iron lines of groups *a* and *b*, which are free from pole effect. If the displacements found by us are complicated by the presence of another effect, not following the cube law, such a close agreement between observed and calculated values would be a remarkable coincidence. Another reason for thinking the 6-ampere arc, 6–7 mm in length, is practically free from the pole effect in the central plane is found in the way the displacements vary with current. Combining with ours some of Royds's[1] data for the differences between the centers of arcs carrying different currents, we find for the following current-changes the corresponding increases in wave-length:

5	to	7	amperes	+0.001 A
4.5	"	9.5	"	+0.003
4	"	12	"	+0.007

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[1] During the progress of this investigation Dr. Royds's interesting paper appeared (*Bulletin* No. 40, Kodaikanal Observatory). The two investigations have proceeded along some common lines, and where the same ground is covered by the observations they are mutually confirmatory.

These indicate approximately displacements of 0.001, 0.002, and 0.004 A for 2-ampere increments to currents of 5, 7, and 10 amperes, respectively, and that decrease of current below 6 amperes is not necessary for the elimination of pole effect in the iron arc employed.

VIII. DISCUSSION

Though our observations show that displacements of the maximum intensity of certain types of lines occur between the center and pole of the iron arc, the cause of the displacements is not evident, nor is the mechanism plain that produces the unsymmetrical broadening which characterizes lines of these types, and with which the displacements are more closely related than with the pressure-shifts. Dr. Goos[1] attributed such displacements to differences of pressure in the arc. We do not find pressure differences of the order necessary to produce them. It may be said that probably all data relative to the pressure displacements of these sensitive lines are more or less affected by pole effect, and cannot serve for the accurate determination of pressure differences. Dr. Royds[2] considers density the predominating influence in producing the displacements. The fact that a tenfold change in vapor density in the furnace is without effect upon the position of the maximum is opposed to the density hypothesis, but the conditions obtaining in the furnace and in the arc are not strictly comparable. Exner and Haschek[3] suggest variability in the intensity of the components of a complex line under varying excitation as an explanation of displacement of the maximum. This would mean that in 18 per cent of the lines examined by us a close-lying satellite to the red of the principal component given by the central zone of the arc increased in relative intensity on approaching the negative pole, and that in 5 per cent of the lines the component of increased relative intensity was to the violet. More observational data than are now available are necessary to determine the probability of such a behavior of complex lines. The possibility of the occurrence of such variability is evident, but the observations of Nutting do not indicate it in the case of iron.[1] Neither of these points of view seems to us to offer a satisfactory explanation of the phenomenon, nor until definite results are obtained from investigations now in progress and to be undertaken do we wish to offer any further suggestion, as we are inclined to sympathize with Nutting when he says: "In conclusion, I wish to enter a plea for a simpler and broader basis for spectroscopy and a basis as free as possible from either assumption or speculation."[2]

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[1] *Loc. cit.*

[2] *Loc. cit.*

[3] *Sitzungsberichte Wiener Akad.*, **116**, Abt. II*a*, 323, 1907.

Though we have not satisfied ourselves as to the explanation of the main phenomenon considered in this investigation, we feel that the data should be accessible to other investigators, particularly in view of the employment of the iron arc as a standard in the redeterminations in progress, and as a basis of sun and arc comparisons in the discussions bearing upon pressure, motion, anomalous dispersion, and Einstein effect in the solar atmosphere, as it is evident that the iron lines given in Tables VII and VIII, and lines of other elements behaving in a similar way, must be given separate consideration, and conclusions based upon them accepted with great caution.

We have suggested the term "pole effect," not only as a convenient designation of the phenomenon, but also as an indication of its dependence upon nearness to the pole. The effects due to increase of current appear to us a projection of the polar influence to a greater distance from the pole, while a shortening of the arc simply brings the central zone nearer to the pole.

We take this opportunity to express our appreciation of the assistance given us by various members of the staff; we are under particular obligation to Miss Ware for her able and unwearying help in the difficult measurements.

IX. SUMMARY

1. A combination of totally reflecting prisms and a rotating sector furnishes a means of making rigorously simultaneous exposures upon different sources.

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[1] *Astrophysical Journal*, **22**, 7, 1906.

[2] *Astrophysical Journal*, **28**, 70, 1908.

2. Displacements of the maxima of certain unsymmetrical iron lines in passing from the center to the negative pole of the arc are shown by the persistence of the displacements when the widths of the lines at the pole are less than at the center of the arc, by the shift of the intensity maxima of the photometric curves, and by the relative position of the maxima at pole and center with respect to the superimposed iodine absorption lines.

3. Observations upon symmetrical lines with large pressure-shifts do not show a general increase of pressure in passing from the center to the negative pole sufficient to produce the observed displacements.

4. The wave-lengths of these sensitive lines are not affected by a tenfold change in the density of the iron vapor in the furnace.

5. Their wave-lengths are independent of a change in furnace temperature over the range of our observation, 2100°–2600° C.

6. Except in a very few special cases, the pole effect disappears *in vacuo* and in so far appears independent of electrical conditions.

7. It does not appear to be intimately related to the differences in luminosity between the positive and negative poles.

8. The variation of pole effect with wave-length does not follow the same law as pressure displacements.

9. Therefore an increase of pressure localized at the pole and in the core of the arc, where a greater proportional contribution is made to the total intensity of these lines than to any other groups, does not alone explain the displacements.

10. Between λ 2979 and λ 6678, of 1570 lines examined 286 show displacements to the red and 80 to the violet.

11. The affected lines are not distributed with any degree of uniformity, but show rather a tendency to cluster in certain regions.

12. In extensive regions of the spectrum, λ 4900–λ 5050, λ 5500–λ 6000, nearly all the lines are of this character.

13. An investigation of the limits of the working conditions in the 6-ampere, 110-volt, 6 mm iron arc of the Pfund form shows that the small fluctuations in current occurring in practice may be neglected, but that even when the arc is running steadily only a narrow equatorial zone is practically undisturbed by the pole effect.

14. Our study of the behavior of iron lines under different conditions furnishes additional ground for regarding the classification suggested by Gale and Adams as resting upon a real physical basis.

15. Emphasis is placed upon the necessity of considering the pole effect when the arc is used in comparisons of intensity, in redetermination of wave-lengths, and in astrophysical investigations.

Mount Wilson Solar Observatory
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NOTE ON THE DENSITIES OF SECOND-TYPE STARS

By HARLOW SHAPLEY

During the last few years our knowledge of stellar densities has been considerably increased through the acquisition of specific values for the mean density of certain classes of double stars; and, at the same time, there has been a growing need in studies of stellar development for more definite information regarding this as well as other physical properties of stellar bodies. From the study of eclipsing variables it is found that the average density of the first-type stars is one-tenth or two-tenths that of the sun. Moreover, the range of values is apparently limited, for from a total of some fifty carefully investigated binaries, no star of spectral type B or A is known to have less than one-hundredth the solar density. Among the second-type systems a number are known to be denser than the white stars; but, on the other hand, there are also solar-type stars of remarkably low density—so rare, in fact, that with solar mass their volumes must be hundreds of times that of the sun. This result has been stated in various recent articles, but only in a more or less summary fashion; and, as the existence of such low densities and the bearing they may have on current astronomical problems is not generally admitted, it is the object of the present communication to give the data upon which the conclusion depends in a manner sufficiently detailed to permit an easy inspection and consideration of its validity.

The point is of some importance in determining the order of stellar evolution. Admitting that stars in growing older contract and become denser, the stars of least density must obviously be the youngest; and, if we suppose that the order of evolution is uniformly that represented by the spectral sequence B, A, F, G, K, and M, we must expect the densities of second-type stars always to be greater than those of the first. In other words, second-type stars of low density find no place in this scheme of stellar evolution except by the assumption that these particular bodies, in spite of gravitation, expand as time goes on. There may be some other explanation of such small values—we might hypothecate the existence of important counter-gravitative forces, or deny the whole of the eclipse theory, or, regardless of the evidence, propose that these are exceptional bodies which do not develop in the normal manner or are preceded by as yet undiscovered whiter stars of still greater volume, rarity, and luminosity.[1] But otherwise, if the existence of the abnormal densities is proved, it would seem that the conventional order of the evolutionary stages must be modified.

The existence of many second-type stars of great intrinsic brightness is not questioned—for instance, Capella, the Cepheid variables, and the bright yellow stars in the Hyades group. With any reasonable assumption as to the surface brightness of these stars, the inevitable conclusion is either that they have enormous masses, or, if the masses are within the limits found in double-star systems, that the mean densities are extremely low. In some cases the dimensions must be so large that there can be no middle-ground adjustment that will keep both mass and density within the limits generally admitted in stellar studies. To account for the great size of the second-type stars of high luminosity, the choice between large mass or small density is generally made in favor of the former in order to maintain a late epoch in stellar development for these objects. But this implies for these stars, when earlier in their history they were of spectral type A or B, still greater dimensions than they now possess; and this circumstance, coupled with the much greater intrinsic light-emitting power of the whiter stars, would demand the presence of giant white forerunners of a magnitude, both absolute and apparent, not at present to be found.

This possible, though apparently improbable, interpretation of the great volume of isolated giant red and yellow stars contributes nothing to the question of whether the first-type stars are denser, in the mean and individually, than some of those of the second type; but, for valuable evidence, recourse may be had to the data of eclipsing binaries in which the mean density of each system can be determined independently of the mass.[1]

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[1] In this connection it is well to keep in mind the group of B-type spectroscopic binaries whose periods exceed 100 days. We have as yet no assurance that their densities may not be peculiarly low. None of them is known to be an eclipsing variable. Of the second-type spectroscopic binaries, however, one-third have periods longer than the longest of the B's, so that, if length of period is to be the criterion for density, here again the white stars have intermediate values (*Mt. Wilson Contr.*, No. 99; *Astrophysical Journal*, 41, 291, 1915).

The questions to be answered are, first: Do these extremely low densities certainly exist? and secondly: Are the stars considered certainly those with the redder spectra? Since the evidence so far presented is by no means to be considered overwhelming, nor perhaps even incontrovertible, a specific record of the data is desirable in order to indicate the present status of the argument.

The equations giving the density in terms of the orbital elements have been developed in various forms, and results for 20 stars, which seem certainly to be of the second type, have been published. Some of these densities are abnormally low, but the fact that their derivation has been involved with the rather complicated orbital theory of binary systems perhaps detracts something from the conviction that the results would otherwise carry. There is, however, a simple relation which gives the upper limit of the mean density of an eclipsing binary without hypothesis as to the depth of the minima or the relative size or brightness of the components—in fact, in a circular orbit or one of small eccentricity the limit involves only the period and the duration of one eclipse. As the results from this relation are of the same order as those previously found, the conclusion as to the existence of very low densities is freed from the orbital theory and given a very direct derivation.

Expressing the period, P, and the semi-duration of eclipse, t, in days, and the mean density, d_0, in terms of that of the sun, we have (as shown in the supplementary note)

$$d_0 \leqq \frac{0.054}{P^2 \sin^3 \frac{2\pi t}{P}} . \tag{1}$$

Table I contains data relative to five eclipsing systems of low mean density. The upper limits in the fifth column were derived by the foregoing relation. The remaining data are from *Contributions from the Princeton University Observatory*, No. 3, pp. 82 ff., 1915.

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[1] The assumption that isolated stars and binaries are physically comparable is necessary, of course, in applying to the general aggregate of stars the conclusions reached in the present case; but there seems to be no reason at present to doubt the validity of this assumption.

TABLE I

DENSITIES AND DIMENSIONS OF FIVE ECLIPSING SYSTEMS

STAR	SPECTRUM	PERIOD	SEMI-DURATION OF MINIMUM	UPPER LIMIT MEAN DENSITY	COMPUTED DENSITY		HYPOTHETICAL LONGEST RADIUS		
					Bright Star	Faint Star	Bright Star	Faint Star	Relative Orbit
SX Cass...	G3	36^{d}572	2^{d}5	0.0005	0.0004	0.0002	15.3	18.6	59.
RX Cass...	Ko	32.316	2.5	0.0005	0.0005	0.0004	14.8	14.8	53.
RZ Oph...	F8	261.9	8.0	0.00012	0.001	0.00003	10.1	33.5	217.
RT Lac....	G5	5.074	0.4	0.02	0.013	0.010	4.6	4.6	15.6
W Crucis..	Gp	198.5	22.0	0.00005	0.000002	0.000025	94.	36.	180.

Only for W Crucis are the observations adequate at present to give a curve and orbit of the first grade; the periods, however, are accurately known in all cases. To indicate how closely the duration of the eclipse can be ascertained, as well as to show how closely the variation of these stars resembles that of typical eclipsing binaries of shorter period, the computed light-curves are given in Figs. 1, 2, 3, and 4. That for W Crucis has already been published in this *Journal*;[1] the peculiarities of its spectrum and the small distance separating the components may entitle it to diminished weight in the present discussion.

The sources of the observations for the other four systems are as follows:

SX Cassiopeiae.—The measures are visual estimates by Luizet; the open circles represent points of low weight. Normal magnitudes are printed in *Contributions from the Princeton University Observatory*, No. 3, p. 132 (1915), and the orbit, *ibid.*, p. 86. The curve is computed from the uniform orbital elements.

RX Cassiopeiae.—Observations are by Wendell using a Harvard polarizing photometer, each point representing the measures of a single night. The curve is computed from the uniform elements (*ibid.*, pp. 86, 135).

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[1] **36,** 148, 1912.

RZ Ophiuchi.—Only one-half of the curve is shown. The x's are normal points from observations made at the Laws Observatory by Seares and Haynes; the dots define co-ordinates of the observed curve by Nijland. The curve is computed from the darkened elements (*ibid.*, pp. 86, 157); the orbit is discussed in *Astronomische Nachrichten*, **194**, 225, 1913; see also *Laws Observatory Bulletin*, No. 16, 1908.

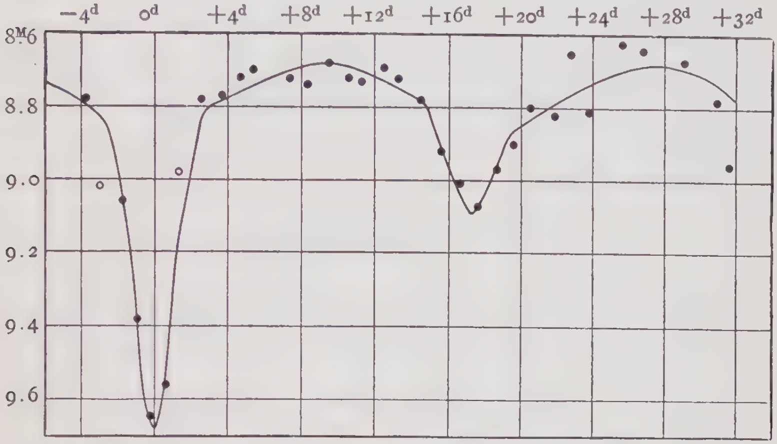

FIG. 1.—The light-curve of SX Cassiopeiae

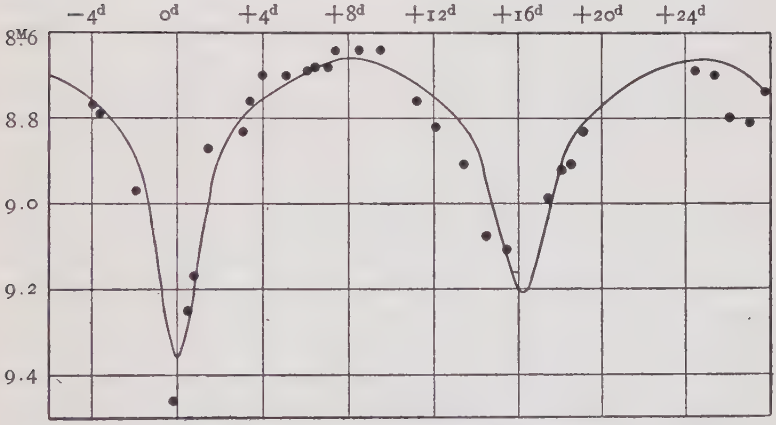

FIG. 2.—The light-curve of RX Cassiopeiae

RT Lacertae.—The *x*'s represent normal magnitudes by Enebo; the dots, normals by Luizet (black squares have quadruple weight). The curve is computed from the darkened elements, the uniform solution being impossible (*Contributions from the Princeton University Observatory*, No. 3, pp. 17, 26, *et passim*).

One other variable similar to these—ϵ Aurigae—might be added to the list, but its marked peculiarities of spectrum make the interpretation of its light-curve uncertain.[1] Similarly β Lyrae must be considered anomalous.[2]

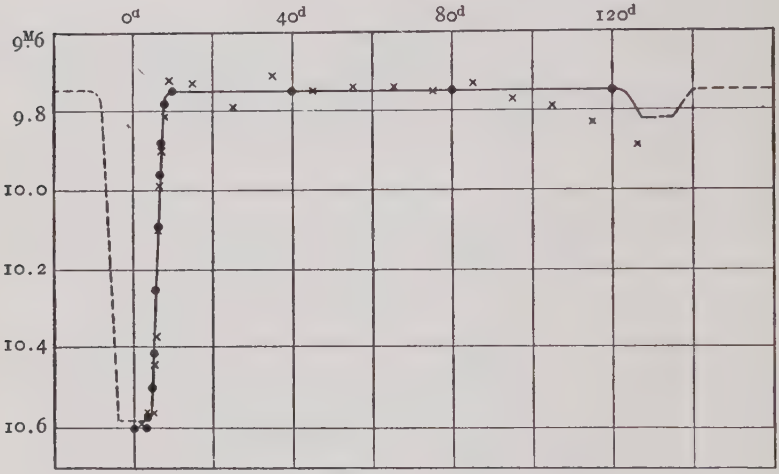

FIG. 3.—The light-curve of RZ Ophiuchi

The faintness of the stars in the table has made the classification of the spectra difficult.[3] To make certain, however, that they are definitely of the second type, three independent and accordant classifications of their spectra were kindly made at the writer's request at Harvard by Miss Cannon.

The tabulated semi-duration of the eclipse, which is used to compute the upper limit of the mean density, was read directly from the plot of the observations. For all but RZ Ophiuchi[1] the position and shape of the observed secondary minimum is sufficient to show that the orbital eccentricity is so small that the formula for the limiting density is valid. The depths and shapes of the secondary minima also show that the components must be of approximately the same dimensions, so that the mean density of the system also indicates the mean density of each component. This is not the case, however, for RZ Ophiuchi, where the steepness of the curve at primary eclipse shows that the component whose spectrum is classified as F8 is the smaller and brighter one for which the actual density, as computed from the orbital elements, is ten times the mean density; the second component is doubtless of a redder spectral type.

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[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 20, 84, 94, 1915.

[2] *Ibid.*, pp. 71 ff.

[3] For the spectrum of W Crucis see *Astrophysical Journal*, **36**, 153, 1912.

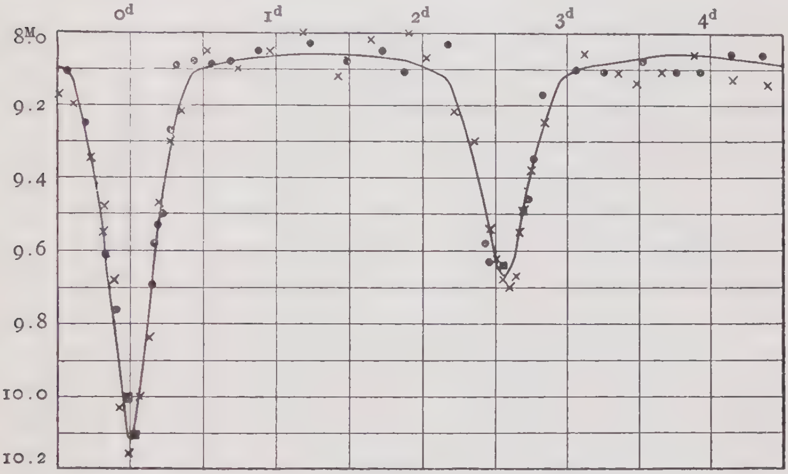

FIG. 4.—The light-curve of RT Lacertae

The computed densities in the fifth and sixth columns were derived from a discussion of the orbital elements and have been adjusted for polar flattening and probable irregular division of masses between the components in each system. They are close to the true values of the mean densities of these individual stars, but of course do not pretend to represent accurately the average density of the giant second-type stars as a whole. For a first-type star the lowest recorded density is 0.012, computed from a provisional curve for RZ Scuti, type B3; it is likely that further observation will tend to increase rather than diminish this value. The next lowest value is 0.017 for UZ Cygni, type A.[1]

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[1] The deviation of the observation near the secondary minimum suggests a deeper secondary eclipse than computed, but one of the comparison stars used for this part of the curve at the Laws Observatory is suspected of slight variation. Graff's measures of the maximum light show no trace of a conspicuous secondary minimum (*Astronomische Nachrichten*, **176**, 79, 1907). A deeper secondary minimum would give a lower mean density for the brighter component. Data are now available for a revision of the orbit, which will be undertaken soon.

Fig. 5 gives a diagrammatic representation of four of the foregoing low-density systems, the major axes of the several stars and orbits being taken from the last three columns of Table I. For convenience the linear dimensions of SX Cassiopeiae and RX Cassiopeiae have been divided by two, those of RZ Ophiuchi by four, and those of W Crucis by eight. (The tabulated dimensions for RT Lacertae show that it is intermediate between the giant systems and the sun.) The diagram shows not only that these binaries are typical in their relative dimensions, but also that the components are distinctly separated. If the components were in contact, as is probably the case with β Lyrae and RR Centauri,[2] we might expect the existence of a gaseous envelope that would give rise to spectral peculiarities; or, of more importance to the present problem, we might question the meaning of the computed densities, as Jeans[3] has done, because of possible interactions through a connecting neck of gas. But in these giant systems we find the stars as definitely and distantly separated as the average eclipsing and spectroscopic binary. The sun is drawn to scale (with the linear dimensions of the binaries reduced as mentioned above) on the assumption that each component of all the systems has solar mass. The masses are more likely to exceed the sun's than to be less. If each component is 8 times as massive as assumed, the sun should be drawn one-half as large; if but one-eighth the solar mass, its diameter should be doubled. The true dimensions of the stars relative to the sun are probably well within these limits. For the purpose of comparison, the two eclipsing systems of highest known density, which are also of the second spectral type, are

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[1] Stebbins has just announced that the mean density of δ Orionis (type B) is 0.006 (*Science*, N.S., **41**, 811, June 4, 1915; see also *Astrophysical Journal*, **42**, 144, 1915).

[2] *Monthly Notices*, **63**, 537, 548, 1903. Roberts finds the components to be actually overlapping in their lines of centers. The computed density, however, is entirely normal for a dwarf F-type system.

[3] *Astrophysical Journal*, **22**, 93, 1905.

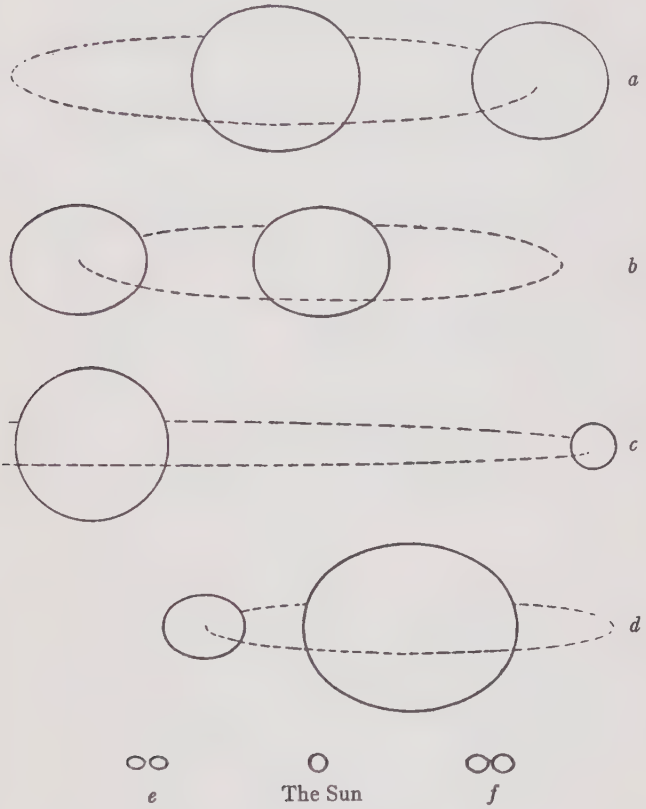

FIG. 5.—Eclipsing binaries of low and high density. The mass of each component is assumed equal to that of the sun.

a) SX Cassiopeiae; period $36\overset{d}{.}572$; spectrum G3. Reduced to one-half.
b) RX Cassiopeiae; period $32\overset{d}{.}316$; spectrum K0. Reduced to one-half.
c) RZ Ophiuchi; period $261\overset{d}{.}9$; spectrum F8. Reduced to one-fourth.
d) W Crucis; period $198\overset{d}{.}5$; spectrum Gp. Reduced to one-eighth.
e) W Ursae Majoris; period $0\overset{d}{.}334$; spectrum G.
f) U Pegasi; period $0\overset{d}{.}375$; spectrum F?

included in the diagram, thus affording a fair illustration of giant and dwarf stars.

SUPPLEMENTARY NOTE ON THE DERIVATION OF EQUATION (1) AND SIMILAR RELATIONS

The equivalent of equation (1) has been given by Russell;[1] expressions for the lower limit of the mean density of a binary, involving only the period, have been given by Stebbins[2] and others. On the basis of what is now known of the elements of eclipsing star orbits the range can be much narrowed, and it may be of value to derive both of the limits from the beginning. The resulting simple relations will then give with considerable accuracy the mean density of an eclipsing variable from the two most easily observed quantities, namely, the length of the period and the duration of minimum.

Selecting the appropriate units of time, length, and density, we derive readily from the equation for elliptic motion the following expression for the mean density of a binary system:

$$d_0 = \frac{0.0134}{P^2(r_1^3+r_2^3)} = \frac{0.0134}{P^2(r_1+r_2)^3 X}\,, \tag{2}$$

in which

$$X = \frac{r_1^3+r_2^3}{(r_1+r_2)^3}\,,$$

and P, r_1, r_2 are the revolution period and the radii of the component stars, relative to the distance between their centers. The ratio X has its maximum value, unity, when one of the radii is zero; its minimum, $\frac{1}{4}$, corresponds to $r_1 = r_2$. Hence,

$$\left.\begin{aligned} &\text{Upper limit of } d_0 = \frac{0.0537}{P^2(r_1+r_2)^3} \qquad \text{(stars equal)} \\ &\text{Lower limit of } d_0 = \frac{0.0134}{P^2(r_1+r_2)^3} \qquad \text{(one star a particle)} \end{aligned}\right\} \tag{3}$$

The stellar radii are determined only by a solution of the orbit, but in the above limiting expressions we may substitute quantities that are determined directly from the light-curve.

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[1] *Astrophysical Journal*, 10, 316, 1899.

[2] *Ibid.*, 34, 105, 1911.

For a circular orbit[1]

$$(r_1+r_2)^2=\cos^2 i \cos^2 \theta'+\sin^2 \theta', \tag{4}$$

where i measures the inclination of the orbit to the plane perpendicular to the line of sight and $\theta'=2\pi t/P$, t as before denoting the semi-duration of eclipse. For any given value of θ'

$$(r_1+r_2)^2=1 \qquad \text{(maximum) for } i=\ 0^\circ$$
$$(r_1+r_2)^2=\sin^2 \theta' \qquad \text{(minimum) for } i=90^\circ$$

Hence,

$$\left.\begin{aligned}
&\text{Maximum upper limit } d_0=\frac{0.0537}{P^2 \sin^3 \theta'} && (r_1=r_2\,,\ i=90^\circ)\\
&\text{Minimum upper limit } d_0=\frac{0.0537}{P^2} && (r_1=r_2\,,\ i=\ 0^\circ)\\
&\text{Maximum lower limit } d_0=\frac{0.0134}{P^2 \sin^3 \theta'} && (r_2=0\ ,\ i=90^\circ)\\
&\text{Minimum lower limit } d_0=\frac{0.0134}{P^2} && (r_2=0\ ,\ i=\ 0^\circ)
\end{aligned}\right\} \tag{5}$$

The first of equations (5) is equivalent to (1).

If the stars are just in contact, $4t=P$, so that $\theta'=\pi/2$ and $(r_1+r_2)^3=1$ for all values of i. The limits then become independent of the inclination and

$$\text{Upper limit } d_0=\frac{0.0537}{P^2} \qquad (r_1=r_2)$$

$$\text{Lower limit } d_0=\frac{0.0134}{P^2} \qquad (r_2=0\,)$$

With the aid of known values of the orbital inclinations we can, in practice, raise the lower limits of d_0 as given by the last two of equations (5). Substituting (4) in (2) we obtain

$$d_0 \geqq \frac{0.0134}{P^2\,(\cos^2 i \cos^2 \theta'+\sin^2 \theta')^{3/2}}\,, \tag{6}$$

which becomes an equality for one star a particle.

The average value of cos i for ninety eclipsing systems, all with ranges at primary eclipse in excess of 0.4 mag. (except β Aurigae, range 0.1 mag. and cos $i=0.23$), is 0.112. On the average then (6) becomes[1]

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[1] The necessary modifications of computations of density because of orbital eccentricity have been noted by Roberts (*Astrophysical Journal*, **10**, 311, 1899) and others. In most cases the changes would be negligible for the present work.

$$d_0 > \frac{0.0134}{P^2(0.013+0.987 \sin^2 \theta')^{3/2}}. \tag{7}$$

For three systems only does cos i exceed 0.4, and in but one exceptional case is the inclination less than 60°. A very safe limit, therefore, is cos $i \leqq 0.5$, and then we have finally

$$d_0 > \frac{0.108}{P^2(1+3 \sin^2 \theta')^{3/2}}. \tag{8}$$

In equations (1) and (8) we have pretty close limits for the mean density which depend not at all on orbital elements. If on the basis of the depths and character of primary and secondary eclipses we can assume equal components, then $X=\frac{1}{4}$ and the right-hand member of (8) may be multiplied by four; and, at any rate, since only in the exceptional case of $r_2 < 0.25 r_1$ is X greater than one-half, this limit of density may be safely doubled in 95 per cent of the eclipsing systems.

Mount Wilson Solar Observatory
May 1915

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[1] Similarly the average value of (1) may be derived, and is found to be just four times as large as (7).

Contributions from the Mount Wilson Solar Observatory, No. 108
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THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRA OF COBALT AND NICKEL

BY ARTHUR S. KING

The treatment in this paper of the electric furnace spectra of cobalt and nickel follows the method previously used for the spectra of iron,[1] titanium,[2] vanadium and chromium,[3] the lines being classified according to the temperature at which they first appear and their rate of increase in intensity as the temperature rises. The range of wave-length covered extends from below λ 3000 to about λ 7100.

APPARATUS AND METHODS

The operation of the tube resistance furnace *in vacuo* has been described in former papers. The spectrum was photographed with a 15-ft. concave grating in the vertical spectrograph.[4] The second order (scale 1 mm = 1.85 A) was used as far as λ 5200, and the first order from this point to λ 6700, a few lines at the red end being recorded on films taken with a 1-meter concave grating.

The three temperatures on which the classification of spectrum lines is based were given by a Wanner pyrometer as 2000–2100° C. for the low-, about 2300° C. for the medium-, and 2500–2600° C. for the high-temperature plates. A meager spectrum, consisting of the stronger low-temperature lines of both elements, mainly in the blue region, was obtained as low as 1850° C.

The metallic cobalt and nickel used in the furnace were highly purified preparations by Kahlbaum, each of them showing but a trace, spectroscopically, of the other element. As the furnace tubes were of regraphitized Acheson graphite, there was little disturbance from impurity lines, the main trouble from this source being the strong carbon bands given at the higher temperatures.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

[2] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

[3] *Mt. Wilson Contr.*, No. 94; *Astrophysical Journal*, **41**, 86, 1915.

[4] *Mt. Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 205, 1914.

EXPLANATION OF THE TABLES

Wave-lengths.—The wave-lengths in Tables I and II are those given by Exner and Haschek[1] for the arc spectrum, supplemented occasionally by those of Hasselberg,[2] designated by "H" usually in cases where close doublets were not resolved by Exner and Haschek.

An asterisk after the wave-length denotes that an explanatory remark for the given line is to be found at the end of the table.

The sign † indicates that the estimates of intensity for the line in the furnace spectrum are disturbed by the presence of a band spectrum, this being usually one of the heads of the "Swan spectrum" of carbon, though a banded structure farther in the red, whose origin has not been definitely fixed, interfered with a few lines.

Arc intensities.—These were estimated by the writer from spectra given by the purified cobalt or nickel in the carbon arc, the exposure being timed to produce as distinct intensity contrasts as possible. Nebulous lines, occurring for the most part in the nickel spectrum, are indicated by "n" after the intensity value. The letters "R" and "r," both for arc and for furnace lines, indicate complete and partial self-reversal, respectively.

Furnace intensities.—The columns devoted to the intensities of furnace lines give the relative strength as estimated for each temperature, a line distinctly outlined on the plate being given the intensity "1," a fainter appearance being indicated as a trace, "tr." There is usually a decided difference in appearance between lines in the furnace at different temperatures, and also between furnace lines and those of the arc, but the relative change of different lines with increase of temperature is shown in the tables.

Classification.—The method of assigning lines to the classes given in the last column of Tables I and II is the same as for the spectra previously treated. Class I lines are relatively strong at low temperature and strengthen slowly at higher temperatures. Class II lines appear at low temperature, but strengthen more rapidly than those of Class I as the tube becomes hotter. The lines of Class III are absent or faint at low temperature, appear at medium temperature, and are usually considerably stronger at high temperature. Class IV lines appear at the highest furnace temperature, sometimes faintly at medium temperature; while those of Class V are usually absent in the furnace, or if present are faint compared with the arc intensity. Arc lines below a certain minimum intensity are not entered in the tables unless they appear also in the furnace.

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[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Kgl. svenska vet. akad. handl.*, **28**, 1896; see also Kayser, *Handbuch der Spectroscopie*, **5**, 310; **6**, 172.

The use of "A" after the class number indicates that the line in question is relatively weak in the arc, being usually not more than half as strong as in the high-temperature furnace.

LEADING CHARACTERISTICS OF THE FURNACE CLASSES

Class I.—The lines of this class are of a well-defined and fairly uniform type. With few exceptions, the scale adopted gives them nearly the same intensity at the three furnace temperatures and in the arc, while the lines of other classes decrease with varying degrees of rapidity from high to low temperature. A large proportion of the Class I lines are of moderate strength and unreversed. These are very similar in appearance at all furnace temperatures. A considerable number, however, reverse at high and sometimes at medium temperature, while at low temperature the lines are sharp but still strong. A few lines of this class maintain their strength at low temperature to an unusual degree, these coming out strongly at the lowest temperature at which the vapor radiates. In general, however, the higher temperatures do not seem to be at a disadvantage in producing the low-temperature lines, the rule prevailing that a line strong at low temperature is strong at all temperatures; though in the case of Class I A lines it may be weak in the arc. The question arises whether, when the furnace is operated at high temperature, the Class I lines may be radiated chiefly by the cooler vapor which is doubtless present near the ends of the tube. This seems improbable since the ratio of exposure times for high and low temperature is of the order of 1:50, so that

TABLE I

Temperature Classification of Cobalt Lines

λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.	
2982.37....	1	1	1	.....	III
2987.28....	15r	15R	10r	5	II
2989.70....	15r	15R	10r	5	II
2996.67....	1	tr	.....	.....	IV
2999.84....	1	tr	.....	.....	IV
3000.66....	7	5r	3	2	II
3005.86....	3	2	1	.....	III
3013.70....	8	10r	6r	2	II
3015.77....	3	3	2	.....	III
3017.33....	3	1	1	.....	III
3017.66....	15r	15R	10r	3	II
3022.47....	3	2	1	.....	III
3026.49....	6	3	2	tr	III
3031.43....	2	1	tr	.....	III
3034.55....	6	6r	4	2	II
3038.42....	2	2	2	.....	III
3039.66....	3	2	1	.....	III
3040.93....	1	1	.....	.....	IV
3042.60....	8	10r	5r	2	II
3044.11....	30R	30R	20R	10r	II
3048.21....	2	1	.....	.....	IV
3049.00....	12r	12R	8r	3	II
3050.64....	3	2	1	.....	III
3054.84....	4	5	3	2	II
3060.17....	5	5	3	tr	III
3061.15....	1	1	tr	.....	III
3061.94....	20r	20R	15R	8r	II
3062.33*...	5	?	3	2	II
3064.49....	5	5r	4	2	II
3070.94....	1	tr	.....	.....	IV
3072.06....	6	5r	4	3	I
3072.45....	15r	15R	12r	6r	II
3073.64....	3	3	2	.....	III
3079.49....	5	5	4	2	II
3082.73....	12r	12R	10R	5r	II
3082.96....	2	2	1	.....	III
3086.46....	4	3	2	tr	III
3086.89....	15r	15R	12R	6r	II
3087.86....	3	3	1	.....	III
3088.76....	1	3	1	.....	III A
3089.68....	10	10R	8r	4	II
3090.36....	4	4	2	.....	III
3095.81....	3	4	2	.....	III
3096.50....	3	3	2	.....	III
3096.80....	2	2	1	.....	III
3098.30....	10	10R	8r	4	II
3099.76....	2	2	1	.....	III
3102.49....	4	3	2	tr	III
3103.82....	5	5	3	tr	III
3104.12*...	5	5	4	tr	III
3106.03....	3	3	3	1	II
3106.22....	1	3	2	tr	III A
3107.15....	3	3	1	.....	III
3107.65....	1	1	.....	.....	IV
3109.60....	4	3	2	.....	III
3110.12....	5	3	2	.....	III
3110.94....	5	4	4	3	I
3111.45....	2	1	tr	.....	III
3113.58....	6	6	4	1	III
3118.35....	5	8r	5	3	II
3118.76....	1	4	3	1	II A
3121.54....	10	12R	10r	5	II
3121.69....	10	12R	10r	5	II
3126.60....	1	1	.....	.....	IV
3126.85....	4	3	1	.....	III
3127.35....	7	8r	6	5	I
3129.10....	3	5	5	2	II
3129.57....	3	3	1	.....	III
3131.96....	1	1	.....	.....	IV
3132.33....	4	6r	6	3	II
3136.81....	5	6r	3	2	II
3137.10....	1	1	tr	.....	III
3137.47*...	10	10R	8r	5	II
	3	3	1	.....	III
3137.75....	4	4	2	.....	III
3140.08*...	12	12R	10r	5	II
3140.83....	2	2	tr	.....	IV
3145.16....	3	3	1	.....	III
3147.19....	15r	15R	12r	5	II
3149.43....	10	8R	6r	4	II
3150.82....	2	.....	.....	.....	V
3150.93....	2	.....	.....	.....	V
3152.84....	6	5	4	tr	III
3154.78....	5	3	1	.....	III
3154.91....	10	5	5	3	II
3157.23....	1	1	tr	.....	III
3158.92....	12	12R	12r	6	II
3159.80....	10	8R	6	5	II
3161.78....	5	4	3	tr	III
3168.19*...	6	8	4	tr	III
3169.90....	9	5	4	tr	III
3173.30....	1	1	tr	.....	III
3174.29....	2	1	tr	.....	III
3175.06....	4	6	3	.....	III
3177.40....	8	4	2	.....	III
3179.98....	1	tr	.....	.....	IV
3180.42....	2	1	1	.....	III
3182.25....	7	5	4	tr	III

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.	
3186.05....	2	3	1	.....	III
3186.46....	5	5	5	4	I
3188.50....	7	6	4	1	III
3189.87....	5	5	5	4	I
3191.44....	4	6	5	3	II
3192.35....	3	2	1	.....	III
3193.30....	5	6	5	3	II
3198.79....	5	5	5	3	II
3199.44....	4	3	3	3	I
3203.15....	4	3	3	3	I
3206.00....	1	1	.....	.....	IV
3210.35....	5	4	2	.....	III
3210.96....	3n	1	.....	.....	IV
3215.46....	1	tr	.....	.....	IV
3217.15....	1	1	tr	.....	III
3219.31....	5	8R	5	4	II
3223.30....	1	3	3	tr	III A
3224.80....	4	4	2	.....	III
3227.15....	4	2	1	.....	III
3227.93....	2	4	4	2	II A
3234.30....	1	1	.....	.....	IV
3235.69....	6	4	3	1	II
3237.18....	8	10R	8r	4	II
3243.70....	2	2	1	.....	III
3243.99....	8	7	6	3	II
3247.13....	4	3	2	.....	III
3247.32....	8	6	6	3	II
3250.17....	6	8r	5	4	II
3253.60....	1	tr	.....	.....	IV
3254.37....	12	10	9	6	II
3258.16....	4	5	3	tr	III
3258.58....	1	1	tr	.....	III
3260.99....	9	6	5	2	II
3263.35....	4	4	2	.....	II
3264.96....	5	5	5	4	I
3265.49....	3	4	2	.....	III
3268.15....	1	tr	.....	.....	IV
3269.04....	1	1	tr	.....	III
3270.35....	2	tr	.....	.....	IV
3271.92....	8	5	5	2	II
3276.60....	4	2	1	.....	III
3277.44....	4	5	3	tr	III
3277.80....	3	1	tr	.....	III
3278.23....	2	tr	.....	.....	IV
3278.96....	6	5	4	1	III
3279.39....	5	4	3	1	II
3281.75....	2	4	5	3	I A
3282.23....	1	tr	.....	.....	IV
3282.37....	1	1	tr	.....	III
3283.49....	4	4	1	.....	III
3283.60....	9	7	7	5	I
3283.95....	3	5	2	1	II
3286.66....	1	1	tr	.....	III
3287.34....	7	4	5	2	II
3287.69....	2	1	tr	.....	III
3292.24....	3	4	2	.....	III
3293.37....	3	1	tr	.....	III
3294.04....	2	2	1	.....	III
3294.69....	2	tr	.....	.....	IV
3298.83....	6	5	4	1	III
3304.03....	4	4	3	1	II
3304.29....	3	tr	.....	.....	IV
3304.95....	1	tr	.....	.....	IV
3305.27....	2	tr	.....	.....	IV
3305.87....	2	1	tr	.....	III
3306.54....	1	tr	.....	.....	IV
3307.31....	7	6	5	2	II
3308.65....	4	2	1	.....	III
3308.96....	4	2	1	.....	III
3312.33....	7	5	5	2	II
3312.99....	3	1	tr	.....	III
3313.27....	2	tr	.....	.....	IV
3314.21....	8	5	3	.....	III
3314.46....	2	.....	.....	.....	V
3315.20....	3	1	tr	.....	III
3318.55....	4	5	3	1	II
3319.31....	4	2	1	.....	III
3319.66....	8	4	3	1	II
3320.00....	4	2	1	.....	III
3322.06....	2	1	1	.....	III
3322.41....	8	8	5	1	III
3325.44....	10	6	6	3	II
3326.72....	2	4	2	.....	III A
3327.14....	8	5	3	tr	III
3328.34....	3	1	tr	.....	III
3329.16....	2	1	tr	.....	III
3329.63....	5	3	1	.....	III
3333.55....	10	8r	8	8	I
3334.31....	30r	30R	20R	15	II
3337.34....	8	8r	8	10	I
3338.68....	1	1	tr	.....	III
3339.97....	8	5	3	tr	III
3341.52....	5	4	2	.....	III
3342.10....	5	4	2	.....	III
3342.88....	8	10r	6	1	III
3344.36*...	2	2	1	.....	III
3346.45....	1	2	1	.....	III A
3347.10....	8	5	2	.....	III
3348.28....	8	7	5	2	II
3351.29....	1	tr	.....	.....	IV

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.	
3351.70....	3	1	tr	.....	III
3354.34*...	4	?	?	.....	III?
3354.51....	20	20R	15R	10	II
3355.27....	3	2	1	.....	III
3356.06....	2	2	1	.....	III
3356.59....	6	8	5	tr	III
3356.97....	3	1	tr	.....	III
3358.13....	3	2	1	.....	III
3359.20....	3	3	1	.....	III
3359.42....	6	6	5	1	III
3361.72*...	5?	3?	2	.....	III
3362.93....	6	4	2	.....	III
3363.41....	4	2	1	.....	III
3363.89....	3	1	.....	.....	IV
3364.38....	4	5	3	.....	III
3365.13....	2	2	1	.....	III
3367.25....	30r	30R	20R	15	II
3370.48....	10	10r	10	10	I
3373.40....	7	6	4	1	III
3374.13....	4	5	4	1	III
3374.42....	5	1	1	.....	III
3376.34....	2	1	tr	.....	III
3377.20....	5	6	6	2	II
3378.50....	3	1	tr	.....	III
3378.86....	5	3	1	.....	III
3381.65....	4	5	4	tr	III
3382.20....	2	2	1	....	III
3384.09....	4	2	1	.....	III
3385.38....	25r	25R	20R	15	II
3387.19....	1	tr	.....	.....	IV
3387.56....	1	tr	.....	.....	IV
3388.29....	30r	30R	20R	15	II
3388.80....	1	1	tr	.....	III
3390.54....	5	5	4	1	III
3390.92....	3	1	tr	.....	III
3395.07....	2	2	1	.....	III
3395.55....	40r	40R	30R	20	II
3396.61....	1	1	tr	.....	III
3398.96....	3	1	tr	.....	III
3399.54....	1	tr	.....	.....	IV
3400.63....	1	1	tr	.....	III
3401.74....	2	3	2	.....	III
3402.14....	4	1	tr	.....	III
3405.27....	150R	150R	100R	60R	II
3409.05*...	2	?	.....	.....	IV?
3409.29....	60r	60R	40R	20r	II
3412.50....	80R	80R	50R	30r	II
3412.79....	80R	80R	50R	40R	II
3415.66....	5	6	6	6	I
3417.32....	50r	50R	35R	20	II
3417.84....	5	4	1	.....	III
3417.93....	6	6	6	6	I
3420.64....	5	5	3	1	II
3420.95....	7	7	5	1	III
3421.77....	3	3	2	.....	III
3422.63....	1	tr	.....	.....	IV
3423.03....	4	5	4	1	III
3424.67....	10	8	7	3	II
3426.60....	3	1	tr	.....	III
3427.90....	2	1	.....	.....	IV
3428.34....	6	3	1	.....	III
3428.89....	3	1	tr	.....	III
3429.82....	1	tr	.....	.....	IV
3431.76....	50r	50R	35R	20r	II
3432.46*...	3	3	2	.....	III
3433.18....	60R	60R	40R	25	II
3437.10....	3	1	tr	.....	III
3437.83....	6n	3	1	.....	III
3438.83....	4	5	3	tr	III
3439.05....	5	2	tr	.....	IV
3441.28....	2	tr	.....	.....	IV
3443.06....	40r	40R	30R	20	II
3443.31*...	5	?	?	.....	III?
3443.79....	80R	80R	60R	30r	II
3445.29....	1	1	1	.....	III
3446.21....	12	4	2	.....	III
3447.43....	3	tr	.....	.....	IV
3448.49....	4	1	tr	.....	III
3449.26....	60R	50R	40R	30r	II
3449.54....	60R	60R	50R	30r	II
3452.44....	3	1	1	.....	III
3453.66....	200R	200R	125R	80R	II
3455.33....	25r	25R	20R	20	I
3456.58*...	1 1	tr tr	....	....	IV IV
3457.05....	9	8r	8	8	I
3458.16....	3	4	2	.....	III
3460.86....	4	5	6	4	I
3461.33....	15	4	3	tr	III
3462.94....	60r	60R	40R	30r	II
3463.62....	3	3	2	.....	III
3465.96....	100R	100R	80R	60R	II
3467.37....	1	1	.....	.....	IV
3468.74....	1	tr	.....	.....	IV
3469.11....	3	1	tr	.....	III
3471.53....	7	4	2	.....	III
3472.34....	1	tr	.....	.....	IV
3473.60....	1	1	1	.....	III
3474.17....	100R	100R	60R	40r	II
3474.40*...	6	?	?	?	?

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3474.68....	6	6	6	6	I	3548.60....	7	7	6	2	II
3476.50....	5n	1	.....	.....	IV	3550.78....	20r	20R	20R	15	I
3478.00....	4	1	tr	.....	III	3551.84....	2	1	1	.....	III
3478.60....	8	6	4	1	III	3552.90....	8	10r	10	10	I
3478.90....	7	7	5	2	II	3553.16....	8	6	6	2	II
3479.74....	1	1	1	.....	III	3553.31....	2	1	tr	.....	III
3480.17....	6	6	4	2	II	3558.93....	12	12r	12	10	I
3483.29....	2	2	1	.....	III	3559.75....	1	tr	.....	.....	IV
3483.58....	20r	20R	20R	15	I	3560.47....	5	6	3	.....	III
3485.51....	15	6	4	1	III	3561.03....	20r	20R	20R	15	I
3485.85....	4	4	3	1	II	3562.25....	6	6	3	.....	III
3487.84....	8	6	5	2	II	3563.09....	7	9	6	2	II
3489.57....	60r	40R	30R	20	II	3564.31....	4	2	1	.....	III
3490.89....	10	8r	8	8	I	3565.09....	25r	25R	25R	20	I
3491.49....	15	20R	20R	15	I	3568.56....	2	2	1	.....	III
3492.12....	3	1	tr	.....	III	3569.59....	80R	70R	50R	25	II
3495.83....	50r	50R	40R	25	II	3570.57....	4n	3	2	1	II
3496.20....	3	1	tr	.....	III	3575.13....	25r	25R	20R	15	I
3496.80....	15	15R	12r	12	I	3575.53....	60r	60R	50R	30R	II
3496.90....	6	6	3	.....	III	3577.39....	3	4	3	1	II
3502.45....	100R	100R	60R	40R	II	3577.82....	2	.....	.....	.....	V
3502.80....	20r	20R	20R	20	I	3578.21....	6	6	3	1	II
3503.85....	3	3	1	.....	III	3579.04....	6	5	4	2	II
3504.89....	5	4	2	.....	III	3579.15....	6	6	5	2	II
3505.29....	3	2	1	.....	III	3582.02....	4	4	2	1	II
3506.47....	80R	80R	50R	30r	II	3584.94....	15	15R	12r	10	I
3510.00....	50r	40R	30R	25	II	3585.33....	25R	25R	20R	15	I
3510.59....	30r	30R	30R	25	I	3585.94....	4	4	3	1	II
3512.80....	60R	50R	40R	25	II	3586.20....	3	2	1	.....	III
3513.61....	50R	50R	40R	30r	II	3587.30....	70R	60R	40R	25	II
3516.78....	1	1	1	.....	III	3591.91....	4	3	2	.....	III
3518.50....	50R	40R	20R	15	II	3595.03....	50R	50R	40R	20	II
3520.23....	15	20R	20R	10	II	3596.67....	5	5	3	.....	III
3521.73....	30r	30R	30R	20	I	3600.97....	3	4	3	tr	III
3521.85....	5	5	5	5	I	3602.22....	40R	40R	30R	20	II
3523.00....	4	1	tr	.....	III	3604.62....	4	1	1	tr	II
3523.55....	25r	20R	20R	20	I	3605.17....	5	6	5	1	III
3523.83....	7	5	5	2	II	3605.52....	20r	20R	15r	15	I
3526.00....	3	4	2	.....	III	3608.45....	3	10	8	3	II A
3526.97....	100R	100R	80R	50R	II	3609.94....	4	1	1	.....	III
3528.09....	5	2	1	.....	III	3611.89....	10	8	6	3	II
3529.19....	30r	30R	25R	20	I	3615.54....	6	7	5	2	II
3529.96....	80R	80R	50R	30r	II	3618.15....	4	9	7	3	II A
3530.70....	1	1	tr	.....	III	3620.56....	5	5	3	tr	III
3533.51....	25r	25R	25R	20	I	3624.54....	5	7	4	1	III
3534.91....	4	4	2	.....	III	3625.18....	8	12r	9	8	I
3537.85....	1	1	tr	.....	III	3626.20....	2	4	2	.....	III A
3543.15....	2	5	5	2	II A	3627.98....	25r	25R	20R	15	I
3543.43....	15	8	8	5	II	3631.59....	20r	25R	25R	18	I
3546.85....	6	5	4	1	III	3632.12....	2	1	1	.....	III

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3633.01	7	5	3	tr	III	3750.07	9	6	6	3	II
3633.49	2	1	1	.....	III	3751.75	5	8	4	1	III
3634.86	7	5	3	tr	III	3754.47	4	2	1	.....	III
3636.84	6	6	5	2	II	3755.60	10	10	7	3	II
3637.44	4	3	2	.....	III	3759.83	3	2	1	.....	III
3638.50	1	1	tr	.....	III	3760.55	4	5	4	2	II
3639.60	10	10	8	5	II	3774.75	8	10	6	2	II
3641.94	6	7	4	3	II	3777.25	1	1	tr	.....	III
3643.35	9	8	6	4	II	3777.68	6	8	5	1	III
3645.34	5	6	4	1	III	3805.94	2	2	1	.....	III
3645.60	3	3	2	tr	III	3808.25	10	10R	10r	12	I
3647.23	5	5	3	tr	III	3811.23	5	7	6	5	I
3647.56	1	1	tr	.....	III	3812.62	4	4	3	tr	III
3647.85	12	15R	15r	15	I	3814.62	5	5	4	1	III
3648.26	3	2	1	.....	III	3816.48	15	6	5	4	I
3649.49	8	8	4	tr	III	3816.61	15	6	5	4	I
3651.41	4	5	4	1	III	3817.01	5	5	3	1	II
3652.70	15	20R	20R	20	I	3820.08	4	5	3	1	II
3654.59	5	6	5	2	II	3823.66	1	tr	.....	.....	IV
3657.10	7	8	8	10	I	3841.60	5	4	4	4	I
3658.06	2	1	1	.....	III	3842.21	30	20R	15r	10	II
3662.32	12	8	7	4	II	3843.85	4	4	3	tr	III
3668.80	1	1	tr	.....	III	3845.60	60	60R	40R	30R	II
3670.20	3	3	2	tr	III	3850.27	5	4	3	.....	III
3676.72	12	10	5	1	III	3851.09	4	10r	8	8	I A
3683.22	20	20	15	10	II	3852.00	2	2	1	.....	III
3684.63	10	12	6	1	III	3856.94	4	2	2	.....	III
3685.11	2	2	1	.....	III	3861.31	20	20R	15r	15	I
3686.62	2	1	tr	.....	III	3863.75	2	1	tr	.....	III
3690.91	7	6	4	2	II	3870.66	4	3	1	.....	III
3693.29	8	8	8	5	I	3873.23	60	60R	40R	30R	II
3693.53 H	2	2	1	.....	III	3874.09	40	40R	30R	20R	II
3693.63	8	6	7	5	I	3877.01	20	20R	15r	15	I
3699.15	2n	1	.....	.....	IV	3882.06	25	20R	18R	15r	I
3702.39	12	8	5	1	III	3884.79	10	8r	8	6	I
3704.22	25	30R	20	20	I	3885.45	6	6	6	4	I
3707.62	6	5	4	2	II	3890.18	2	2	1	.....	III
3709.00	12	9	8	5	II	3891.85	2	.....	.....	.....	V
3711.83	3	2	2	tr	III	3892.30	3	2	1	.....	III
3712.35	6	5	4	1	III	3893.20	2	1	1	.....	III
3726.79	5	6	4	1	III	3893.45	2	.....	.....	.....	V
3728.96	3	2	1	.....	III	3894.25	60	50R	30R	20R	II
3730.63	20	15	12	8	II	3895.15	20	20R	15R	8	II
3731.42	2	2	2	tr	III	3898.54	4	5	5	1	III
3732.59	20	15	12	12	I	3904.23	2	.....	.....	.....	V
3733.65	12	10	8	5	II	3904.94	3	.....	.....	.....	V
3734.30	7	7	5	2	II	3905.70	2	.....	.....	.....	V
3736.08	12	12	9	4	II	3906.46	10	12r	12r	10	I
3740.34	5	4	3	1	II	3910.13	15	20R	20R	15r	I
3745.65	25	30R	20r	20	I	3917.80	8	6	5	2	II

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.	
3920.32....	2	1	1	.....	III
3920.75....	2	1	1	.....	III
3920.90....	4	3	3	1	II
3921.27....	1	tr	.....	.....	IV
3922.90....	7	8	8	7	I
3925.33....	3	1	tr	.....	III
3929.43....	3	1	tr	.....	III
3934.07....	6	8	6	6	I
3934.85....	1	1	tr	.....	III
3935.44....	1	1	1	.....	III
3936.13....	30	30R	15R	10	II
3939.00....	3	.....	.....	.....	V
3939.21....	2	.....	.....	.....	V
3941.06....	12	12r	10r	8	I
3941.91....	20	20R	15r	10	II
3942.84....	2	tr	.....	.....	IV
3945.07....	1	tr	.....	.....	IV
3945.51....	15	15r	12r	12	I
3946.75....	2	2	2	1	II
3947.26....	3	3	3	1	II
3952.46....	8	8r	8	7	I
3953.10....	25	25R	15r	12	II
3958.10....	15	15R	12r	8	II
3961.15....	6	4	2	1	II
3965.15....	1	2	2	1	II A
3965.37....	2	4	4	2	II A
3968.75....	1	2	2	1	II A
3969.28....	8	5	4	1	III
3972.69....	6	1	tr	.....	III
3973.31....	10	10	10	3	II
3974.90....	10	15R	12r	12	I
3975.45....	3	2	2	tr	III
3977.34....	3	3	2	tr	III
3978.78....	10	10r	10	9	I
3978.99....	4	.....	.....	.....	V
3979.67....	10	15R	15R	12	I
3987.25....	6	8	8	8	I
3990.45....	6	5	4	2	II
3991.68*...	4	2?	tr	.....	IV
3991.83*...	6	8?	6	6	I
3994.70....	6	6	5	5	I
3995.45....	60	60R	40R	20R	II
3998.09....	40	40R	20R	15	II
4003.85....	2	1	1	.....	III
4008.07....	1	1	tr	.....	III
4011.25....	2	4	4	4	I A
4014.09....	7	6	5	2	II
4016.95....	2	.....	.....	.....	V
4019.45....	5	8	8	6	I
4021.07....	20	20R	15R	15	I
4023.55....	4	4	4	1	III
4027.18....	10	10r	10r	10	I
4035.74....	8	3	1	.....	III
4040.95....	2	2	2	tr	III
4045.56....	20	20R	15r	15	I
4053.10....	3	1	1	.....	III
4054.08....	1	1	.....	.....	IV
4057.10....	2	.....	.....	.....	V
4057.36....	5	8r	8	8	I
4058.36....	8	10r	10	10	I
4058.76....	6	6	6	2	II
4066.56....	15	12r	10	10	I
4068.72....	8	7	6	3	II
4069.71....	1	tr	.....	.....	IV
4076.30....	3	8	8	7	I A
4077.56*...	2	2	1	.....	III
	2	.....	.....	.....	V
4081.64....	2	.....	.....	.....	V
4082.75....	2	5	5	4	I A
4083.78....	1	1	.....	.....	IV
4086.49....	15	15	12	6	II
4088.45....	1	8	8	6	I A
4092.56....	25	20R	15r	15	I
4093.03....	3	2	1	.....	III
4093.22....	2	.....	.....	.....	V
4096.11....	2	.....	.....	.....	V
4104.57....	2	2	2	tr	III
4104.91....	4	3	2	tr	III
4110.70....	25	20R	15r	15	I
4118.96....	50	50R	30R	15r	II
4121.52....	60	60R	40R	20r	II
4122.43....	2	1	tr	.....	III
4132.00....	3	.....	.....	.....	V
4132.30....	4	6	6	4	I
4139.60....	3	3	2	tr	III
4150.62....	2	4	4	2	II A
4158.59....	4	1	1	.....	III
4162.32....	2	1	.....	.....	IV
4179.34....	2	1	tr	.....	III
4187.46....	4	4	4	2	II
4190.88....	20	20R	20r	25	I
4195.03....	1	1	tr	.....	III
4225.28....	2	2	2	tr	III
4234.15*...	2	4	4	6	I A
	2	4	4	6	I A
4237.50....	1	1	1	.....	III
4241.69....	2	.....	.....	.....	V
4242.03....	2	2	1	.....	III
4245.70....	2	.....	.....	.....	V
4248.30....	2	.....	.....	.....	V

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4252.48....	12	12r	12r	15	I	4728.61....	3	8	8	12	I A
4268.61....	2	1	1	.....	III	4735.01†...	2	?	?	.....	III ?
4285.92....	6	8	8	10	I	4737.92....	2	5	3	1	II A
4303.41....	3	10	9	8	I A	4749.89....	10	8	4	1	III
4307.58....	2	.....	.....	.....	V	4754.60....	3	2	1	.....	III
4331.43....	3	tr	.....	.....	IV	4768.26....	5	3	1	.....	III
4339.80....	5	4	2	tr	III	4771.30....	6	5	2	.....	III
4371.31†...	5	6 ?	3 ?	tr	III	4776.51....	6	4	2	.....	III
4373.81....	6	1	.....	.....	IV	4778.42....	2	tr	.....	.....	IV
4375.11....	3	2	1	.....	III	4780.20....	10	6	3	tr	III
4375.70....	2	2	1	.....	III	4781.64....	3	10	6	2	II A
4380.29†...	5	1 ?	1 ?	.....	III ?	4793.10....	15	8	4	1	III
4391.78....	4	2	1	.....	III	4796.06....	2	.....	.....	.....	V
4392.11....	3	3	2	.....	III	4796.60....	1	15	10	6	II A
4396.09....	1	2	1	.....	III A	4813.70....	20	10	5	1	III
4402.86....	3	1	tr	.....	III	4814.22....	2	1	tr	.....	III
4405.08....	3	1	1	.....	III	4816.09....	1	1	tr	.....	III
4417.55....	5	2	1	.....	III	4840.50....	25	12	5	1	III
4421.51....	4	2	1	.....	III	4843.68....	3	1	tr	.....	III
4431.79....	3	1	1	.....	III	4868.08....	25	12	8	3	II
4445.22....	2	1	.....	.....	IV	4882.89....	2	1	tr	.....	III
4445.90....	4	2	1	.....	III	4899.70....	2	8	5	1	III A
4467.09....	10	6	3	tr	III	4904.38....	1	1	tr	.....	III
4469.75....	15	12	5	1	III	4912.58....	1	5	4	4	I A
4471.76....	5	3	2	.....	III	4920.40....	1	3	2	.....	III A
4472.00....	1	1	1	.....	III	4928.47....	2	3	2	.....	III
4478.50....	4	2	1	.....	III	4953.32*...	2	10 ?	10 ?	10	I A
4484.11....	3	2	1	.....	III	4966.72....	2	8	8	7	I A
4494.94....	2	tr	.....	.....	IV	4972.09....	2	1	.....	.....	IV
4514.35....	1	1	.....	.....	IV	4988.10....	2	6	6	6	I A
4517.26....	4	3	1	.....	III	5067.73†...	2	?	?	.....	IV ?
4526.95....	2	1	tr	.....	III	5077.57†...	3	4 ?	2 ?	.....	III ?
4528.08....	2	1	1	.....	III	5087.97....	3	.....	.....	.....	V
4531.12....	30	20	10	3	II	5095.13†...	8	3 ?	1	.....	III
4534.18....	7	5	2	tr	III	5109.02†...	10	1 ?	.....	.....	V
4543.99....	6	6	4	1	III	5113.39†...	6	?	?	.....	IV ?
4546.15....	1	1	tr	.....	III	5122.93†...	8	?	.....	.....	IV ?
4549.89....	10	8	4	1	III	5124.89†...	2	?	?	.....	IV ?
4565.79....	15	12	5	1	III	5125.84†...	7	?	.....	.....	IV ?
4570.20....	2	tr	.....	.....	IV	5126.36†...	10	?	?	.....	IV ?
4580.34....	4	10	8	8	I A	5133.60....	15	.....	.....	.....	V
4581.80....	20	15	4	1	III	5145.63†...	2	?	.....	.....	IV ?
4588.90....	1	10	7	7	I A	5146.89†...	15	?	.....	.....	IV ?
4594.82....	4	1	.....	.....	IV	5149.21†...	2	?	?	.....	IV ?
4597.10....	5	1	.....	.....	IV	5149.93†...	4	3 ?	2 ?	1	II
4625.91....	2	2	1	.....	III	5154.20†...	8	?	.....	.....	IV ?
4629.58....	15	8	4	1	III	5156.49†...	10	?	.....	.....	IV ?
4663.60†...	12	8 ?	4 ?	1	III	5158.57†...	2	?	?	.....	IV ?
4682.55†...	9	?	2 ?	.....	III ?	5159.00†...	2	1 ?	?	.....	IV ?
4693.36†...	6	?	?	.....	IV ?	5165.30†...	3	?	?	.....	IV ?
4698.56†...	3	?	?	.....	IV ?	5166.27....	2	.....	.....	.....	V

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
5176.22	20	15	12	2	III	5390.62†	2	1?	.....	.....	IV?
5192.53	4	.....	.....	.....	V	5402.20	3	.....	.....	.....	V
5210.20	3	tr	.....	.....	IV	5407.70	5	.....	.....	.....	V
5210.97	3	tr	.....	.....	IV	5408.35†	2	2?	1?	.....	III?
5211.91	3	tr	.....	.....	IV	5413.43	2	.....	.....	.....	V
5212.85	25	4	3	.....	III	5431.22	2	.....	.....	.....	V
5219.20	2	.....	.....	.....	V	5434.72†	2	3?	2?	.....	III?
5222.67	4	1	.....	.....	IV	5437.17†	3	1?	1?	.....	III?
5230.40	25	15	12	8	II	5444.80*	20	?	?	.....	IV?
5235.32	15	12	10	4	II	5452.53	3	.....	.....	.....	V
5248.02	15	15	12	5	II	5454.81	20	1	.....	.....	V
5250.12	7	.....	.....	.....	V	5469.49	4	8	7	1	III A
5254.78	8	1	.....	.....	IV	5470.69	4	.....	.....	.....	V
5257.75	10	1	1	.....	III	5477.30	5	.....	.....	.....	V
5264.40	2	1	.....	.....	IV	5483.59	40	20	20	20	I
5266.00	4	tr	.....	.....	V	5484.19	10	tr	.....	.....	V
5266.51 H*	10	2?	1?	tr?	II?	5489.81†	5	2?	1?	.....	III?
5266.71 H*	25	15	12	8	II	5495.90†	2	1?	1?	.....	III?
5268.70	10	2	2	.....	III	5523.51†	8	4?	3?	2?	II?
5276.32	8	.....	.....	.....	V	5525.23	4	.....	.....	.....	V
5280.80	20	4	3	.....	III	5530.99	10	12	10	6	II
5283.60	4	.....	.....	.....	V	5546.60†	2	2?	?	.....	III?
5287.78 H	3	tr?	.....	.....	V	5559.02†	2	2?	?	.....	III?
5288.02 H	5	tr	.....	.....	V	5590.99	10	10	10	3	II
5301.20	15	15	12	8	II	5636.30	3	.....	.....	.....	V
5312.80	8	1	tr	.....	III	5637.91	3	.....	.....	.....	V
5316.90	7	1	tr	.....	III	5640.22	1	2	1	tr	III A
5321.89	2	.....	.....	.....	V	5647.47	12	10	10	3	II
5325.40	10	2	1	.....	III	5659.36	3	4	3	2	II
5326.06	4	1	tr	.....	III	5688.82	2	3	2	1	II
5326.39	3	tr	.....	.....	IV	5770.62	2	.....	.....	.....	V
5328.20	2	.....	.....	.....	V	5830.32	4	.....	.....	.....	V
5331.62	15	15	12	6	II	5846.78	2	.....	.....	.....	V
5332.85	5	1	1	.....	III	5881.32*	2	3?	3?	3?	I?
5333.82	5	1	tr	.....	III	5890.71	12	8	8	2	III
5335.02	6	.....	.....	.....	V	5915.74	10	8	8	2	III
5336.30	3	.....	.....	.....	V	5935.61	6	6	6	1	III
5339.61	4	.....	.....	.....	V	5946.73	5	2	1	.....	III
5341.34	7	.....	.....	.....	V	5984.40*	3	2	2	?	III?
5342.89	50	4	4	.....	III		3	2	1	?	III?
5343.58	20	2	2	.....	III	5992.11*	20	20	20	4	III
5347.63	4	.....	.....	.....	V	6000.91	5	3	3	3	I
5349.23	4	.....	.....	.....	V	6006.50	5	1?	.....	.....	IV?
5352.30	20	4	4	.....	III	6007.85	5	.....	.....	.....	V
5353.69	25	5	4	.....	III	6049.34	6	.....	.....	.....	V
5359.16 H	2	tr	.....	.....	IV	6070.80	2	.....	.....	.....	V
5359.41 H	6	1	1?	.....	III?	6082.67	15	3	2	.....	III
5362.95	15	1	1	.....	III	6086.84†	7	4	3	1?	II?
5369.83	20	15	15	12	I	6093.35	10	12	10	8	I
5381.31	5	5	4	1	III	6108.12	2	.....	.....	.....	V
5381.92	6	.....	.....	.....	V	6117.20	8	10	8	7	I

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
6122.90....	8	1 ?	.....	.....	IV ?	6450.51....	80	60	50	50	I
6189.20....	10	20	15	7	II A	6451.38....	3n	.....	.....	.....	V
6211.34†...	2	1 ?	?	.....	IV ?	6455.30....	40	4	3	.....	III
6231.20*...	10	12	10	2 ?	III	6478.10....	10	.....	.....	.....	V
6232.70....	2	1 ?	.....	.....	IV ?	6490.50....	6	7	6	1	III
6249.70....	8	6	6	2	II	6551.69....	3	4	4	.....	III
6257.81†...	6	2 ?	1	.....	III	6563.61....	40	20	20	6	II
6273.28†...	4	1 ?	tr ?	.....	III ?	6596.17....	12	.....	.....	.....	V
6282.89....	40	30	20	20	I	6617.30....	6n	.....	.....	.....	V
6320.62....	8	1	.....	.....	IV	6617.70....	3n	.....	.....	.....	V
6348.00....	10	.....	.....	.....	V	6624.00*...	2	3 ?	3 ?	.....	III ?
6351.66....	2	.....	.....	.....	V	6632.69....	15	10	8	4	II
6395.40....	8	.....	.....	.....	V	6679.03....	4	5	4	2	II
6396.71....	2	.....	.....	.....	V	6771.29*...	20	20	15	15	I
6417.99....	15	7	6	tr	III	6815.20*...	15	15	10	10	I
6421.91....	2	.....	.....	.....	V	6872.62*...	10	10	6	6	I
6430.10....	4	4	5	1	III	7016.82*...	3	3	3	3	I
6430.51....	2	.....	.....	.....	V	7053.11*...	8	8	6	6	I
6444.89....	2	.....	.....	.....	V	7085.25*...	15	12	10	12	I

REMARKS ON TABLE I

λ	
3062.33	Concealed at high temperature by λ 3061.94.
3104.12	Probably double.
3137.47	Doublet, just resolved.
3140.08	Close doublet. Companion makes reversal unsymmetrical.
3168.19 and 3344.36	Both probably double.
3354.34	Concealed by reversal of λ 3354.51.
3361.72	Coincides with strong Ni line.
3409.05	Furnace line may be concealed by λ 3409.29.
3432.46	Probably double.
3443.31	Concealed by adjacent lines.
3474.40	Concealed by reversal of λ 3474.17.
3991.68 and 3991.83	Blend at high temperature.
4077.56	Doublet in arc. Only violet component appears in furnace.
4234.15	Close doublet, not fully resolved.
4953.32	Blend with Ni. Furnace line probably all Co.
5266.51 and 5266.71	Close blend.
5444.80	Very weak in furnace if present.
5881.32	Furnace line may belong to band spectrum.
5984.40	Doublet. Disturbed by band at low temperature.
5992.11	May be close doublet.
6231.20	Low-temperature line may belong to band.
6624.00	Furnace line may belong to band.
6771.29 to 7085.25	Photographed with 1-meter concave grating.

TABLE II

Temperature Classification of Nickel Lines

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
2981.80....	20R	20R	15R	8r	II	3207.05....	4n	.....	.....	.....	V
2983.56....	4	2	1	.....	III	3210.00....	5	tr	.....	.....	V
2984.28....	12R	12R	10r	5	II	3213.53....	5n	.....	.....	.....	V
2991.22....	4	2	2	1	II	3214.17....	7	1	tr	.....	III
2992.71....	20R	20R	15R	8r	II	3216.93....	5	1	.....	.....	IV
2994.58....	25R	25R	15R	8r	II	3217.93....	8	2	1	.....	III
3002.58....	100R	100R	75R	40R	II	3219.92....	3	.....	.....	.....	V
3003.70....	60R	60R	40R	20R	II	3221.41....	5	tr	.....	.....	V
3012.11....	75R	75R	30R	15r	II	3221.81....	10r	15R	12r	6	II
3019.28....	20R	20R	15R	8r	II	3223.66....	3	1	.....	.....	IV
3029.36....	3	1	.....	.....	IV	3225.19....	10r	12R	10r	5	II
3031.98....	10r	10r	5	2	II	3227.11....	5	7	4	3	II
3038.04....	60R	60R	40R	20R	II	3233.06....	25R	25R	20R	15r	II
3045.15....	10r	10r	6	3	II	3233.28*...	4	?	.....	.....	IV ?
3050.92....	100R	100R	75R	40R	II	3234.00....	2	.....	.....	.....	V
3054.42....	50R	50R	30R	15R	II	3234.78....	10r	12R	10r	5	II
3057.76....	50R	50R	30R	15R	II	3235.86....	4	4	3	3	I
3064.75....	25R	25R	20R	10r	II	3243.20....	25R	25R	20R	15r	I
3066.59....	3	1	.....	.....	IV	3245.47....	4n	.....	.....	.....	V
3080.91....	20R	20R	15r	7	II	3248.56....	8	10r	6	4	II
3097.27....	15r	15R	12r	6	II	3249.55....	6	5	4	4	I
3099.25....	12r	12r	8	4	II	3250.90....	9	10r	6	4	II
3101.67....	100R	100R	60R	30R	II	3264.56....	2n	.....	.....	.....	V
3101.99....	40R	40R	25R	15r	II	3268.21....	4n	.....	.....	.....	V
3105.60....	15r	15r	10r	5	II	3269.08....	2n	.....	.....	.....	V
3107.83....	4	3	2	2	I	3271.25....	10	10r	6	4	II
3114.26....	20R	20R	12r	8	II	3282.03....	5	tr	.....	.....	V
3116.84....	2	.....	.....	.....	V	3282.81....	8	8	5	4	II
3129.42....	7	4	3	3	I	3282.96....	5	.....	.....	.....	V
3134.22....	60R	60R	40R	20R	II	3284.56....	4	1	.....	.....	IV
3145.23....	3	2	2	2	I	3287.08....	8	10r	6	4	II
3145.82....	8	8r	4	2	II	3287.36....	2	1	tr	.....	III
3151.33....	4n	.....	.....	.....	V	3305.10....	6	1	.....	.....	IV
3154.68....	2	1	.....	.....	IV	3307.16....	2	.....	.....	.....	V
3159.65....	3	2	2	2	I	3309.56....	2n	.....	.....	.....	V
3164.30....	2	1	.....	.....	IV	3310.35....	5	5	5	4	I
3165.64....	3	2	2	2	I	3312.49....	10	2	1	.....	III
3170.86....	2	1	.....	.....	IV	3313.15....	4	.....	.....	.....	V
3176.44....	2	1	.....	.....	IV	3315.82....	30R	30R	25R	12r	II
3181.89....	5	3	2	1	II	3320.42....	20R	20R	15R	8r	II
3183.14....	3	1	tr	.....	III	3320.92....	6	tr	.....	.....	V
3183.40....	4	2	1	tr	II	3321.36....	2	.....	.....	.....	V
3184.50....	8	8r	4	2	II	3322.50....	15r	15R	10r	6	II
3191.97....	2	.....	.....	.....	V	3326.80....	4	1	.....	.....	IV
3195.67....	6	6	4	2	II	3327.52....	4	.....	.....	.....	V
3197.22....	10r	10r	8r	4	II	3328.85....	5	4	4	4	I
3199.44....	3n	.....	.....	.....	V	3332.31....	6n	.....	.....	.....	V
3200.50....	5	6	3	2	II	3335.72....	2n	.....	.....	.....	V
3202.21....	5	1	.....	.....	IV	3337.15....	4	5	4	3	I

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3338.90	3	1	1	.....	III	3476.80	2n	.....	.....	.....	V
3339.20	4n	.....	.....	.....	V	3478.00	2	.....	.....	.....	V
3359.30	8	1	.....	.....	IV	3478.42	3	tr	.....	.....	IV
3361.44	5	5	3	1	II	3479.36	3	.....	.....	.....	V
3361.75	20R	20R	15R	8r	II	3480.30	4	.....	.....	.....	V
3362.97	6	5	5	4	I	3483.98	25R	25R	20R	12r	II
3363.76	4	tr	.....	.....	IV	3485.25	2n	.....	.....	.....	V
3364.75	5	tr	.....	.....	IV	3486.09	10	10r	7	5	II
3365.92	15r	15R	10r	6	II	3488.43	2	.....	.....	.....	V
3366.32	20R	20R	15R	8r	II	3493.11	150R	150R	100R	60R	II
3366.95	10	2	1	.....	III	3496.47	5	tr	.....	.....	V
3368.05	8	8r	6	5	II	3501.02	25R	25R	20R	12r	II
3369.71	80R	80R	60R	30R	II	3502.73	8	8r	6	5	I
3372.19	15r	15R	12r	8	II	3507.85	8	8r	6	5	I
3374.42	15r	15R	12r	8	II	3510.52	80R	80R	50R	25R	II
3374.82	15	3	2	1	II	3511.76	2	2	1	.....	III
3375.70	2n	.....	.....	.....	V	3514.10	15	15R	10r	8	II
3376.46	4	.....	.....	.....	V	3515.21	150R	150R	100R	60R	II
3380.71	80R	80R	50R	25R	II	3516.33	8	1	1	.....	III
3381.01	15r	15R	12r	8	II	3518.80	8	1	1	.....	III
3387.54	3	4	4	3	I	3519.97	20R	20R	15R	8r	II
3391.20	50R	50R	40R	20R	II	3523.23	4	4	4	4	I
3393.10	100R	100R	70R	40R	II	3523.61	10	10r	8r	6	II
3396.31	6	.....	.....	.....	V	3524.68	200R	200R	125R	80R	II
3401.31	8	1	tr	.....	III	3526.67	3	.....	.....	.....	V
3403.58	8	2	1	.....	III	3528.13	15	15R	10r	8	II
3409.74	8	12R	10r	6	II	3528.70	3	.....	.....	.....	V
3413.66	25R	25R	20R	12r	II	3530.73	4	1	1	.....	III
3414.12	12r	15R	12r	8	II	3548.32	20r	20R	15R	10	II
3414.91	150R	150R	100R	50R	II	3551.71	8	8r	7	6	I
3420.88	5	6	5	4	I	3553.64	7	6	5	5	I
3421.49	7	1	tr	.....	III	3560.05	2	.....	.....	.....	V
3422.47	4	.....	.....	.....	V	3561.90	10	10r	8	6	II
3423.00	4	.....	.....	.....	V	3566.51	100R	100R	60R	30R	II
3423.87	50R	50R	30R	15R	II	3572.02	50R	50R	40R	20R	II
3433.74	70R	70R	50R	30R	II	3576.08	2	.....	.....	.....	V
3435.63	2	1	1	tr	II	3577.36	2	5	5	4	I A
3437.45	30R	30R	25R	15R	II	3588.07	12	12r	10r	7	II
3442.17	5	1	tr	.....	III	3597.86	50R	50R	40R	25R	II
3442.67	4n	.....	.....	.....	V	3602.41	15	15r	10r	8	II
3443.03	2n	.....	.....	.....	V	3604.41	1	3	2	.....	III A
3444.36	5	.....	.....	.....	V	3607.00	4	.....	.....	.....	V
3446.40	100R	100R	60R	35R	II	3609.48	15	15r	10r	8	II
3453.02	40R	40R	25R	15R	II	3610.61	60R	60R	50R	30R	II
3458.60	125R	125R	70R	40R	II	3612.90	30R	30R	15R	10r	II
3461.80	125R	125R	70R	45R	II	3619.52	150R	150R	80R	50R	II
3467.61	12	12R	8	6	II	3624.89	15	15r	8	6	II
3467.77	4	.....	.....	.....	V	3630.03	5	.....	.....	.....	V
3469.65	15	15R	10r	8	II	3635.10	12	8	6	6	I
3472.71	70R	70R	40R	20R	II	3641.75	4	5	5	5	I

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3642.52....	2	.....	.....	.....	V	4064.55....	2	.....	.....	.....	V
3644.07....	2n	.....	.....	.....	V	4075.00....	2	5	4	2	II A
3647.85....	2	2	2	1	II	4164.80*...	1	6	5	2	II A
3657.87....	2	.....	.....	.....	V	4195.72....	4	.....	.....	.....	V
3662.10....	8	6	6	6	I	4200.60*...	5	1	1	1	I
3664.27....	20	20R	12r	9	II	4201.89*...	5	1	1	1	I
3666.16....	2	1	1	.....	III	4231.19....	5	.....	.....	.....	V
3668.36....	3	.....	.....	.....	V	4284.84....	6	tr	.....	.....	V
3669.40....	12	12r	8	6	II	4288.15....	15	tr	.....	.....	V
3670.60....	20	20R	12r	9	II	4296.05....	8	.....	.....	.....	V
3674.30*...	10	8?	8?	8	I	4325.52....	2	.....	.....	.....	V
	15	15r?	8?	5	II	4325.78....	6	tr	.....	.....	V
3688.59....	15	15R	12r	8	II	4330.90....	2	.....	.....	.....	V
3689.43....	2	.....	.....	.....	V	4331.83....	12	8	6	3	II
3694.07....	8	8	6	6	I	4356.07....	3	6	3	1	II A
3697.05....	3	.....	.....	.....	V	4359.76....	10	1	.....	.....	V
3713.84....	1	tr	.....	.....	IV	4368.47....	2	1	.....	.....	IV
3715.65....	2	.....	.....	.....	V	4384.70....	5	.....	.....	.....	V
3722.64....	15	15R	12r	7	II	4390.05....	3	.....	.....	.....	V
3724.95....	4	.....	.....	.....	V	4398.80....	3	.....	.....	.....	V
3730.90....	4	5	5	5	I	4399.78....	3	.....	.....	.....	V
3736.95....	15	15R	10	8	II	4401.02....	3	.....	.....	.....	V
3739.40....	10	8	8	7	I	4401.75....	30	8	4	tr	III
3739.94....	3n	.....	.....	.....	V	4410.66....	4	1	tr	.....	III
3744.72....	5	.....	.....	.....	V	4437.15....	5	tr	.....	.....	V
3749.19....	8	8	8	7	I	4437.78....	2	.....	.....	.....	V
3772.67....	6	6	6	6	I	4459.19....	20	6	3	.....	III
3775.75....	30r	30R	15r	10	II	4462.63....	10	2	1	.....	III
3778.20....	5	5	5	5	I	4470.64....	15	4	2	.....	III
3783.72....	30r	30R	15r	10	II	4520.15....	4	4	3	1	II
3792.48....	5	5	5	5	I	4547.11....	5	1	tr	.....	III
3793.79....	8	8	7	6	I	4547.38....	3	.....	.....	.....	V
3807.35....	35r	35R	20R	12r	II	4592.72....	10	2	1	tr	III
3831.89....	20	20R	12r	6	II	4600.56....	6	tr?	.....	.....	V
3833.00....	5	5	5	5	I	4605.17....	12	3	1	.....	III
3858.51....	40r	40R	30R	20R	II	4606.38....	3	.....	.....	.....	V
3889.84....	15	5	3	2	II	4648.85....	15	3	2	tr	III
3909.10....	8n	.....	.....	.....	V	4667.12†...	2	?	1?	.....	III?
3912.47....	8n	.....	.....	.....	V	4667.92....	3	.....	.....	.....	V
3913.14....	5	5	5	5	I	4686.39....	5	2	1	.....	III
3944.29....	12n	.....	.....	.....	V	4701.70....	3	.....	.....	.....	V
3962.25....	3n	.....	.....	.....	V	4703.97....	4	.....	.....	.....	V
3970.65....	10n	.....	.....	.....	V	4714.60†...	25	6?	4?	2	II
3972.32....	10	6	5	5	I	4715.95†...	8	2?	1?	tr	II
3973.71....	25	20R	10r	5	II	4731.98†...	3	?	.....	.....	IV?
3974.82....	10n	.....	.....	.....	V	4732.63....	3	.....	.....	.....	V
3984.29....	8n	.....	.....	.....	V	4752.59....	4	1	1	.....	III
3994.15....	3n	.....	.....	.....	V	4754.92....	3	.....	.....	.....	V
4006.30....	3	.....	.....	.....	V	4756.70....	10	4	2	.....	III
4017.67....	6n	.....	.....	.....	V	4762.82....	3	6	4	2	II A

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4764.15....	4	1	tr	.....	III	5176.72....	5	tr	.....	.....	V
4786.75....	15	5	3	1	II	5184.78....	4	1	.....	.....	IV
4800.01....	1	1	tr	.....	III	5192.66....	2	.....	.....	.....	V
4807.20....	4	1	1	.....	III	5197.35....	2	.....	.....	.....	V
4829.20....	15	4	3	.....	III	5216.58....	2	.....	.....	.....	V
4831.37....	10	3	2	.....	III	5220.30....	2	.....	.....	.....	V
4832.90....	2	.....	.....	.....	V	5235.61....	2	.....	.....	.....	V
4838.78....	2	.....	.....	.....	V	5268.52....	2	.....	.....	.....	V
4855.59....	15	4	2	tr	III	5353.60*...	3	?	?	1	II ?
4857.57....	2	1	.....	.....	IV	5371.60....	4	2	.....	.....	IV
4866.47....	10	3	2	.....	III	5411.41....	4	.....	.....	.....	V
4871.01....	2	.....	.....	.....	V	5424.87....	4	5	3	2	II
4873.62....	4	2	1	.....	III	5436.08....	5	5	3	2	II
4904.61....	10	3	2	.....	III	5462.69....	4	.....	.....	.....	V
4912.19....	2	.....	.....	.....	V	5477.12....	50	20	15	15	I
4914.10....	3	1	tr	.....	III	5495.15....	2	.....	.....	.....	V
4918.52....	4	2	1	.....	III	5510.20†...	4	1 ?	.....	.....	IV ?
4925.72....	2	1	tr	.....	III	5553.93†...	2	1 ?	.....	.....	IV ?
4936.00....	4	1	.....	.....	IV	5578.94....	5	5	5	4	I
4937.45....	4	4	2	.....	III	5588.09....	5	4	3	3	I
4945.62....	2	tr	.....	.....	IV	5589.58†...	2	1 ?	.....	.....	IV ?
4953.38*...	3	.....	.....	.....	V ?	5592.49....	8	4	3	2	II
4971.50....	2	.....	.....	.....	V	5593.99....	4	2	2	.....	III
4980.34....	12	3	2	.....	III	5615.01....	5	.....	.....	.....	V
4984.31....	10	2	1	.....	III	5625.52....	4	.....	.....	.....	V
4998.40....	2	1	tr	.....	III	5637.33....	2	.....	.....	.....	V
5000.51....	4	1	1	.....	III	5649.89....	2	.....	.....	.....	V
5012.64....	2	1	tr	.....	III	5664.23....	3	.....	.....	.....	V
5017.73†...	10	?	1 ?	.....	III ?	5682.42....	8	.....	.....	.....	V
5018.48....	3	tr	.....	.....	IV	5695.19†...	6	1 ?	.....	.....	IV ?
5035.52....	12	2	1	.....	III	5709.76....	12	12	10	10	I
5042.37....	4	.....	.....	.....	V	5712.10....	5	5	4	3	II
5049.00....	4	.....	.....	.....	V	5715.29....	6	.....	.....	.....	V
5080.70....	30	1	1	.....	III	5748.58....	2	2	2	1	II
5081.29....	25	1	1	.....	III	5754.89....	10	8	7	4	II
5084.20....	15	1	1	.....	III	5761.03....	4	1	.....	.....	IV
5097.07....	2	.....	.....	.....	V	5805.40....	5	.....	.....	.....	V
5099.51....	5	.....	.....	.....	V	5831.82....	2	.....	.....	.....	V
5100.12....	10	.....	.....	.....	V	5858.00†...	7	1 ?	.....	.....	IV ?
5115.57....	8	.....	.....	.....	V	5893.11....	12	8	6	2	II
5125.38†...	4	?	?	.....	IV ?	5997.02....	3n	.....	.....	.....	V
5129.55....	5	.....	.....	.....	V	5997.80....	2n	.....	.....	.....	V
5131.93....	3	.....	.....	.....	V	6007.54....	3	3	2	1	II
5137.23....	8	6	5	5	I	6053.91....	2	.....	.....	.....	V
5139.43....	3	.....	.....	.....	V	6086.53....	5n	.....	.....	.....	V
5142.91....	10	.....	.....	.....	V	6108.36....	8	8	5	4	II
5146.61....	12	.....	.....	.....	V	6111.22....	2n	.....	.....	.....	V
5155.31....	4	.....	.....	.....	V	6116.35....	6n	.....	.....	.....	V
5155.90....	9	.....	.....	.....	V	6163.60....	5n	.....	.....	.....	V
5168.81....	6	1	tr	.....	III	6175.69....	8	.....	.....	.....	V

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
6177.00....	12	.....	.....	.....	V	6366.61....	4	tr	.....	.....	IV
6187.00....	2	.....	.....	.....	V	6378.40†...	5	1?	.....	.....	IV?
6191.48....	12	12	10	8	I	6483.08....	5	.....	.....	.....	V
6204.78....	2	2	1	.....	III	6586.52....	6	6	4	2	II
6224.18....	3	.....	.....	.....	V	6598.74....	3	tr	.....	.....	IV
6230.33....	2	.....	.....	.....	V	6635.32....	3	.....	.....	.....	V
6256.60....	15	15	12	10	I	6643.89....	20	20	20	20	I
6258.87....	2	.....	.....	.....	V	6767.99*...	20	15	12	10	I
6259.79....	2	.....	.....	.....	V	6772.55*...	5	.....	.....	.....	V
6314.89....	15	12	8	6	II	6914.83*...	3	3	3	3	I
6327.79†...	5	6?	4?	2?	II?	7122.54*...	5	.....	.....	.....	V
6339.40....	7	1	.....	.....	IV						

REMARKS ON TABLE II

λ	
3233.28	May be concealed at high temperature by λ 3233.06.
3674.30	Close doublet. Resolved at low temperature.
4164.80	Measured as λ 4164.70 in furnace spectrum. May be due to impurity.
4200.60 4201.89	Very unusual type of Class I lines.
4953.38	Furnace line probably due to Co.
5353.60	Blend with Co at high and medium temperatures.
6677.99 to 7122.54	Photographed with 1-meter concave grating.

the cooler vapor near the ends of the tube would be very ineffective in recording its spectrum during the brief exposure at high temperature.

Class II.—This class includes a large proportion of the stronger arc lines. In the region of shorter wave-length, wide reversals of these lines are frequent, in some cases the reversal persisting even at low temperature. The reversals are usually much wider in the furnace than in the arc at moderate current, and the appearance of the lines from the two sources is very different. The estimates of relative intensity are made independently for arc and furnace, the scales being adjusted so that the stronger lines are given the same intensity in the arc and in the high-temperature furnace. Lines of Class II usually remain strong at medium temperature in these spectra, but weaken at low temperature more than the lines of Class I, the distinction between the two classes being often based on this feature.

Class III.—These lines, whose characteristic is an initial appearance at medium temperature, form the most numerous class in the cobalt and nickel spectra. A large part of them show but slight change of intensity between the arc and the high and medium furnace temperatures, but some interesting exceptions appear, especially in the nickel spectrum. λλ 5080.70, 5081.29, 5084.20 are examples of very strong arc lines which appear only faintly in the furnace, but show at both high and medium temperature.

Classes IV and V.—These high-temperature lines, which are faint or absent in the furnace, are much more common in the nickel than in the cobalt spectrum. This is due partly to the frequent occurrence of the nebulous type among the nickel lines, but a large proportion of other strong arc lines, especially in the visible region, have not appeared in the furnace spectrum. In the cobalt spectrum, the lines in Classes IV and V are usually among the weaker arc lines which, however, are not given by furnace temperatures which show other lines as weak as these in the arc.

Lines relatively weak in the arc spectrum.—The number of lines, designated by "A" after the class number, for the production of which the arc appears to be less favorable than the furnace, is quite different for the cobalt and nickel spectra, the numbers being 40 and 6, or 5 per cent and 1½ per cent, respectively, of the whole number of lines listed. This is in harmony with the greater relative richness of the cobalt spectrum in the furnace as compared with the arc, the proportion of lines in Classes IV and V being much smaller than for nickel.

DISTRIBUTION OF CLASSES ACCORDING TO WAVE-LENGTH

As the detailed examination of the cobalt and nickel spectra covers 4000 A, it seemed of interest to see how the classes are divided within successive equal intervals, as of 500 A, throughout this range. This is shown in Table III, which gives the percentage of each class of the total number of lines within the given 500 A.

TABLE III

Division of Lines in Each 500 A among Furnace Classes
(The figures give percentage belonging to each class of the total number of lines within 500 A)

Class	Cobalt					Nickel				
	I	II	III	IV	V	I	II	III	IV	V
λ										
3000–3500	7	28	51	13	1	10	46	7	10	25
3500–4000	23	34	36	3	4	25	40	7	1	27
4000–4500	22	9	46	9	13	7	13	17	3	60
4500–5000	13	10	60	15	2	0	11	49	9	30
5000–5500	2	9	28	27	34	5	8	20	10	58
5500–6000	4	26	44	0	26	12	19	4	23	42
6000–6500	18	6	24	14	38	9	17	4	13	56
6500–7000	43	21	14	0	21	33	11	0	11	44

The columns of Table III do not show a regular change in the percentages of any class as we pass along the spectrum. In each spectrum there is a grouping of Class I lines in the extreme red, while at the other end we find a large proportion of the lines from λ 3000 to λ 4000 belonging to the low-temperature Classes I and II, the lines of Classes IV and V being in a decided minority at the violet end, especially for cobalt. This is in harmony with the feature noted for other elements, that the furnace spectrum, at least for the medium and high temperatures, is relatively rich in the region of shorter wave-length as compared with the arc. The greater part of the lines which require arc conditions occur farther toward the red. There is a definite tendency in the furnace, as in the spectra of other light-sources, for lines exhibiting a similar behavior to group in certain regions, this probably resulting from series relations which have not as yet been worked out, and the occasional preponderance of a single class in a given region may be a consequence of such groupings.

OCCURRENCE OF ENHANCED LINES IN THE FURNACE SPECTRUM

The behavior in the furnace spectrum of a number of lines listed by Lockyer[1] as enhanced seems to render it questionable whether these are to be considered as of this class, or at least as a pronounced type of enhanced lines. The list follows with the furnace class of each line taken from Table I:

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[1] *Tables of Wave-Lengths of Enhanced Lines*, Solar Physics Committee, 1906.

λ	Class	λ	Class
3817.01	II	3947.26	II
3843.85	III	3961.15	II
3851.09	I A	3977.34	III
3852.00	III	3987.25	I
3863.75	III	4014.09	II
3870.66	III	4023.55	III
3878.90	...	4077.56	III V (double)
3898.54	III	4145.31	...
3904.23	V	4160.86	...
3925.33	III	4244.42	...
3929.43	III	4414.09	...
3946.75	II	4569.48	...

In the range from λ 3800 to λ 4100, the distribution of these lines as to furnace class is about what would be expected of the same number of lines taken at random, two lines of Class I even being found among them; while previous investigations have shown that typical enhanced lines, for which the difference between arc and spark intensity is large, are among the most difficult lines to obtain in the furnace. Exner and Haschek give most of these cobalt lines, as far as λ 4100, as of the same intensity in arc and spark. From the evidence at hand it seems probable that they are not enhanced lines of the regular type, but may be similar to a group of iron lines discussed in a former paper,[1] which appear in the furnace and in the spectra of flames whose temperature does not seem to be very high.

Among the enhanced lines of nickel as given by Lockyer, only λ 3889.84 appears in the furnace, this being a Class II line, of moderate strength in the arc spectrum.

EXPLANATION OF PLATE III

In this plate, the cobalt spectrum from λ 3356 to λ 3720 is shown in two sections for the arc and for three furnace temperatures. Leading features are the variations in relative intensity for lines

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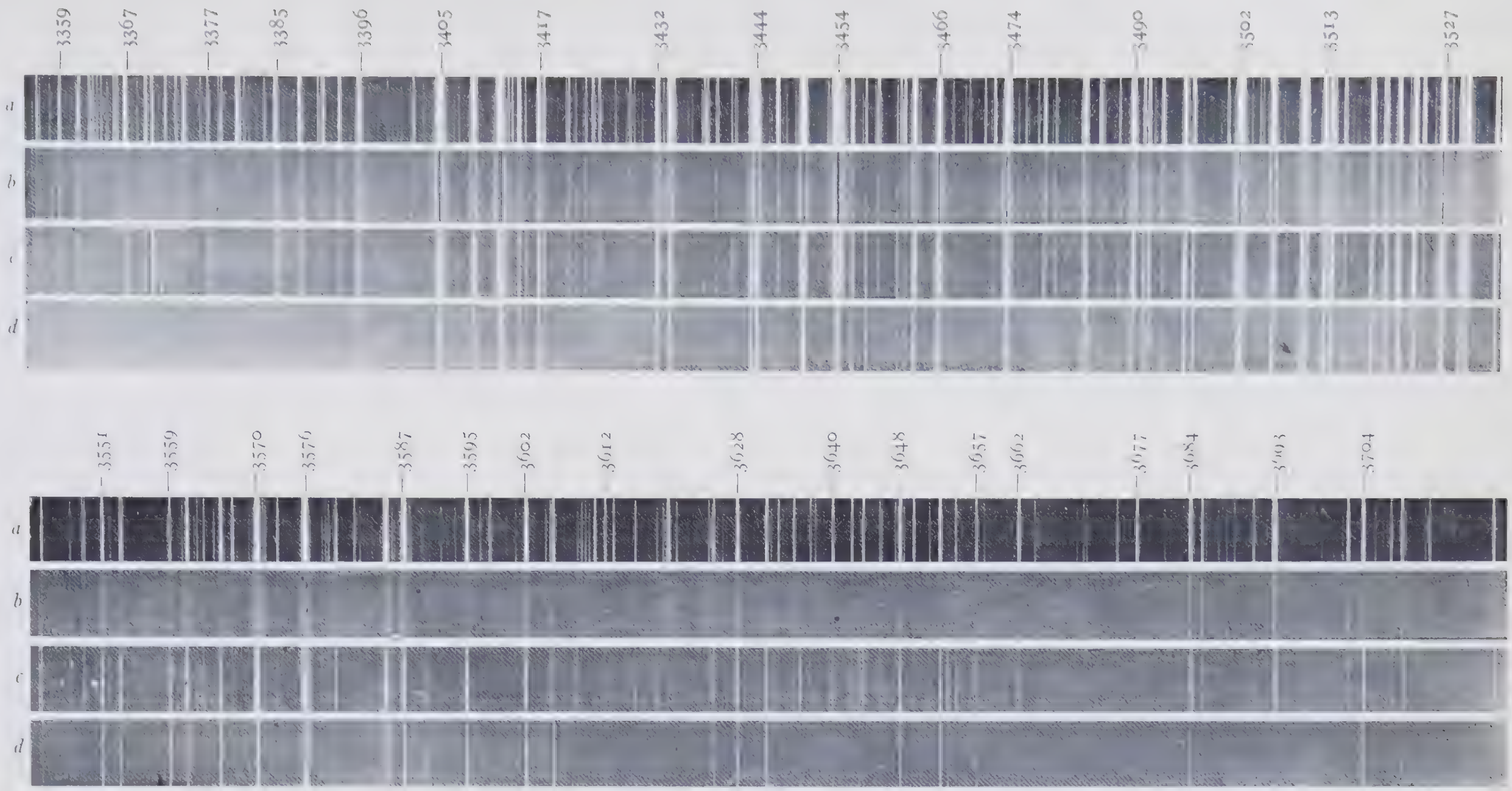
[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

PLATE III



Furnace and Arc Spectra of Cobalt

a. Arc spectrum of cobalt.
b. Spectrum of the electric furnace at 2600° C.
c. Spectrum of the electric furnace at 2300° C.
d. Spectrum of the electric furnace at 2100° C.

of different classes, the numerous wide reversals in the high-temperature spectrum, and the general richness of the low-temperature spectrum in this region.

SUMMARY

1. The furnace spectra of cobalt and nickel have been examined from λ 3000 to λ 7100 with regard to the temperature at which a given line appears and its rate of increase in intensity as the temperature rises. The classification of lines on this basis includes 840 lines in the cobalt spectrum and 423 in that of nickel.

2. The leading features of the various furnace classes are discussed, these being in the main similar to those observed in the furnace spectra of other elements.

3. The number of lines relatively fainter in the arc than in the furnace is larger for cobalt than for nickel; while the nickel spectrum shows a large proportion of lines, many of them of nebulous type, which require the arc conditions to give them strongly.

4. An examination of the distribution of furnace classes through the spectrum shows a relative richness of the furnace spectrum at the violet end, and a tendency for lines of similar character to group in certain regions.

5. A number of lines, especially of cobalt, which have been classified as enhanced appear in the furnace spectra, thus indicating that they may not be enhanced lines of pronounced type.

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RESEARCHES ON SOLAR VORTICES

By CARL STÖRMER[1]

I. INTRODUCTION

Professor Hale, as is well known, has done fundamental work on sun-spots in discovering the Zeeman effect in their spectra, thus proving the existence of very strong magnetic fields in the spots and in the surrounding whirl revealed by the spectroheliograms. In an attempt to employ mathematical methods in the investigation of these phenomena, I spent some months of the summer of 1912 at the Mount Wilson Solar Observatory.

The detailed study of the collection of spectroheliograms made at the Observatory suggested to me an application of the classical researches on terrestrial cyclones made in 1876 by my countrymen Guldberg and Mohn.[2] The hydrogen flocculi seemed to be arranged around a single spot in curved paths that were very similar to logarithmic spirals; and just the same curves were found by Guldberg and Mohn as trajectories of the air particles in the outer part of cyclones. It therefore seemed advisable to start with the hypothesis that the motion of charged electric gas molecules takes place along such spirals around the sun-spot center, and compute the resulting magnetic field.

This investigation which I made in Pasadena showed that the lines of magnetic force due to a plane whirl of the kind mentioned above are space-curves whose projection on the plane of the whirl are also logarithmic spirals intersecting the first at right angles.

This result led me to the idea first advanced by Brester[3] in 1909, that the hydrogen flocculi resemble terrestrial auroras, thus implying that the visible whirls are not real current-lines, but lines of magnetic force due to a magnetic field at a lower level—a view expressed by Deslandres[1] in 1910. Professor Hale had previously stated in his papers[2] that the Zeeman effect was probably due to a low-level vortex. In accordance with the mathematical result mentioned above, the current-lines of this hypothetical whirl at a low level would be logarithmic spirals tending more and more to become circles according as the structure of the hydrogen whirl tended to be radial. This would much better explain the strength of the Zeeman effect of sun-spots of radial structure.

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[1] Research associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[2] "Etudes sur les mouvements de l'atmosphère, Première partie"; *Programme de l'Université pour le 2^re semestre*, 1876, Christiania, 1876; and *Meteorologische Zeitschrift*, XII. Band, 1877, No. 14.

[3] "The Solar Vortices of Hale," *Proceedings of the Amsterdam Academy of Sciences*, January 30, 1909.

In the following pages is given a detailed account of my researches on the mathematical theory of solar vortices.

II. DEFINITION OF THE WHIRL

Let us first consider a logarithmic spiral in the XY-plane of a system of rectangular co-ordinates in space. If we use polar co-ordinates, R and ϕ, in that plane (see Fig. 1), the equation of the spiral is

$$R=Ae^{\phi \tan \omega}$$

where A and ω are constants. A only fixes the position of the spiral; for if we introduce

$$\phi=-\cot \omega \, ln \, A$$

we get

$$A=e^{-\phi_0 \tan \omega}$$

and consequently

$$R=e^{(\phi-\phi_0) \tan \omega},$$

which is the spiral

$$R=e^{\phi \tan \omega},$$

turned through the angle ϕ_0 around the Z-axis.

The other constant, ω, determines the shape of the spiral. In order to get all possible values of $\tan \omega$ between $-\infty$ and $+\infty$, we can suppose ω to be within the interval $-\pi/2$ to $+\pi/2$.

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[1] *Comptes Rendus*, January 10, 1910. See also G. Hale, "Notes on Solar Magnetic Fields and Related Phenomena," *Publications of the Astronomical Society of the Pacific*, **22**, 71, 1910.

[2] *Annual Report of the Solar Observatory of the Carnegie Institution of Washington*, 1909, p. 166 (24).

On the other hand, ω is the constant angle between the tangent MT and the normal MN on the radius vector, both reckoned in the direction of the increasing angle ϕ. ω is negative if MT lies between MN and the origin, positive if MT lies on the other side. If ω is negative, R will decrease with increasing ϕ, and we approach nearer and nearer to the origin in following the spiral. If ω is positive, R will increase with increasing ϕ, so that we recede from the origin in following the spiral in the direction of increasing ϕ.

In Fig. 3 are shown various types of spirals corresponding to various values of ω. Each separate figure contains a series of congruent spirals seen from the positive side of the Z-axis, with the origin in the center; the corresponding value of ω is written above the figure. $\omega=0$ corresponds to circles concentric around the origin. If ω approaches $-\pi/2$ or $+\pi/2$ the spirals tend to become straight lines through the origin.

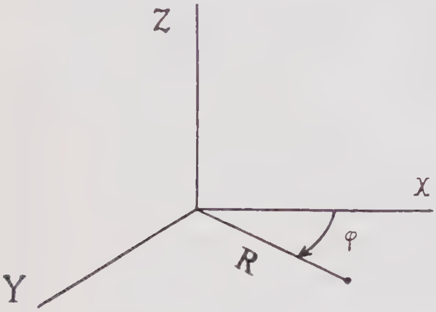


FIG. 1

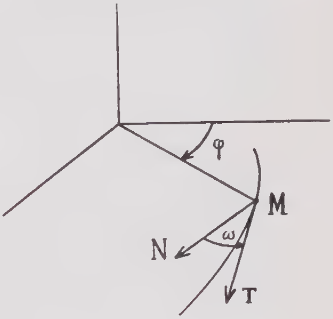


FIG. 2

After this review of the well-known properties of logarithmic spirals, we will consider an ideal vortex defined as follows:

The whirl shall be bounded by two cylinders of revolution with the Z-axis as axis and with radii equal to ρ_1 and ρ_2, by the XY-plane, and by a parallel plane at the distance h above it. We consider h small as compared with ρ_2 in order to have a flat whirl. The motion of the gas in the space mentioned is supposed to be permanent, and the trajectories of the gas particles are supposed to be logarithmic spirals parallel to the XY-plane, whose projections on that plane are

$$R=Ae^{\phi \tan \omega}$$

where ω is the same for all particles, and where A is determined by the position of the spiral. Let us suppose further that the velocity and electric charge in unit volume are functions of the distance from the Z-axis only.

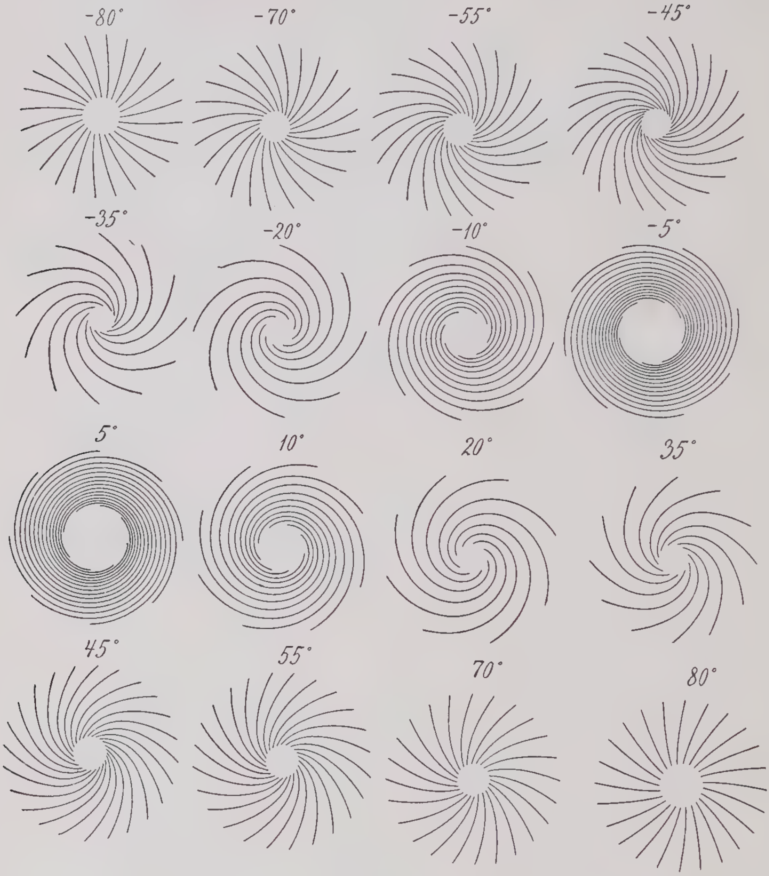


FIG. 3

It is clear that such a whirl does not exist in reality; there must be a source and a sink for the coming and going masses of gas in the whirl, but in the first approximation we will neglect the electric action of masses outside this hypothetical whirl; their action will be calculated later, when we have discussed the formulae for this first simple case.

To fix the motion without ambiguity, we will call the velocity v positive if the motion corresponds to increasing ϕ, negative if it corresponds to decreasing ϕ. Thus the motion seen from the positive side of the Z-axis will be clockwise for positive v, counterclockwise for negative v; and the gas will approach the Z-axis if the product ωv is negative, and recede from the Z-axis if it is positive, as is seen in Fig. 4.

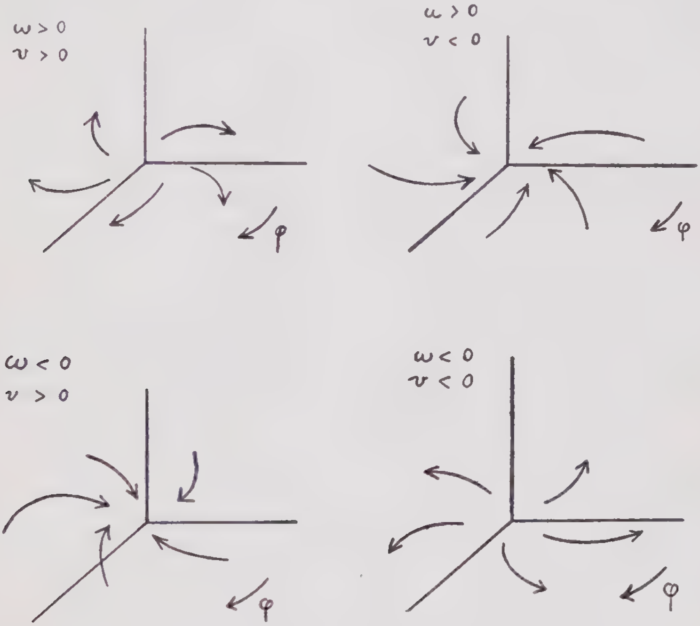


FIG. 4

Let us now calculate the magnetic action of this hypothetical whirl on a unit of north magnetism placed at a point outside the whirl. For the sake of brevity we first suppose ω, v, and the electric charge to be positive. In order to have a solenoid element bounded by current-lines, let us consider an element of the whirl, bounded by the following surfaces:

The two planes

$$z=\zeta\,, \qquad z=\zeta+\Delta\zeta\,;$$

the two cylinders

$$R=\rho\,, \qquad R=\rho+\Delta\rho\,;$$

and the two cylinders

$$R = Ae^{\phi \tan \omega}, \qquad R = Ae^{(\phi+\Delta\theta) \tan \omega},$$

where $\Delta\zeta$, $\Delta\rho$, and $\Delta\theta$ are positive infinitesimal quantities, and $\Delta\theta$ and $\Delta\zeta$ infinitesimal compared with $\Delta\rho$.

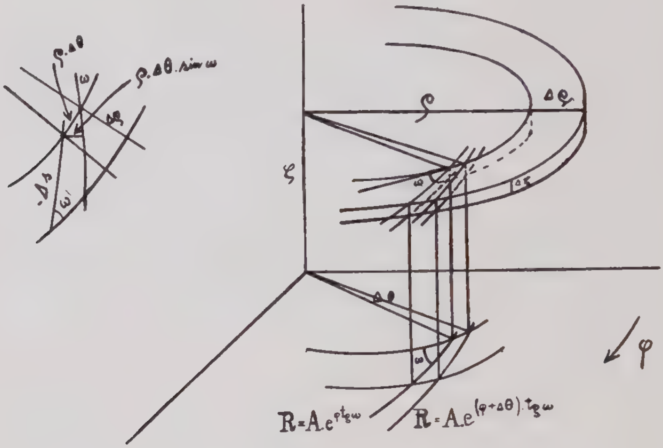


FIG. 5

Let us make use of the c.g.s. system of units, and the *electrostatic* system for the electric charge. The cross-section of our element will be

$$\rho\Delta\theta \sin \omega \, \Delta\zeta \, .$$

If ε is the electric charge in a unit volume of the gas, there will pass as a convection current through this section in one second

$$\varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta \text{ electrostatic units,}$$

and consequently

$$\frac{1}{3\times 10^{10}} \varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta \text{ electromagnetic units.}$$

If by i we denote the current in amperes passing through the element, we have

$$i = \frac{1}{3\times 10^{9}} \varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta \, .$$

On the other hand, the length of the element is

$$\Delta s = \frac{\Delta\rho}{\sin\omega}\ .$$

Now, as is well known, the magnetic action of this current-element on one unit of north magnetism situated at the point M with co-ordinates x, y, and z will have the following components parallel with X-, Y-, and Z-axes:

$$p_x = \frac{1}{10}\, i\, \frac{\Delta s}{r_1^2}\,(\cos\beta_1 \cos\gamma_2 - \cos\gamma_1 \cos\beta_2)\ ,$$

$$p_y = \frac{1}{10}\, i\, \frac{\Delta s}{r_1^2}\,(\cos\gamma_1 \cos\alpha_2 - \cos\alpha_1 \cos\gamma_2)\ ,$$

$$p_z = \frac{1}{10}\, i\, \frac{\Delta s}{r_1^2}\,(\cos\alpha_1 \cos\beta_2 - \cos\beta_1 \cos\alpha_2)\ ,$$

where r_1 is the distance from the element to the point M; α_1, β_1, γ_1, the angles between the current-element and the positive directions of the X-, Y-, and Z-axes; and α_2, β_2, γ_2, the angles between the direction from (M) to the current-element and the same three axes.

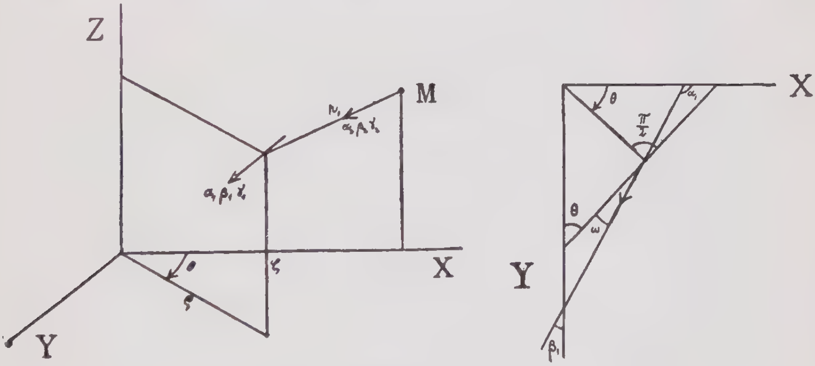


FIG. 6

Now choose the point M in the XZ-plane (Fig. 6) so that:

$$y = 0\ .$$

The co-ordinates of the current-element are

$$\rho\cos\theta, \quad \rho\sin\theta, \quad \zeta\ ,$$

and consequently

$$r_1^2=(x-\rho\cos\theta)^2+(\rho\sin\theta)^2+(z-\zeta)^2$$

and

$$\cos\alpha_2=\frac{\rho\cos\theta-x}{r_1}\,,\qquad \cos\beta_2=\frac{\rho\sin\theta}{r_1}\,,\qquad \cos\gamma_2=\frac{\zeta-z}{r_1}\,.$$

On the other hand, we have

$$\alpha_1=\frac{\pi}{2}-\omega+\theta\,,\qquad \beta_1=\theta-\omega\,,\qquad \gamma_1=\frac{\pi}{2}\,,$$

which give

$$\cos\alpha_1=\sin(\omega-\theta)\,,\qquad \cos\beta_1=\cos(\omega-\theta)\,,\qquad \cos\gamma_1=0\,.$$

If we substitute the values of the angles α and β, we get

$$p_x=-\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\cos(\omega-\theta)$$

$$p_y=\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\sin(\omega-\theta)$$

$$p_z=\frac{1}{10}\,i\,\frac{\Delta s}{r_1^3}\left\{x\cos(\omega-\theta)-\rho\cos\omega\right\}.$$

These formulae are valid for ε, v, and ω positive. But if we change the sign of ε or of v, the magnetic action will change direction, that is, p_x, p_y, and p_z will change their signs. But this will also be the case with i and consequently the formulae are valid for both signs of the quantities v and ε. In an analogous manner we see that they are valid for all values of the angle ω.

The components of action of an element situated symmetrically with respect to the XZ-plane are obtained by changing θ into $-\theta$; we find for these components

$$p'_x=-\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\cos(\omega+\theta)\,,$$

$$p'_y=\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\sin(\omega+\theta)\,,$$

$$p'_z=\frac{1}{10}\,i\,\frac{\Delta s}{r_1^3}\left\{x\cos(\omega+\theta)-\rho\cos\omega\right\}.$$

If we introduce the values of i and of Δs and add, we get

$$p_x+p'_x=-\frac{2\cos\omega}{3\times 10^{10}}\cdot(z-\zeta)\,\varepsilon\rho v\cos\theta\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,,$$
$$p_y+p'_y=\frac{2\sin\omega}{3\times 10^{10}}\cdot(z-\zeta)\,\varepsilon\rho v\cos\theta\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,,$$
$$p_z+p'_z=\frac{2\cos\omega}{3\times 10^{10}}\cdot(x\cos\theta-\rho)\,\varepsilon\rho v\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,.$$

From these the action of the part of the whirl bounded by the two planes

$$z=\zeta, \qquad z=\zeta+\Delta\zeta\,,$$

and the two circular cylinders

$$R=\rho, \qquad R=\rho+\Delta\rho$$

is obtained by an integration with respect to θ, which gives the components

$$-\frac{2\cos\omega}{3\times 10^{10}}(z-\zeta)\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\,,$$
$$\frac{2\sin\omega}{3\times 10^{10}}(z-\zeta)\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\,,$$
$$\frac{2\cos\omega}{3\times 10^{10}}\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{x\cos\theta-\rho}{r_1^3}\,d\theta\,.$$

Here ε and v are independent of the angle θ according to the supposition mentioned above that they are functions of ρ alone.

The action of a ring bounded by the same two planes and the two cyclinders $R=\rho_1$ and $R=\rho_2$ is obtained by an integration with respect to ρ, which gives the components

$$-\frac{2\cos\omega}{3\times 10^{10}}(z-\zeta)\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\right)d\rho\,,$$
$$\frac{2\sin\omega}{3\times 10^{10}}(z-\zeta)\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\right)d\rho\,,$$
$$\frac{2\cos\omega}{3\times 10^{10}}\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{x\cos\theta-\rho}{r_1^3}\,d\theta\right)d\rho\,.$$

Here ε and v are functions of ρ only.

A final integration with respect to ζ gives the components of the action of the whirl itself:

$$-\frac{2\cos\omega}{3\times 10^{10}}\int_0^h (z-\zeta)\left[\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi \frac{\cos\theta\, d\theta}{r_1^3}\right)d\rho\right]d\zeta$$

with analogous expressions for the other components.

But here we suppose the product $\varepsilon\rho v$ to be a continuous function of ρ; and the point $(x,0,z)$ being outside the whirl, r_1 is positive over the entire field of integration. This field being also finite, the succession of the integrations is indifferent, so that we can write more simply

$$-\frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi (z-\zeta)\,\varepsilon\rho v\,\frac{\cos\theta}{r_1^3}\,d\theta d\rho d\zeta.$$

On the other hand, as everything is symmetrical around the Z-axis, we get the components in an arbitrary point (x,y,z) by substituting $R=\sqrt{x^2+y^2}$ for x; hence

$$\left.\begin{aligned}
H_R &= -\frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\\
H_\phi &= \frac{2\sin\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\\
H_z &= \frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(R\cos\theta-\rho)\,\varepsilon\rho v}{D^3}\,d\theta d\rho d\zeta
\end{aligned}\right\}\quad (1)$$

where

$$D^2=R^2-2R\rho\cos\theta+\rho^2+(z-\zeta)^2.$$

H_R is the component normal to the Z-axis, H_z the component parallel to that axis, and H_ϕ the component normal to H_R and H_ϕ. As regards the signs, H_R is positive in the direction from the

Z-axis, H_z is positive in the direction of increasing z, and H_ϕ positive in the direction of increasing ϕ (see Fig. 7).

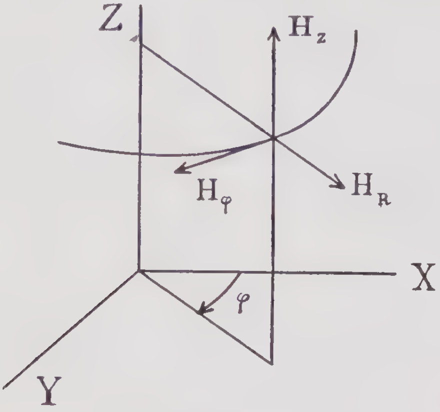


FIG. 7

III. PROPERTIES OF THE COMPONENTS OF THE MAGNETIC ACTION. EQUATIONS OF THE LINES OF MAGNETIC FORCE IN SPACE

Let us, in the formula for H_R, H_ϕ, and H_z, introduce the function

$$\Phi=\frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi\frac{\varepsilon\rho v\cos\theta}{D}\,d\theta d\rho d\zeta \qquad (2)$$

As the point (x,y,z) is outside the field of integration, and consequently $D>0$, we obtain the derivatives $\frac{\delta\Phi}{\delta R}$ and $\frac{\delta\Phi}{\delta z}$ as follows:

$$\frac{\delta\Phi}{\delta R}=\frac{2}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left[\frac{\delta}{\delta R}\int_0^\pi\frac{R\cos\theta}{D}\,d\theta\right]d\rho d\zeta\,,$$

$$\frac{\delta\Phi}{\delta z}=\frac{2}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left[\frac{\delta}{\delta z}\int_0^\pi\frac{R\cos\theta}{D}\,d\theta\right]d\rho d\zeta\,.$$

But

$$\frac{\delta}{\delta R}\int_0^\pi\frac{R\cos\theta}{D}d\theta=\int_0^\pi\frac{\cos\theta\,d\theta}{D}+R\int_0^\pi\frac{(\rho-R\cos\theta)\,d\theta}{D^3}-\int_0^\pi\frac{R\rho\sin^2\theta\,d\theta}{D^3}\,.$$

Integrating by parts, the last integral in the right member becomes equal to the first and we get

$$\frac{\delta}{\delta R}\int_0^\pi \frac{R\cos\theta}{D}\,d\theta = R\int_0^\pi \frac{\rho - R\cos\theta}{D^3}\,d\theta\,.$$

Further

$$\frac{\delta}{\delta z}\int_0^\pi \frac{R\cos\theta}{D}\,d\theta = -R\int_0^\pi \frac{(z-\zeta)\cos\theta\,d\theta}{D^3}\,.$$

This gives us

$$\frac{\delta\Phi}{\delta R} = \frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(\rho - R\cos\theta)\,\varepsilon\rho v}{D^3}\,d\theta d\rho d\zeta\,,$$

$$\frac{\delta\Phi}{\delta z} = -\frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\,,$$

and we can therefore write

$$\left.\begin{aligned} H_R &= \frac{\cos\omega}{R}\,\frac{\delta\Phi}{\delta z} \\ H_\phi &= -\frac{\sin\omega}{R}\,\frac{\delta\Phi}{\delta z} \\ H_z &= -\frac{\cos\omega}{R}\,\frac{\delta\Phi}{\delta R} \end{aligned}\right\} \qquad (3)$$

From this it is easy to find the lines of magnetic force in space outside the whirl. These are the integral curves of the system

$$\frac{dx}{H_x} = \frac{dy}{H_y} = \frac{dz}{H_z}\,,$$

where H_x, H_y, and H_z are the components of the magnetic force, parallel to the X-, Y-, and Z-axes respectively.

But we have

$$H_R = \frac{x}{R}\,H_x + \frac{y}{R}\,H_y\,,$$

$$H_\phi = \frac{x}{R}\,H_y - \frac{y}{R}\,H_x\,,$$

and if we take R and ϕ as variables instead of x and y, we have

$$xdx+ydy=RdR\,,$$
$$xdy-ydx=R^2d\phi\,.$$

Hence

$$\frac{dx}{H_x}=\frac{dy}{H_y}=\frac{xdx+ydy}{xH_x+yH_y}=\frac{xdy-ydx}{xH_y-yH_x}=\frac{dz}{H_z}$$

and from the last three equations we get

$$\frac{dR}{H_R}=\frac{Rd\phi}{H_\phi}=\frac{dz}{H_z}\,,$$

which are the differential equations of the lines of force in the variables R, ϕ, and z.

Now if we substitute in the equation

$$\frac{dR}{H_R}=\frac{Rd\phi}{H_\phi}$$

the above-developed expressions for H_R and H_ϕ, we get

$$\frac{dR}{R}=-\cot\omega\; d\phi\,,$$

which, integrated, gives us

$$R=Be^{-\phi\cot\omega}\,, \tag{4}$$

B being a constant of integration.

On the other hand, if we substitute in the equations

$$\frac{dR}{H_R}=\frac{dz}{H_z}$$

the values of H_R and H_z, we get

$$\frac{\delta\Phi}{\delta R}\,dR+\frac{\delta\Phi}{\delta z}\,dz=0$$

and Φ being a function of R and z only, we can integrate and obtain

$$\Phi=C \tag{5}$$

where C is a constant.

From equations (4) and (5) we have the following fundamental theorem:

The lines of magnetic force due to the spiral whirl are curves in space lying on the surfaces of revolution $\Phi = C$. *Their projections on the plane of the whirl are logarithmic spirals cutting the current-lines of the whirl everywhere in right angles.*

IV. FIRST APPLICATION TO THE THEORY OF SOLAR VORTICES

Let us first consider the various cases that can occur for different signs of ω, v, and ε. In order to find the signs of the integrals, let us consider

$$\int_0^\pi \frac{\cos\theta\, d\theta}{D^3},$$

where

$$D^2 = R^2 - 2R\rho\cos\theta + \rho^2 + (z-\zeta)^2 .$$

This integral may be written

$$\int_0^\pi \frac{\cos\theta}{D^3}\, d\theta = \int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{D^3} + \int_{\frac{\pi}{2}}^{\pi} \frac{\cos\theta\, d\theta}{D^3} .$$

By taking $\theta_1 = \pi - \theta$ as variable in the last integral, we get

$$\int_0^\pi \frac{\cos\theta}{D^3}\, d\theta = \int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{D^3} - \int_0^{\frac{\pi}{2}} \frac{\cos\theta_1\, d\theta_1}{D_1^3},$$

where

$$D_1^2 = R^2 + 2R\rho\cos\theta_1 + \rho^2 + (z-\zeta)^2 .$$

But for $\theta = \theta_1$, and different from $\pi/2$, we have $D_1 > D$ and therefore the first integral will be greater than the second, that is,

$$\int_0^\pi \frac{\cos\theta\, d\theta}{D^3} > 0 .$$

On the other hand, ε and v have the same sign throughout the whirl, and if we are on the positive side of the XY-plane and above the whirl, $z-\zeta$ will be positive. Farther along the Z-axis, we have

$$H_z = H_{zo} = -\frac{2 \cos \omega}{3 \times 10^{10}} \int_0^h \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{\varepsilon \rho^2 v}{D^3} \, d\theta d\rho d\zeta \, .$$

Hence, from this equation and from equation (1) we conclude that:

H_R and H_{zo} will have the opposite sign to εv, and H_ϕ the same sign as $\varepsilon \omega v$.

We will apply this to solar vortices around sun-spots, assuming that the hydrogen flocculi represent lines of magnetic force due to a whirl in a lower level of the sun's atmosphere. We will choose the Z-axis along a solar radius, with its positive direction pointing away from the sun's center and toward the observer. For different signs of ε, ω, and v, we have the 8 combinations shown in Table I, with corresponding signs for the components H_R, H_ρ, and H_{zo}.

TABLE I

	ε	ω	v	H_R	H_ϕ	H_{zo}
1	+	+	+	−	+	−
2	+	+	−	+	−	+
3	+	−	+	−	−	−
4	+	−	−	+	+	+
5	−	+	+	+	−	+
6	−	+	−	−	+	−
7	−	−	+	+	+	+
8	−	−	−	−	−	−

This gives the diagrams shown in Fig. 8, where the whirl is seen from a distant point outside the sun. The current lines are plain curves with arrows in the direction of motion, and the projection of the lines of force situated between the whirl and the observer are dotted lines with arrows in the direction of the magnetic force. When these last arrows point to the center, H_{zo} is negative and the magnetic force along the Z-axis is directed away from the observer; when they point out, that force is directed toward the observer.

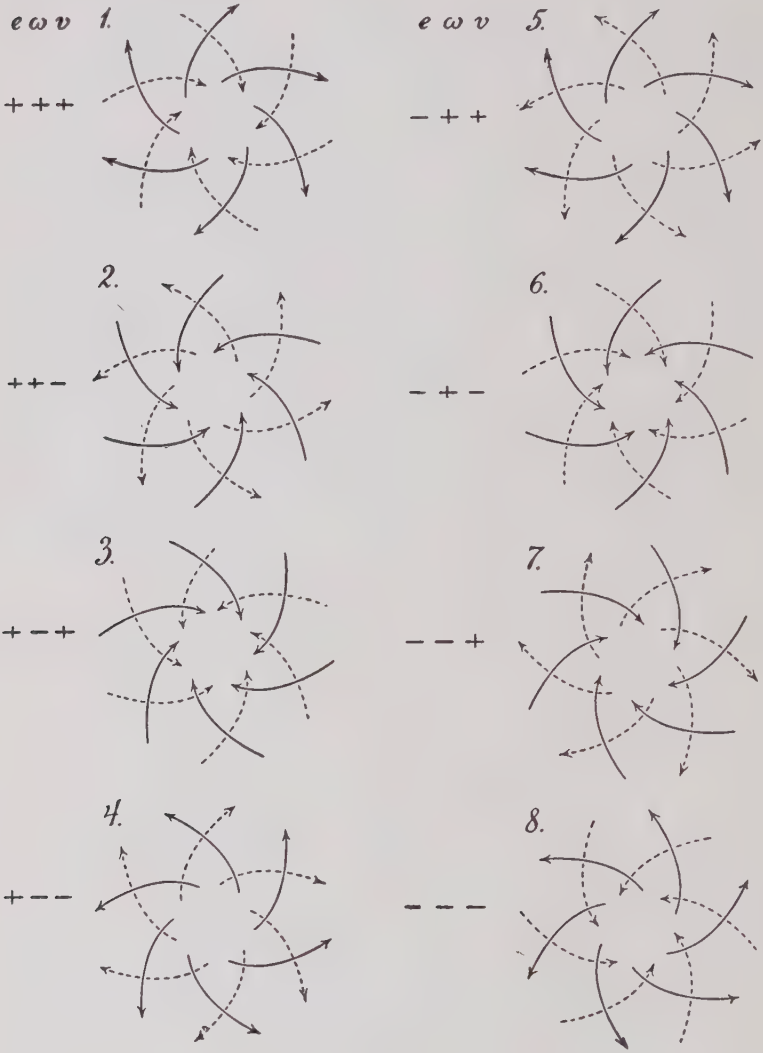
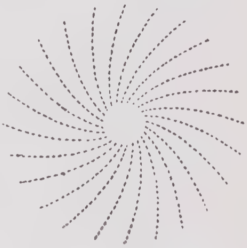


Fig. 8

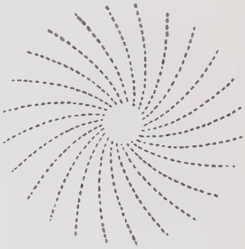
We now have to find which of these 8 cases corresponds with the observed facts. This can be done by combining Hale's classical researches on the Zeeman effect with the recent investigations on radial motion in sun-spots by St. John.[1]

In his paper "The Probable Existence of a Magnetic Field in Sun-Spots" Hale states the following facts: "The magnetic force in the center of the whirl is directed toward the observer, when the direction of motion indicated by the hydrogen flocculi whirl is clockwise, and directed away from the observer, when the direction of motion is counter-clockwise."[2]

The meaning of the definition of clockwise and counter-clockwise motion will be seen[3] in Fig. 9. There is here no question about direction of motion along the current-lines; the definition is purely geometrical. According to this, cases Nos. 1, 4, 6, and 7 must be rejected. On the other hand, through the recent researches of St. John,[4] the existence of a whirl at a lower level giving the Zeeman effect and causing the observed arrangement of the hydrogen flocculi at the higher levels is rendered extremely probable. In this low-level whirl, probably situated at or below the levels oo and ooo in the figure in St. John's paper, St. John finds a motion outward from the axis of the whirl. This excludes cases Nos. 2 and 3, and thus only cases Nos. 5 and 8 remain as corresponding to observed facts. In both cases ε is negative, that is to say, we come to the same conclusion as Professor Hale: *The Zeeman effect is due to a whirl of negatively charged electric particles.*



Clockwise



Counter-clockwise

FIG. 9

This whirl, however, is distinct from the observed hydrogen whirl, and is situated in a much lower level of the sun's atmosphere.

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[1] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

[2] *Mt. Wilson Contr.*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[3] *Op. cit.*

[4] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

The sign of electricity in the whirl being determined, we have to choose the probable value of the angle ω, which determines the form of the current-lines. This can be done by comparing Fig. 10 with a series of photographs of hydrogen flocculi whirls taken at Mount Wilson. It will thus be seen that values of ω lying between 15° and 25°, or between -15° and -25°, agree well with the reality; the dotted lines must then be identified with the hydrogen flocculi. The current-lines and projection of lines of magnetic force (hydrogen

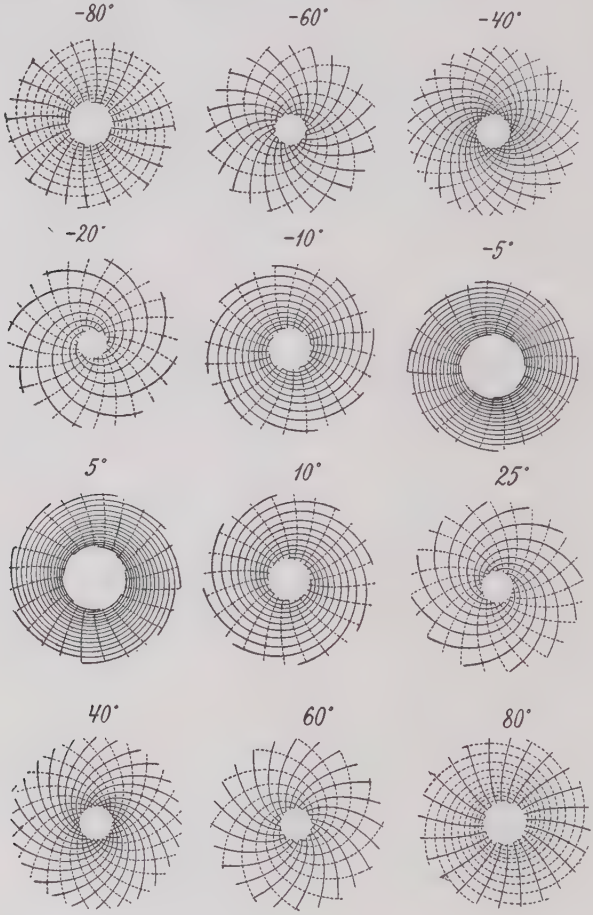


Fig. 10

flocculi) for the values $\omega = 20^\circ$ and $\omega = -20^\circ$ are shown on a larger scale in Fig. 11. This agrees well with the strong magnetic effect shown by solar vortices where the structure of the hydrogen flocculi is nearly radial; for the more nearly radial this structure is, the more nearly circular are the current-lines of the whirl.

In order to get some idea regarding the amount of electricity in motion in the whirl, more quantitative calculations are necessary.

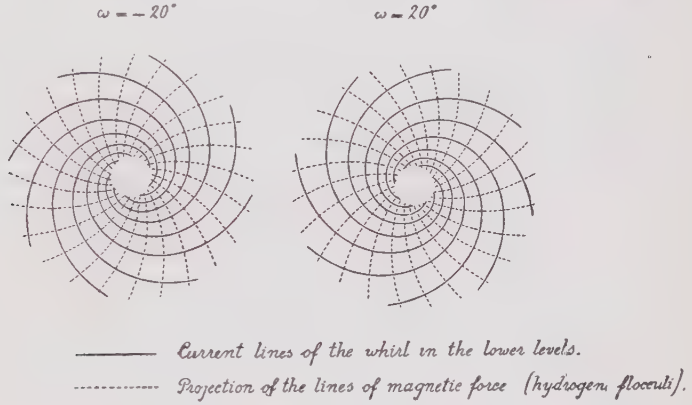

ω = −20°
ω = 20°
Current lines of the whirl in the lower levels.
Projection of the lines of magnetic force (hydrogen flocculi).

FIG. 11

For the sake of simplicity we will put forward another hypothesis concerning ε and v as functions of ρ, as will be seen in the next paragraph.

V. HYPOTHESIS REGARDING THE PRODUCT $\varepsilon\rho v$ AND THE CORRESPONDING FORMULAE FOR NUMERICAL CALCULATIONS

We will now, for the sake of simplicity, make the following assumptions:

1. The density of the gas in the whirl shall be constant throughout the whirl, and the charge ε proportional to the density; that is, ε is supposed to be a constant.

2. The velocity v shall be *inversely proportional* to the distance ρ from the axis. This hypothesis, as already known, was made by Guldberg and Mohn for the *outer* part of terrestrial cyclones. Thus $\varepsilon\rho v$ will be constant throughout the whirl. Let us put

$$\frac{\varepsilon\rho v}{3\times10^{10}} = c\ .$$

Hence

$$\left.\begin{aligned} H_R &= -2c\cos\omega \int_0^h \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{(z-\zeta)\cos\theta}{D^3}\,d\theta d\rho d\zeta \\ H_\phi &= \quad 2c\sin\omega \int_0^h \int_{1}^{\rho_2} \int_0^\pi \frac{(z-\zeta)\cos\theta}{D^3}\,d\theta d\rho d\zeta \\ H_z &= \quad 2c\cos\omega \int_0^h \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{R\cos\theta-\rho}{D^3}\,d\theta d\rho d\zeta \end{aligned}\right\} \qquad (6)$$

and

$$\Phi = 2cR \int_0^h \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{\cos\theta}{D}\,d\theta d\rho d\zeta \qquad (7)$$

where

$$D^2 = R^2 - 2R\rho\cos\theta + \rho^2 + (z-\zeta)^2 .$$

We now have

$$\int \frac{z-\zeta}{D^3}\,d\zeta = \frac{1}{D} + \text{constant} ,$$

$$\int \frac{R\cos\theta-\rho}{D^3}\,d\rho = \frac{1}{D} + \text{constant} .$$

Hence we may write

$$\begin{aligned} H_R &= -2c\cos\omega\,[f(h)-f(0)] \\ H_\phi &= \quad 2c\sin\omega\,[f(h)-f(0)] \\ H_z &= \quad 2c\cos\omega\,[g(\rho_2)-g(\rho_1)] \end{aligned}$$

where

$$f(\zeta) = \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{\cos\theta}{D}\,d\theta d\rho , \quad g(\rho) = \int_0^h \int_0^\pi \frac{d\theta d\zeta}{D} .$$

But we have further, ln being the natural logarithm, with base $e=2.71828\ \ldots\ldots$:

$$\int \frac{d\rho}{D} = \quad ln\ (\rho - R\cos\theta + D) + \text{constant}\,,$$

$$\int \frac{d\zeta}{D} = -ln\ (z - \zeta + D) + \text{constant}\,,$$

which gives

$$f(\zeta) = f(\rho_2,\ \zeta) - f(\rho_1,\ \zeta)\,,$$

$$g(\rho) = g(\rho,\ h) - g(\rho,\ 0)\,,$$

where

$$\left.\begin{aligned} f(\rho,\zeta) &= \quad \int_0^\pi ln\ (\rho - R\cos\theta + D)\cos\theta\ d\theta \\ g(\rho,\zeta) &= -\int_0^\pi ln\ (z-\zeta+D)\ d\theta \end{aligned}\right\} \qquad (8)$$

Substituting into the expressions for the components, we finally get

$$\left.\begin{aligned} H_R &= -2c\cos\omega\,[f(\rho_2,h) - f(\rho_1,h) - f(\rho_2,0) + f(\rho_1,0)] \\ H_\phi &= \quad 2c\sin\omega\,[f(\rho_2,h) - f(\rho_1,h) - f(\rho_2,0) + f(\rho_1,0)] \\ H_z &= \quad 2c\cos\omega\,[g(\rho_2,h) - g(\rho_1,h) - g(\rho_2,0) + g(\rho_1,0)] \end{aligned}\right\} \qquad (9)$$

For each system of values, R,z,ρ,ζ, the integrals $f(\rho,\zeta)$ and $g(\rho,\zeta)$ can now be easily calculated numerically. Integraphs and planimeters may be employed, or the approximate formula of Simpson, which is well known.

We thus have the means of calculating in every case the magnetic field in space outside the whirl.

If the point (x,y,z) be on the axis of the whirl, R is zero and

$$D^2 = \rho^2 + (z-\zeta)^2\,,$$

which gives

$$f(\rho,\zeta)=ln\ (\rho+D)\int_{0}^{\pi}\cos\theta\ d\theta=0\ ,$$

$$g(\rho,\zeta)=-\pi\ ln\ (z-\zeta+D)\ .$$

Hence $H_R=0$, $H_\phi=0$, and $H_z=H_{z0}$, where

$$H_{z0}=-2\pi c\cos\omega\ ln\ \frac{(z-h+\sqrt{(z-h)^2+\rho_2^2})(z+\sqrt{z^2+\rho_1^2})}{(z-h+\sqrt{(z-h)^2+\rho_1^2})(z+\sqrt{z^2+\rho_2^2})} \tag{10}$$

It will be interesting to obtain a verification. Let h and k be infinitesimal, and

$$\rho_2=\rho+k\ ,\quad \rho_1=\rho\ ,\quad \omega=0\ .$$

If we put

$$\sqrt{\rho^2+z^2}=u$$

we have

$$z-h+\sqrt{(z-h)^2+(\rho+k)^2}=$$

$$(z+u)\Big[1-\frac{1}{u}h+\frac{\rho}{(z+u)u}k+\frac{\rho^2}{2(z+u)u^3}h^2+\frac{\rho z}{(z+u)u^3}hk$$

$$+\frac{z^2}{2(z+u)u^3}k^2+\ \ldots\ \Big].$$

Hence

$$ln\ \frac{z-h+\sqrt{(z-h)^2+(\rho+k)^2}}{z-h+\sqrt{(z-h)^2+\rho^2}}=\frac{\rho}{(z+u)u}k+\frac{\rho}{u^3}hk-\frac{u^2+uz+z^2}{(z+u)u^3}k^2+\ \ldots\ .$$

This gives

$$ln\ \frac{z+\sqrt{z^2+\rho^2}}{z+\sqrt{z^2+(\rho+k)^2}}=-\frac{\rho}{(z+u)u}k-\frac{u^2+uz+z^2}{(z+u)u^3}k^2+\ \ldots\ .$$

and finally

$$H_z=-2\pi c\,\frac{\rho}{u^3}\,hk+\ \ldots\ .$$

where

$$c = \frac{\varepsilon \rho v}{3 \times 10^{10}} \; .$$

If we suppose ε and v to be positive, εhkv electrostatic units will pass through the cross-section of the current in one second, which corresponds to a current of

$$i = \frac{\varepsilon hkv}{3 \times 10^{9}}$$

amperes. If we introduce this value,

$$H_z = -\frac{1}{10} \, i \, \frac{2\pi\rho^2}{u^3} + \; \dots \; .$$

and this first term represents exactly the magnetic action in the point (0,0,z) of the circular current whose section is the infinitesimal product hk, and where the direction of the current is that of increasing ϕ (see Fig. 12).

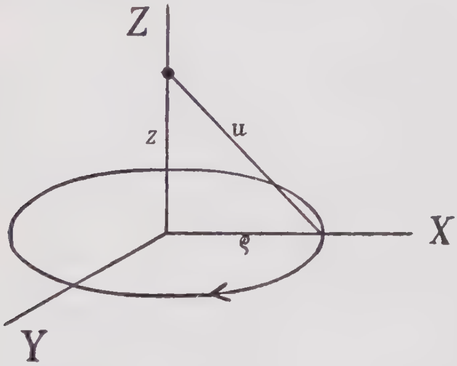


FIG. 12

VI. SPECIAL CASE IN WHICH THE THICKNESS OF THE WHIRL IS VERY SMALL COMPARED WITH ITS DIAMETER

We will now consider the special case in which the thickness of the whirl is very small compared with its diameter. Then for points (x,y,z) not very near the whirl, we can put

$$f(\rho,h) - f(\rho,0) = hf'(\rho,0) \; ,$$
$$g(\rho,h) - g(\rho,0) = hg'(\rho,0) \; ,$$

where $f'(\rho,0)$ and $g'(\rho,0)$ are the partial derivatives $\frac{\delta f}{\delta \zeta}$ and $\frac{\delta g}{\delta \zeta}$ for the value $\zeta=0$.

Now

$$f'(\rho,\zeta)=-\int_0^\pi \frac{z-\zeta}{\rho-R\cos\theta+D}\cdot\frac{\cos\theta\, d\theta}{D}\,,\quad g'(\rho,\zeta)=\int_0^\pi \frac{d\theta}{D}$$

and

$$f'(\rho,0)=-z\int_0^\pi \frac{\cos\theta\, d\theta}{(\rho-R\cos\theta+D_0)D_0}=z\int_0^\pi \frac{(\rho-R\cos\theta)\cos\theta\, d\theta}{(R^2\sin^2\theta+z^2)D_0}$$

$$g'(\rho,0)=\int_0^\pi \frac{d\theta}{D_0}\,,$$

where

$$D_0^2=R^2-2R\rho\cos\theta+\rho^2+z^2\,.$$

This must be substituted in the formulae

$$\left.\begin{aligned} H_R&=-2ch\cos\omega\,[f'(\rho_2,0)-f'(\rho_1,0)]\\ H_\phi&=\quad 2ch\sin\omega\,[f'(\rho_2,0)-f'(\rho_1,0)]\\ H_z&=\quad 2ch\cos\omega\,[g'(\rho_2,0)-g'(\rho_1,0)]\end{aligned}\right\}\qquad (11)$$

The same result can also be obtained by starting from formula (6), and considering h as infinitesimal. Then

$$H_R=-2ch\cos\omega\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{z\cos\theta}{D^3}\,d\theta d\rho$$

and an integration with respect to ρ gives the foregoing result.

The function Φ can easily be found in this case. We have

$$\Phi=2ch\,R\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{\cos\theta}{D_0}\,d\theta d\rho\,,$$

and as

$$\int\frac{d\rho}{D_0}=ln\,(\rho-R\cos\theta+D_0)+\text{constant}\,,$$

we get

$$\Phi = 2chR\left(f(\rho_2, 0) - f(\rho_1, 0)\right) \tag{12}$$

where the function $f(\rho,\zeta)$ is given by equation (8), when $D=D_0$.

The integrals occurring in the formulae for the components can be expressed by elliptic integrals of the first and second kind; but only the reduction of the integral for $g'(\rho,0)$ is sufficiently simple to be of any importance in the present problem. The integral in this case simplifies by the well-known substitution[1]

$$\frac{\pi}{2} - \frac{\theta}{2} = \psi$$

as follows:

$$\int_0^\pi \frac{d\theta}{\sqrt{R^2 - 2R\rho\cos\theta + \rho^2 + z^2}} = \frac{2}{\lambda}\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1 - k^2\sin^2\psi}} \tag{13}$$

where

$$k^2 = \frac{4R\rho}{(R+\rho)^2 + z^2}, \qquad \lambda = \sqrt{(R+\rho)^2 + z^2}.$$

As this integral is well tabulated in Legendre's *Traité des Fonctions Elliptiques* (Paris, 1826), **2**, 223, the calculation is rather easy.

The other integrals are most conveniently computed by Simpson's formula, or by machines.

VII. NUMERICAL APPLICATION TO A SOLAR VORTEX

We will now apply the preceding formula to an ideal solar vortex, in order to see how many of the observed facts can be accounted for.

According to the researches mentioned above by St. John, the vortex giving the Zeeman effect is probably situated at a level of the sun's atmosphere equal to or lower than the level Fe, 00, where the pressure is probably about 10 atmospheres. As the pressure increases very rapidly in descending into the sun's atmosphere, it seems probable that the thickness of the outflowing whirl in question can be only a very small fraction of the diameter indicated by the hydrogen flocculi whirl in the upper levels. This is also analogous to terrestrial cyclones, whose thickness is very small compared with their diameter. In other words, h must be very small compared with ρ_2. As to the size of ρ_1 compared with h and ρ_2, we can make a probable assumption by starting from the fact observed by Hale, namely, that the magnetic effect of the vortex decreases very rapidly upward along the axis of the vortex, so that there is only a very small fraction of its original value when we come to the level of 5000 miles above the photosphere.

---

[1] See, for instance, Appell, *Traité de Mécanique Rationnelle*, **3**, chap. xxxv.

In order to do this, we will start from the formula

$$H_{z0} = -2\pi \cos\omega \, ln \, \frac{(z-h+\sqrt{(z-h)^2+\rho_2^2})(z+\sqrt{z^2+\rho_1^2})}{(z-h+\sqrt{(z-h)^2+\rho_1^2})(z+\sqrt{z^2+\rho_2^2})} \, .$$

This gives

$$\frac{dH_{z0}}{dz} = 2\pi c \cos\omega \left\{ \frac{1}{\sqrt{z^2+\rho_2^2}} - \frac{1}{\sqrt{(z-h)^2+\rho_2^2}} - \frac{1}{\sqrt{z^2+\rho_1^2}} + \frac{1}{\sqrt{(z-h)^2+\rho_1^2}} \right\}$$

The geometrical signification of the radicals will be seen in Fig. 13.

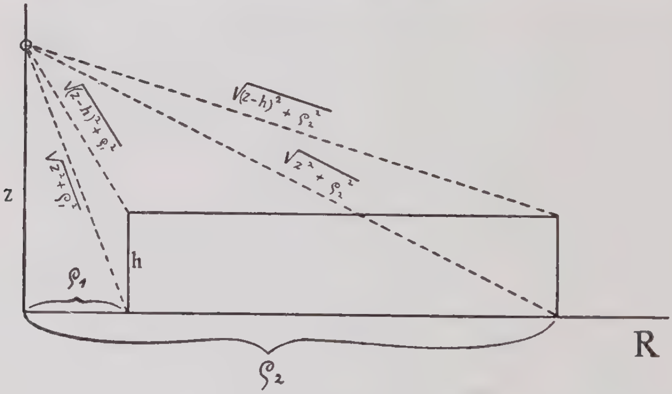


FIG. 13

On studying this figure, it will be seen that if we choose h small compared with the elevation of the high levels of the sun's atmosphere, and, on the other hand, suppose ρ_1 to be of the same size as, or smaller than, h, the magnetic force at these high levels will be only a very small fraction of what it is at the level $z=h$.

We will choose as a numerical example

$$h=\rho_1=500\ \text{km}=5\times 10^7\ \text{cm}\,,\quad \rho_2=50{,}000\ \text{km}=5\times 10^9\ \text{cm}\,.$$

The last value is chosen to correspond with the size of the observed hydrogen whirls around sun-spots.

Further we take $\omega=20^\circ$. With regard to the constant c, we will start from the value of about 3000 gausses observed in sun-spots by the Zeeman effect, and suppose that this is the value of H_{z0} for $z=h$. We then get $c=-583.1$. Starting from these values we find the following values of H_{z0} along the Z-axis:

z in km	H_{z0} in Gausses	z in km	H_{z0} in Gausses
500	3000	10,000	142.6
1000	1901	20,000	55.33
2000	916.7	30,000	28.46
3000	583.2	40,000	16.48
5000	318.4	50,000	10.62

A probable system of values for h, ρ_1, ρ_2, ω, and c being found, it is of the greatest interest to compute the magnetic field above the spot. We have done this for each point encircled in Fig. 14.

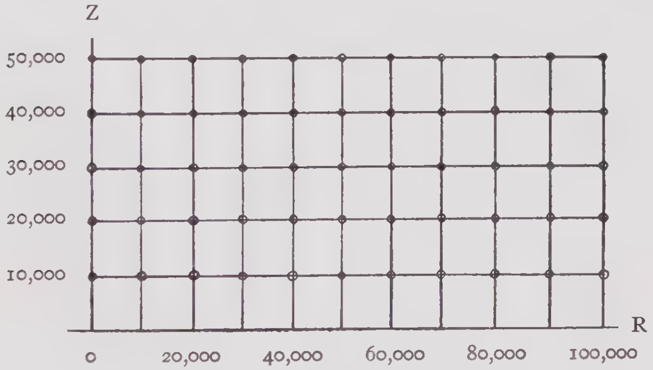


Fig. 14

We have used the formula given in Section VI for the case in which h is small compared with ρ_2. In the following paragraphs we will give these computations in detail.

VIII. COMPUTATION OF THE COMPONENTS H_z AND H_R

We have

$$H_z=4hc\cos\omega\left[\frac{T_2}{\lambda_2}-\frac{T_1}{\lambda_1}\right]$$

where the subscripts 1 and 2 denote the values for $\rho=\rho_1$ and $\rho=\rho_2$ of the functions

$$\lambda=\sqrt{(R+\rho)^2+z^2}\,, \quad T=\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}\,,$$

where

$$k=\sqrt{\frac{4R\rho}{(R+\rho)^2+z^2}}$$

and further

$$\begin{array}{ll} h=5\times10^7 & \omega=20^\circ \\ \rho_1=5\times10^7 & c=-583.1. \\ \rho_2=5\times10^9 & \end{array}$$

By means of Legendre's well-known tables, we find the results given in Table II.

TABLE II

VALUES OF H_z

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	9.47	7.89	5.84	3.84	+2.22	+1.09	+0.39	+0.01	−0.17	−0.25
40,000....	14.98	11.74	8.01	4.64	+2.18	+0.68	−0.08	−0.39	−0.48	−0.38
30,000....	24.86	18.09	10.94	5.22	+1.51	−0.31	−0.92	−0.99	−0.88	−0.73
20,000....	44.84	28.27	14.49	4.85	−0.65	−2.34	−2.21	−1.73	−1.31	−0.99
10,000....	87.65	42.00	17.72	2.33	−6.58	−5.95	−3.76	−2.43	−1.66	−1.18

The calculation of the other component, H_R, has been much more arduous. We have used Simpson's formula for the approximate computation of the definite integrals:

$$\int_a^b f(x)\,dx=\frac{h}{3}\left[y_0+4y_1+2y_2+4y_3+2y_4+\;\ldots\;+2y_{n-2}+4y_{n-1}+y_n\right],$$

where the interval of integration $b-a$ is divided into n parts, each of them equal to h, and where $y_0, y_1, y_2 \ldots\ldots y_{n-1}, y_n$ are the values of $f(x)$ for the points of division. Now

$$H_R=-2chz\cos\omega\,[Y_2-Y_1]$$

where Y_2 and Y_1 are the values for $\rho=\rho_2$ and $\rho=\rho_1$ of the integral

$$Y=\int_0^{\pi}\frac{(\rho-R\cos\theta)\cos\theta}{(R^2\sin^2\theta+z^2)\,D}\,d\theta\,,$$

and

$$D^2=R^2-2R\rho\cos\theta+\rho^2+z^2\,.$$

Here the interval from 0 to π has been divided into 12 parts, which has involved the calculation of the function under the sign of integration for each combination of the values mentioned above of R,z, and ρ, with the values

$$\theta=0°,\ 15°,\ 30°,\ 45°,\ \ldots\ 150°,\ 165°;\ 180°\,.$$

As the value of H_R is not affected if we divide each of the quantities R, Z, ρ_1, ρ_2, and h by the same number, that is, if we choose another unit of length, we have chosen h as this unit. The values of the quantities will then be

$$h=1\,,\quad \rho_1=1\,,\quad \rho_2=100$$

$$R=20,\ 40,\ 60,\ 80,\ 100,\ 120,\ 140,\ 160,\ 180,\ 200$$
$$Z=20,\ 40,\ 60,\ 80,\ 100.$$

The calculation has been made with the "Brunswiga" machine. The results for the function

$$\frac{(\rho-R\cos\theta)\cos\theta}{(R^2\sin^2\theta+z^2)\,D}$$

were integrated by Simpson's formula. Multiplication by the constant $-2chz\cos\omega$ gave finally the results shown in Table III.

TABLE III

VALUES OF H_R

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	2.12	3.69	4.44	4.44	3.95	3.20	2.47	1.86	1.37	0.73
40,000....	3.82	6.37	7.24	6.79	5.61	4.20	2.99	2.08	1.45	1.02
30,000....	7.50	11.65	12.25	10.57	7.93	5.30	3.36	2.15	1.40	0.91
20,000....	17.04	22.71	21.17	16.58	10.92	6.12	3.33	1.90	1.15	0.73
10,000....	49.68	45.86	35.56	25.48	13.83	5.30	2.28	1.16	0.66	0.42

By the formula

$$H_\phi = -H_R \tan \omega ,$$

where

$$\tan \omega = \tan 20^\circ = 0.36397,$$

we find the tabulated values of H_ϕ (Table IV).

TABLE IV

VALUES OF $H\phi$

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000 .	− 0.77	− 1.34	− 1.62	−1.62	−1.44	−1.17	−0.90	−0.68	−0.50	−0.27
40,000 .	− 1.39	− 2.32	− 2.64	−2.47	−2.04	−1.53	−1.09	−1.76	−0.53	−0.37
30,000 .	− 2.73	− 4.24	− 4.46	−3.85	−2.89	−1.93	−1.22	−0.78	−0.51	−0.33
20,000 .	− 6.20	− 8.27	− 7.70	−6.03	−3.97	−2.23	−1.21	−0.69	−0.42	−0.27
10,000 .	−18.08	−16.69	−12.94	−9.27	−5.03	−1.93	−0.83	−0.42	−0.24	−0.15

By means of these values we can calculate the magnetic force

$$H = \sqrt{H_R^2 + H_z^2 + H_\phi^2} .$$

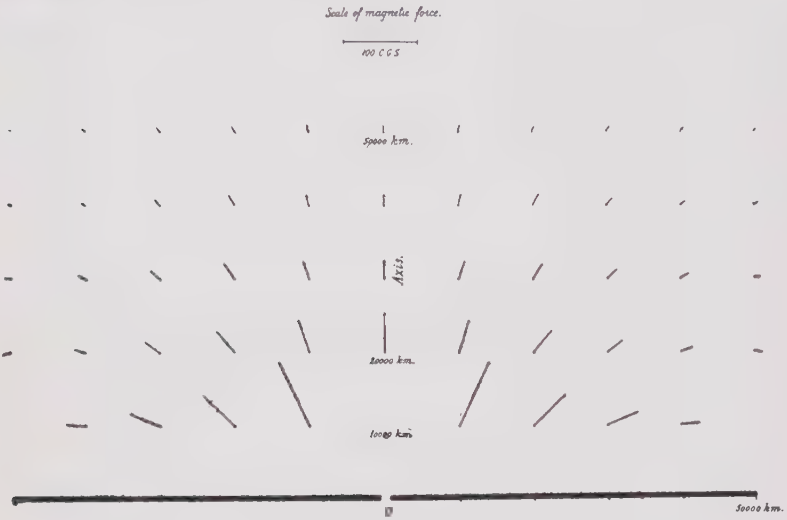
TABLE V

VALUES OF H

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	9.74	8.80	7.51	6.09	4.75	3.58	2.66	1.98	1.47	0.81
40,000....	15.52	13.56	11.11	8.59	6.36	4.52	3.18	2.25	1.61	1.15
30,000....	26.10	21.93	17.02	12.40	8.57	5.65	3.69	2.49	1.73	1.21
20,000....	48.37	37.20	26.79	18.30	11.64	6.92	4.18	2.66	1.79	1.26
10,000....	102.4	64.39	41.78	27.21	16.13	8.20	4.48	2.73	1.81	1.26

The component in the meridian plane through the axis of the whirl has the components H_R and H_z and is everywhere tangent to the curves Φ = constant, where Φ is the function given by equation (12).

The component in the meridian plane for some of the points (R,z) in the tables mentioned above is shown in Fig. 15. It will be seen that the magnetic force decreases rapidly upward from the whirl.



Magnetic field of a solar vortex.

FIG. 15

IX. CALCULATION AND CONSTRUCTION OF THE LINES OF FORCE

We have seen that the lines of force in space are the curves of intersection between cylinders through the logarithmic spirals

$$R=Be^{-\phi \cot \omega}$$

and the surfaces of revolution, whose meridian curves are

$$\Phi=C\,.$$

In the case in question we have

$$\Phi=2\,chR\,[f(\rho_2,0)-f(\rho_1,0)]$$

where

$$f(\rho,0)=\int_0^\pi ln\,(\rho-R\cos\theta+D)\cos\theta\,d\theta\,,$$

$$D^2=R^2-2R\rho\cos\theta+\rho^2+z^2\,.$$

In order to draw the lines Φ=constant, the following method has been used: For the computation of the integral by Simpson's formula, we have first computed the function

$$\log(\rho - R\cos\theta + D)\cos\theta,$$

where the logarithm is to the base 10 and where ρ, R, z have the values given before, with h as unit of length.

If we denote by

$$y_0,\ y_1,\ y_2,\ \ldots\ldots\ y_{12}$$

the values of this function for

$$\theta = 0,\ 15^\circ,\ 30^\circ,\ 45^\circ,\ \ldots\ldots\ 165^\circ \text{ and } 180^\circ$$

and S_1 and S_2 the values of the expression

$$y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots\ldots + 4y_{11} + y_{12}$$

for $\rho = \rho_1$ and $\rho = \rho_2$, we then have approximately

$$\frac{\pi}{36} S_1 = \int_0^\pi \log(\rho_1 - R\cos\theta + D_1)\cos\theta\, d\theta$$

$$\frac{\pi}{36} S_2 = \int_0^\pi \log(\rho_1 - R\cos\theta + D_2)\cos\theta\, d\theta$$

where D_1 and D_2 are the values of D for $\rho = \rho_1$ and $\rho = \rho_2$. As

$$\log A = \log e \cdot \ln A = 0.43429\, \ln A$$

where e is the base of the hyperbolic logarithms, we get approximately

$$\Phi = \frac{\pi c h}{18 \log e} \cdot R\,(S_2 - S_1)\,.$$

Thus the function $R(S_2 - S_1)$ is a function of R and of z, which differs from the function Φ only by a constant factor and this even if we express R, ε, ρ_1, ρ_2, and h with h as unit of length.

Thus the curves Φ=constant are identical with the curves Φ_1=constant, where $\Phi_1 = R(S_2 - S_1)$, if Simpson's formula is assumed to be accurate enough for our purpose. From this we have then drawn the curves with R as abscissa and Φ_1 ordinate, and where z has a constant value for each curve.

The points (R_{01}, z_0) and (R_{02}, z_0) where Φ has a given value, Φ_0, can then be found graphically by the intersection of the curve and the line $\Phi=\Phi_0$ (Fig. 16). Similar curves have been drawn with z as abscissa and Φ as ordinate, for controlling the results. This has finally given enough points for the construction of the curves $\Phi_1=$constant.

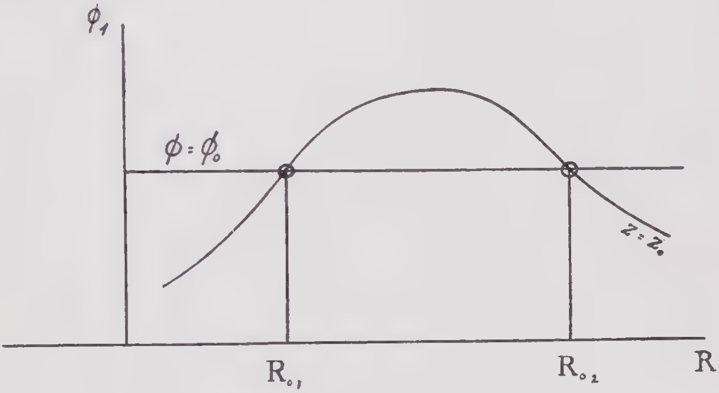


Fig. 16

In order to control the results, the function Φ_1 has also been calculated for $z=0$, in which case the integrals can be reduced to elliptic integrals of the first and second kind by a partial integration.

The curves $\Phi_1=$constant are seen in Fig. 17, and the directions of the components of magnetic force in the meridian plane are also given. It will be seen that the agreement is fairly good, in spite of the approximate method of computation and construction.

With the aid of the curves $\Phi_1=$constant and the logarithmic spirals

$$R=Be^{-\phi \cot \omega},$$

where B is a constant and $\omega=\pm 20°$, I have constructed wire models of the lines of magnetic force in space over the whirl. The dimensions are the same as for the case already mentioned.

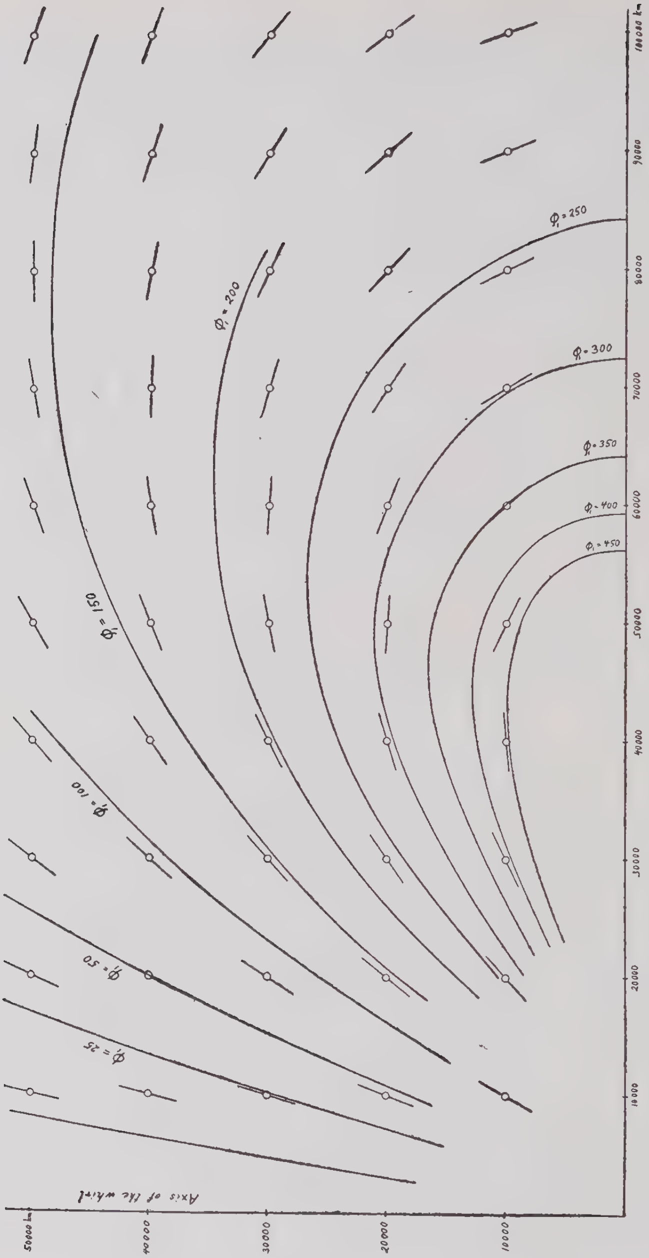
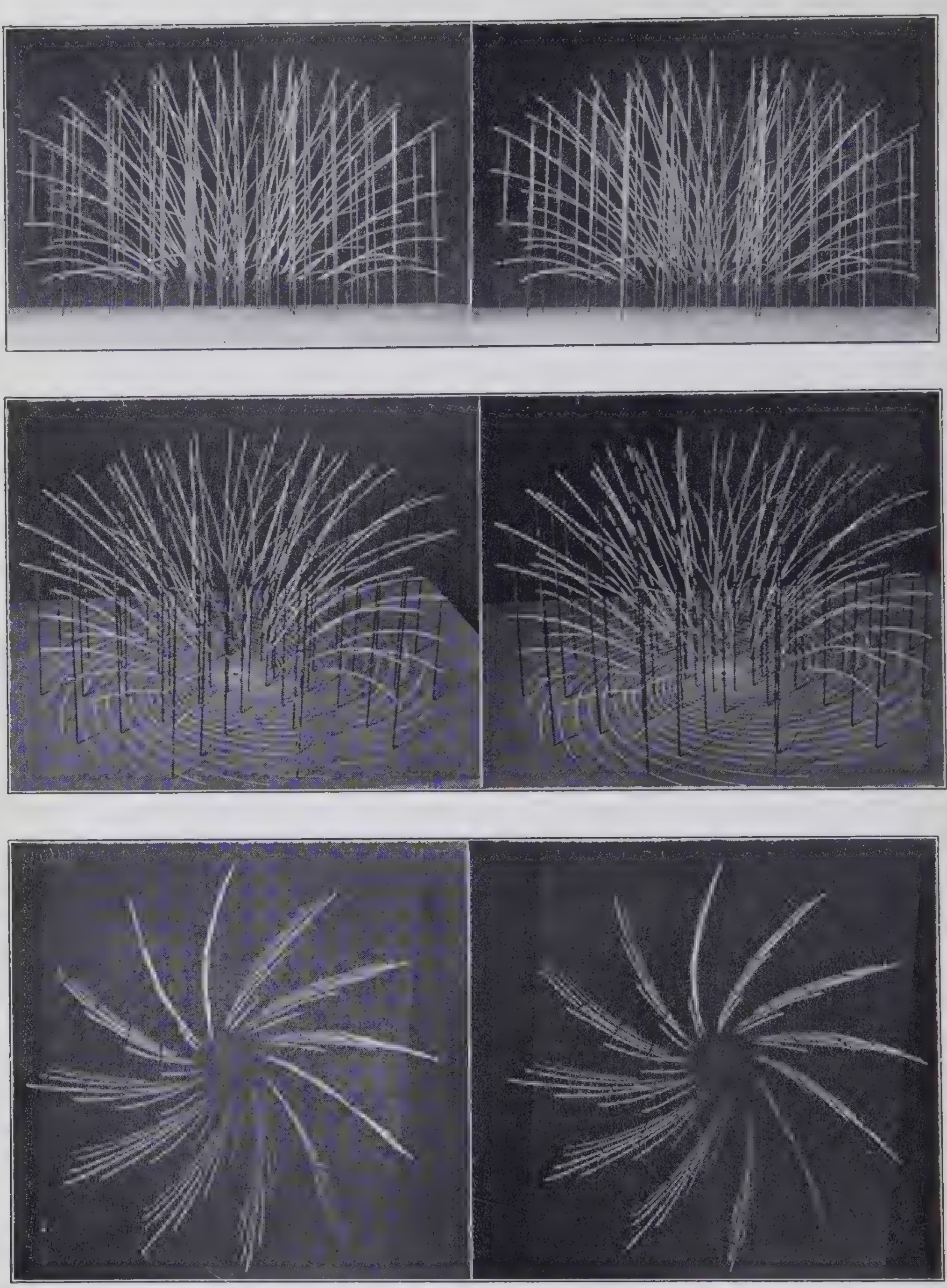

$\Phi_1 = 250$
$\Phi_1 = 300$
$\Phi_1 = 350$
$\Phi_1 = 400$
$\Phi_1 = 450$
$\Phi_1 = 200$
$\Phi_1 = 150$
$\Phi_1 = 100$
$\Phi_1 = 50$
$\Phi_1 = 25$
Axis of the whirl
100000 km
90000
80000
70000
60000
50000
40000
30000
20000
10000
50000 km
40000
30000
20000
10000

Fig. 17

PLATE IV



Stereoscopic Views of Model Showing Distribution of Lines of Magnetic Force for $\omega = -20°$. Current Lines in Lower Levels of Whirl Are Traced on the Base of the Model

In Plate IV are seen some stereoscopic representations of these models for the case $\omega = -20^\circ$, viewed from various points. On the base are drawn the current-lines, and the lines of force in space are supported by dark pins, standing perpendicular to the plane of the whirl.

If we compare the curves $\Phi_1 =$ constant with the well-known spectroheliograph pictures of calcium prominences around a sun-spot at the sun's limb, shown by Professor Slocum in his paper[1] "Attractions of Sun-Spots for Prominences," a very striking resemblance appears. It seems as if these prominences are arranged along lines of magnetic force from a whirl round the sun-spot.

X. THE MAGNETIC FIELD ROUND A BIPOLAR SPOT-GROUP

Let us now consider two whirls and their combined magnetic field of force. For simplicity we will suppose that each of the whirls is of the kind just mentioned, and that they are lying in the same plane with parallel axes perpendicular to the plane. The magnetic resultant force in each point of space over the whirls can then be found by the parallelogram law, because we know the force due to each of the whirls considered separately.

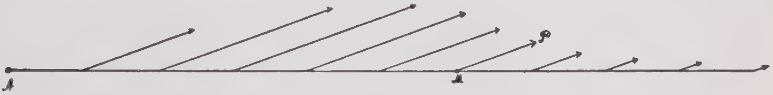


FIG. 18

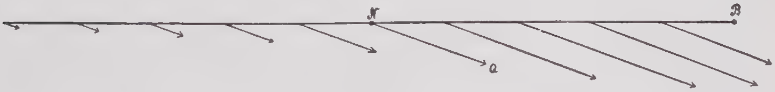


FIG. 19

The graphical method used for finding the resultant components at a given level above the plane of the whirls is as follows: on a sheet of tracing paper are marked the components of the magnetic force due to the whirl *A* (Fig. 18), and on another sheet the components due to the whirl *B* (Fig. 19), both components for points along a radius vector in the selected level $z=$ constant. The two sheets are then placed with the points *A* and *B* coinciding with the centers of the whirls (Fig. 20), and turned to produce coincidence of the points *M* and *N* on the sheets. On a third underlying sheet of tracing paper the points *M* and *R* are marked by pricking with a pin, and the resultant force *MR* is then constructed.

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[1] *Astrophysical Journal*, **36**, 265, 1912.

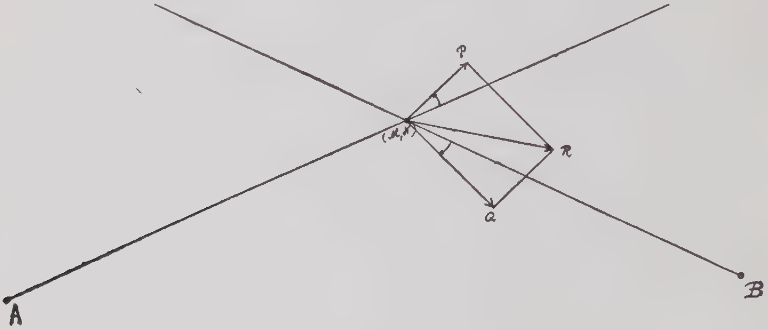

FIG. 20

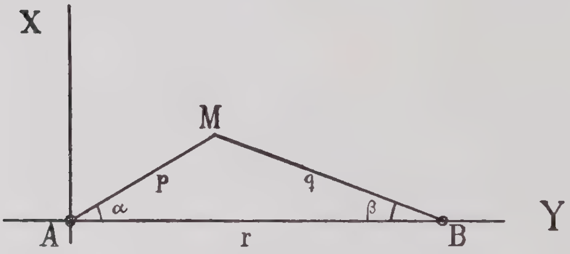

FIG. 21

With regard to the calculation of the resultant force the necessary formula may be developed as follows: Let us consider a given plane parallel to the plane of the whirls, and let *A* and *B* be the points where the axes of the whirls intersect this plane. Let *r* be their mutual distance. Let *M* be a point in the plane and H_R, H_ϕ, and H_z the components at *M* due to the whirl with axis *A*, and K_R, K_ϕ, and K_z the components due to the whirl *B*. Let us finally denote the distances AM and BM by p and g and the angles MAB and MBA by α and β. Then

$$\cos \alpha = \frac{p^2+r^2-g^2}{2pr} ,$$

$$\cos \beta = \frac{g^2+r^2-p^2}{2gr} .$$

If we choose a system of rectangular co-ordinates with the origin in the center of the whirl A, and the X-, Y-, and Z-axes parallel to AX (perpendicular to AB, see Fig. 21), to AY, and to the axis of the whirl, and if we denote by P_x, P_y, and P_z the components of the resulting magnetic force at the point M, parallel with the X-, Y-, and Z-axes, we find

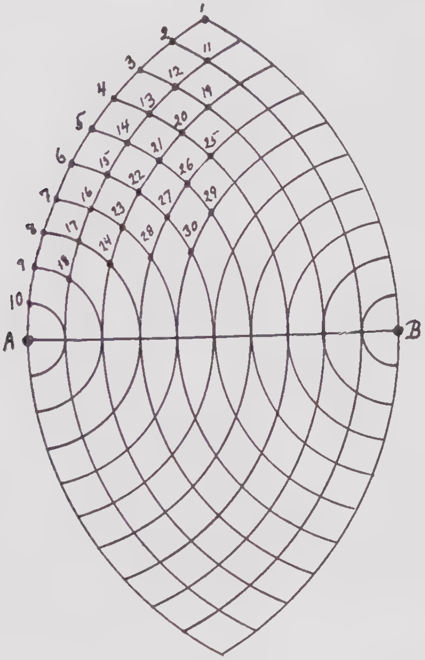

FIG. 22

$$P_x = H_R \sin \alpha - H_\phi \cos \alpha + K_R \sin \beta + K_\phi \cos \beta ,$$
$$P_y = H_R \cos \alpha + H_\phi \sin \alpha - K_R \cos \beta + K_\phi \sin \beta ,$$
$$P_z = H_z + K_z .$$

For the practical computation of P_x, P_y, and P_z, sin α, cos α, sin β, and cos β can first be calculated. If, for instance, $r = 10{,}000$ km, the distances p and g have the values:

10,000, 20,000, ... 90,000 and 100,000 km,

and we obtain the values in the following tables corresponding to the numbered points in Fig. 22. The values of these functions will be useful for the calculation of P_x, P_y, and P_z in special cases.

TABLE VI

$g = 100{,}000$

	p									
	100,000	90,000	80,000	70,000	60,000	50,000	40,000	30,000	20,000	10,000
sin α	0.8660	0.8930	0.9165	0.9367	0.9539	0.9682	0.9798	0.9887	0.9950	0.9987
cos α.	0.5000	0.4500	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500
sin β.	0.8660	0.8037	0.7332	0.6557	0.5723	0.4841	0.3919	0.2965	0.1990	0.0998
cos β.	0.5000	0.5950	0.6800	0.7550	0.8200	0.8750	0.9200	0.9550	0.9600	0.9950
Point no. . .	1	2	3	4	5	6	7	8	9	10

TABLE VII

$g = 90{,}000$

	p							
	90,000	80,000	70,000	60,000	50,000	40,000	30,000	20,000
sin α.	0.8315	0.8549	0.8741	0.8888	0.8980	0.8992	0.8843	0.8181
cos α	0.5556	0.5187	0.4857	0.4583	0.4400	0.4375	0.4668	0.5750
sin β.	0.8315	0.7599	0.6798	0.5924	0.4988	0.3995	0.2947	0.1814
cos β.	0.5556	0.6501	0.7334	0.8056	0.8667	0.9167	0.9556	0.9834
Point no.	11	12	13	14	15	16	17	18

TABLE VIII

$g = 80{,}000$

	p					
	80,000	70,000	60,000	50,000	40,000	30,000
sin α.	0.7806	0.7945	0.8000	0.7924	0.7599	0.6613
cos α	0.6250	0.6072	0.6000	0.6100	0.6500	0.7501
sin β.	0.7806	0.6953	0.6000	0.4953	0.3799	0.2480
cos β.	0.6250	0.7187	0.8000	0.8687	0.9250	0.9687
Point no.	19	20	21	22	23	24

TABLE IX

$g=70{,}000$

	p			
	70,000	60,000	50,000	40,000
sin α	0.6998	0.6887	0.6500	0.5464
cos α	0.7143	0.7250	0.7600	0.8375
sin β	0.6998	0.5903	0.4642	0.3122
cos β	0.7143	0.8072	0.8857	0.9500
Point no.	25	26	27	28

TABLE X

$g=60{,}000$

	p	
	60,000	50,000
sin α	0.5528	0.4560
cos α	0.8333	0.8900
sin β	0.5528	0.3799
cos β	0.8333	0.9250
Point no.	29	30

XI. SOME SIMPLE CASES OF BIPOLAR SPOT-GROUPS

There are two interesting cases which we will study in greater detail, namely, the case of two identical whirls, and the case of two equal and opposite whirls. In both cases there is a certain symmetry in the arrangement of the lines of force, and their projection on the plane of the whirls.

First case: two identical whirls.—Let us consider the components P_x and P_y and their resultant P_h in a plane parallel to the plane of the whirls. Let C (Fig. 23) be the middle point of the line AB joining points A and B where the axes of the whirls cut the plane. Let M and M' be two points in the plane situated symmetrically with regard to C, and let us denote by accents the letters corresponding to the point M'. We then have

$$\begin{aligned} \alpha' &= -\beta\ , & \beta' &= -\alpha \\ H'_R &= K_R\ , & H'_\phi &= K_\phi \\ K'_R &= H_R\ , & K'_\phi &= H_\phi \end{aligned}$$

which gives

$$P'_x = -H_R \sin \alpha + H_\phi \cos \alpha - K_R \sin \beta - K_\phi \cos \beta = -P_x$$
$$P'_y = -H_R \cos \alpha - H_\phi \sin \alpha + K_R \cos \beta - K_\phi \sin \beta = -P_y ,$$

i.e., the *component* P'_h *in the point* M' *is obtained by turning the component* P_h *in the point* M *through an angle of* 180° *around* C *as center.*

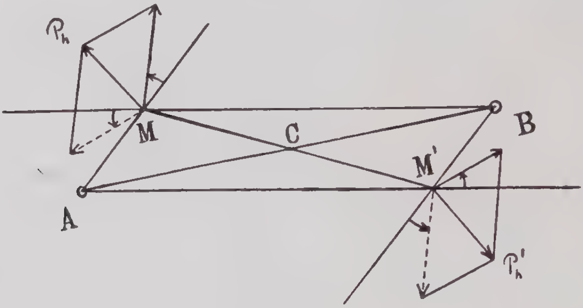

FIG. 23

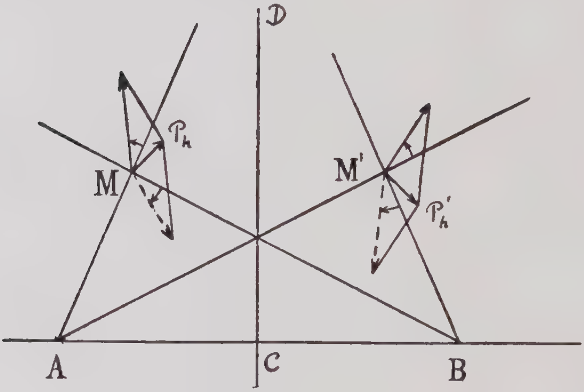

FIG. 24

In Fig. 25 are seen the directions of the components P_h in a plane, corresponding to the level 20,000 km over the two whirls in the sun's atmosphere. For the two identical whirls we have chosen case No. 5 of Fig. 8, with

$$h = 500 \text{ km}, \qquad \omega = 20^\circ$$
$$\rho_1 = 500 \text{ km}, \qquad c = -581.3$$
$$\rho_2 = 50{,}000 \text{ km}, \qquad r = 100{,}000 \text{ km}.$$

Second case: two whirls corresponding to cases Nos. 5 and 8 in Fig. 8.—For the whirl A we have ε negative and ω and v positive, and for the whirl B,

$$\varepsilon' = \varepsilon\,, \qquad \omega' = -\omega\,, \qquad v' = -v\,,$$

consequently

$$c' = \rho\varepsilon' v' = -c\,.$$

Further, for both whirls h, ρ_1, and ρ_2 are supposed to be the same.

Let us consider (Fig. 24) as before a given level above the plane of the two whirls, and let CD be the perpendicular through the central point C of the line AB. Let M and M' be two points situated symmetrically with respect to this line, and let us denote by accents the letters corresponding to the point M'.

We have

$$\alpha' = \beta\,, \qquad \beta' = \alpha\,.$$

Further by formula (11), H_R positive, H_ϕ negative and

$$\begin{aligned} H'_R &= -K_R\,, & K'_R &= -H_R \\ H'_\phi &= K_\phi\,, & K'_\phi &= H_\phi \end{aligned}$$

we find

$$\begin{aligned} P' &= +H'_R \sin\alpha' - H'_\phi \cos\alpha' + K'_R \sin\beta' + K'_\phi \cos\beta' \\ &= -H_R \sin\alpha + H_\phi \cos\alpha - K_R \sin\beta - K_\phi \cos\beta = -P_x \end{aligned}$$

and

$$\begin{aligned} P'_y &= H'_R \cos\alpha' + H'_\phi \sin\alpha' - K'_R \cos\beta' + K'_\phi \sin\beta' \\ &= H_R \cos\alpha + H_\phi \sin\alpha - K_R \cos\beta + K_\phi \sin\beta = P_y\,. \end{aligned}$$

Hence the straight lines passing through M and M' in the direction of the force lie symmetrically with respect to the line CD.

If M is on the line CD, we have

$$H_R = -K_R\,, \qquad H_\phi = K_\phi\,, \qquad \alpha = \beta$$

and

$$P_x = 0\,, \quad P_y = 2R\cos\alpha + 2H_\phi \sin\alpha\,,$$

i.e., the component P_h is parallel to the line AB.

In Fig. 26 are seen the directions of the component P_h in a plane corresponding to a level of 20,000 km, in the sun's

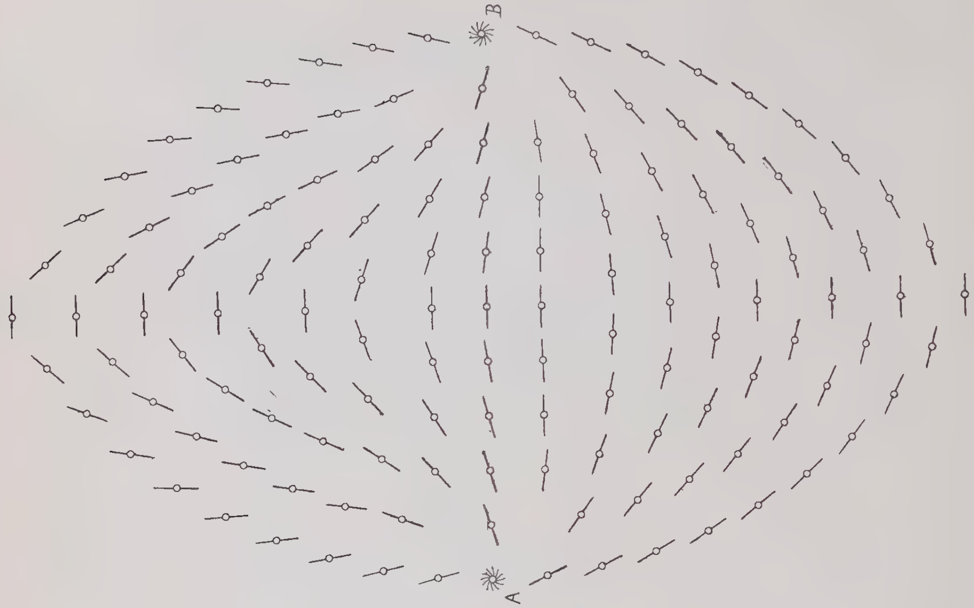

FIG. 26

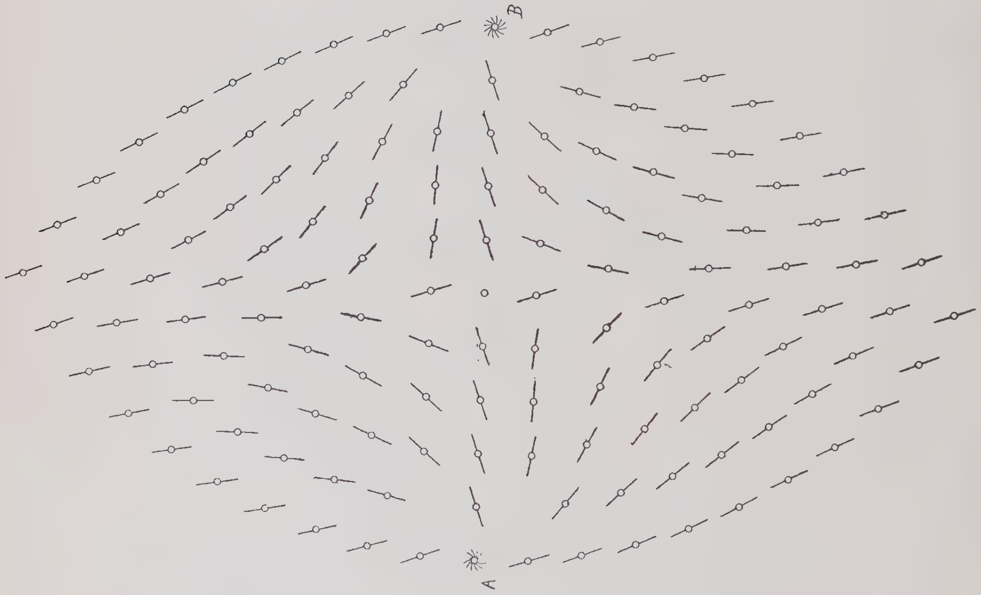

FIG. 25

atmosphere, where the hydrogen flocculi are probably to be found. The dimensions and the distance of the whirls are chosen as in the first case.

It is naturally of considerable interest to compare this with spectroheliograms of bipolar sun-spot groups. This will be done later, when more material has been obtained.

XII. THE ELECTRIC CHARGE OF THE SOLAR VORTEX AND THE ELECTROSTATIC FIELD OVER IT. DIFFICULTIES IN EXPLAINING THE ABSENCE OF THE STARK EFFECT. HYPOTHESIS OF GALVANIC CURRENTS

We have seen that the charge of the low-level whirl probably causing the Zeeman effect is negative. The value of the constant c will give us the means of determining this charge and of calculating the electrostatic field in space over the whirl. We had, by definition (see Section V),

$$c=\frac{\rho\varepsilon v}{3\times 10^{10}},$$

where ε is the charge of electricity in 1 cubic cm of the whirl, measured in electrostatic units, and ρ and v two corresponding values of the distance from the axis and the velocity of a gas-particle of the whirl, measured in the c.g.s. system of units. ε was supposed constant throughout the whirl.

If we measure directly the velocity at a given distance from the axis of the whirl, we can compute ρv, and c being known from the Zeeman effect, we can find ε.

Let us make an estimate of the probable value of the charge ε. As stated in his paper "Radial Motions in Sun-Spots,"[1] St. John has found, at the level Fe, 00 and at a distance of 11,250 km from the axis, an *outward* motion of about 1 km per second. If we assume the current-line to be a logarithmic spiral corresponding to $\omega=20°$, the velocity will then be

$$v=\frac{10^5}{\sin 20°}\text{ cm},$$

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[1] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

i.e., about 3×10^5 cm. If we choose the value of c determined in Section VII, namely, $c = -583.1$ we find $\varepsilon = -0.052$. From this we can compute the number N of corpuscles to a cubic cm of the gas, as we know the elementary charge of a negative corpuscle, which is[1] -4.65×10^{-10}. We then find $N = 1.1 \times 10^8$.

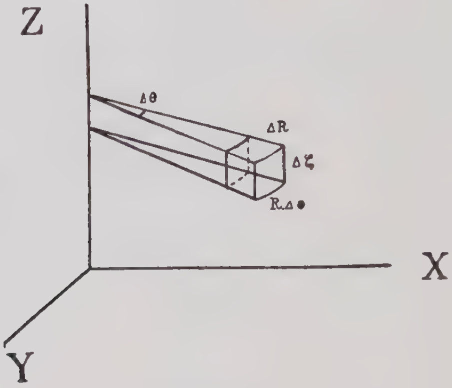

FIG. 27

Let us now compute the electrostatic field above the whirl. Consider an element of the whirl bounded by the planes (Fig. 27)

$$z = \zeta, \qquad z = \zeta + \Delta\zeta,$$

the planes

$$\phi = \theta, \qquad \phi = \theta + \Delta\theta,$$

and the cylinders

$$R = \rho, \qquad R = \rho + \Delta\rho.$$

The volume of this element is equal to $\rho\Delta\rho\Delta\theta\Delta\zeta$ and its charge is $\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta$.

Let δE_x, δE_y, and δE_z be the components parallel to the axes OX, OY, and OZ of the electrostatic action of this element on one electrostatic unit of positive electricity in the point $(x,0,z)$ of the XZ-plane. Then

---

[1] See Rutherford, *Radioactive Substances and Their Radiations*, § 20, Cambridge, 1913.

$$\delta E_x = \frac{x-\rho\cos\theta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,,$$

$$\delta E_y = \frac{-\rho\sin\theta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,,$$

$$\delta E_z = \frac{z-\zeta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,.$$

The action of an element lying symmetrically with respect to the XZ-plane is obtained by changing the sign of θ. Here r_1 is the distance from the point $(x,0,z)$ to the element.

The components of the resultant action of the two elements will be

$$2\delta E_x\,,\qquad 0\,,\qquad 2\delta E_z\,.$$

If we suppose the whirl to be of the same shape as in Section II, the components of its action on an arbitrary point (x,y,z) will be

$$\left.\begin{aligned} E_R &= 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{(R-\rho\cos\theta)\rho d\rho d\zeta d\theta}{D^3} \\ E_z &= 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{(z-\zeta)\rho d\rho d\zeta d\theta}{D^3} \end{aligned}\right\}\qquad(14)$$

Here E_R and E_z are the components normal to and parallel to the Z-axis, lying in the meridian plane and with the same orientation as the components H_R and H_z derived in Section II; further

$$D^2 = R^2+\rho^2-2R\rho\cos\theta+(z-\zeta)^2\,.$$

If we introduce the potential

$$W = 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{\rho d\rho d\zeta d\theta}{D}\qquad(15)$$

we can write

$$E_R = -\frac{\delta W}{\delta R}\,,\qquad E_z = -\frac{\delta W}{\delta z}\,.$$

The components of action of the whirl on an electric charge of ε' electrostatic units in the point (x,y,z) are consequently $\varepsilon' E_R$ and $\varepsilon' E_z$.

If the thickness h of the whirl is very small compared with its radius ρ_2, we can use the formula corresponding to h infinitesimal, namely,

$$W = 2\varepsilon h \int_{\rho_1}^{\rho_2} \int_0^{\pi} \frac{\rho d\rho d\theta}{D_0}$$

with

$$E_R = -\frac{\delta W}{\delta R}, \qquad E_z = -\frac{\delta W}{\delta z},$$

where

$$D_0^2 = R^2 + \rho^2 - 2R\rho \cos\theta + z^2.$$

The expression for W can easily be transformed into another form that is better adapted to numerical computation. We have

$$\int \frac{\rho d\rho}{D_0} = D_0 + R \cos\theta \, ln(\rho - R\cos\theta + D_0) + \text{constant},$$

which gives

$$\left.\begin{aligned} &W = W(\rho_2) - W(\rho_1)\,, \\ &\text{where} \\ &W(\rho) = 2\varepsilon h \int_0^{\pi} D_0 d\theta + 2\varepsilon h R \int_0^{\pi} \cos\theta \, ln(\rho - R\cos\theta + D_0) d\theta \end{aligned}\right\} \qquad (16)$$

Here the second integral is the same as in formula (12) for the function Φ, and has been denoted by $f(\rho, 0)$. The first integral can be expressed by elliptic functions by the substitution quoted in Section VI, which gives

$$\int_0^{\pi} D_0 d\theta = 2\lambda \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2\psi} \, d\psi$$

λ and k having the same definition as before. This integral can then be calculated by Legendre's tables.

Let us now consider the components E_R and E_z. We must first find the partial derivatives of $W(\rho)$ with respect to R and to z.

We have

$$\frac{\delta W(\rho)}{\delta R}=2\varepsilon h\int_0^\pi \frac{R-\rho\cos\theta}{D_0}\,d\theta+2\varepsilon h\,\frac{\delta}{\delta R}\left[R\int_0^\pi \cos\theta\, ln(\rho-R\cos\theta+D_0)d\theta\right].$$

But the partial derivative in the right member becomes

$$\int_0^\pi \cos\theta\, ln(\rho-R\cos\theta+D_0)d\theta+R\int_0^\pi \frac{R\cos\theta-(\rho+D_0)\cos^2\theta}{D_0(\rho-R\cos\theta+D_0)}\,d\theta\,.$$

Integrated by parts, the first term of this becomes

$$-\int_0^\pi \frac{(R\rho+RD_0)\sin^2\theta}{D_0(\rho-R\cos\theta+D_0)}\,d\theta$$

which, combined with the second term, gives for the partial derivative

$$-R\int_0^\pi \frac{d\theta}{D_0}.$$

Consequently,

$$\frac{\delta W(\rho)}{\delta R}=-2\varepsilon h\rho\int_0^\pi \frac{\cos\theta\, d\theta}{D_0}\,.$$

If we use the transformation of Section VI, we further obtain this integral expressed in terms of elliptic functions in the form

$$\left.\begin{aligned} E_R&=2\varepsilon h[T(\rho_2)-T(\rho_1)]\,, \\ \text{where}\qquad & \\ T(\rho)&=\frac{2\rho}{\lambda k^2}\left[(2-k^2)\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}-2\int_0^{\frac{\pi}{2}} \sqrt{1-k^2\sin^2\psi}\;d\psi\right] \end{aligned}\right\}\quad(17)$$

Finally we have to determine E_z. By partial differentiation

$$\frac{\delta W(\rho)}{\delta z}=2\varepsilon hz\int_0^\pi \frac{d\theta}{D_0}+2\varepsilon hRz\int_0^\pi \frac{\cos\theta\, d\theta}{(\rho-R\cos\theta+D_0)D_0}\,.$$

Here we have the integrals which in Section VI we have denoted by $f'(\rho,0)$ and $g'(\rho,0)$. We find

$$E_z = -2\varepsilon h z[g'(\rho_2, 0) - g'(\rho_1, 0)] + 2\varepsilon h R[f'(\rho_2, 0) - f'(\rho_1, 0)] \qquad (18)$$

By formulae (16), (17), and (18), the functions W, E_R, and E_z can easily be computed.

Along the axis of the whirl,

$$\left.\begin{aligned} W &= 2\pi\varepsilon h\left[\sqrt{\rho_2^2+z^2} - \sqrt{\rho_1^2+z^2}\right] \\ E_R &= 0 \\ E_z &= 2\pi\varepsilon h\left[\frac{z}{\sqrt{z^2+\rho_1^2}} - \frac{z}{\sqrt{z^2+\rho_2^2}}\right] \end{aligned}\right\} \qquad (19)$$

Let us apply these to the solar vortex, studied in Section VII. We have

$$h = 5\times 10^7 \text{ cm}, \qquad \rho_1 = 5\times 10^7 \text{ cm}, \qquad \rho_2 = 5\times 10^9 \text{ cm},$$
$$\varepsilon = -0.052 \text{ electrostatic units.}$$

Let us choose $z = 10^9$, 2×10^9, 3×10^9, 4×10^9, and 5×10^9 cm. We then find:

z = 10,000 km	20,000	30,000	40,000	50,000
$E_z = -1.3\times 10^7$	-10^7	-0.8×10^7	-0.6×10^7	-0.5×10^7

Here, as we have seen, E_z is the force acting on one electrostatic unit of positive electricity.

The immense values of this force compared with the absence of any appreciable Stark effect over sun-spots[1] seem quite improbable. Even with a much smaller convection whirl the observed magnitude of the Zeeman effect leads us, as Salet has already remarked,[2] to electric forces corresponding to several million volts per centimeter. We have tried to explain this paradox by assuming with Deslandres that the sun's atmosphere is composed of different levels of opposite charges. If one of these levels is in rapid motion about a sun-spot and the corresponding layer relatively quiet, we shall have large magnetic action and a very small resultant electric field over the layers; we must then assume their location to be below the layers accessible to spectroscopic investigations. In this manner we succeed in bringing the electric field to a very small fraction of the value given above, but yet not small enough to explain the absence of the Stark effect.

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[1] According to a letter from Professor Hale dated June 30, 1914.

[2] *Bulletin de la Société Astronomique de France*, 28, 220, 1914.

It seems as if the only possible explanation will be the hypothesis that the Zeeman effect is due to galvanic currents instead of convection currents. This hypothesis will be developed in another paper.

XIII. THE MOTION OF ELECTRIC CORPUSCLES IN THE SUPERPOSED ELECTRIC AND MAGNETIC FIELDS OVER A SOLAR VORTEX

There is considerable interest in the study of the motion of electric corpuscles over sun-spots because of various applications to the theory of prominences and to the theory of the aurora, especially the latter, if we suppose with Birkeland and others that the aurora is caused by such corpuscles coming from the neighborhood of sun-spots.

If we limit our studies to the highest stratum of the sun's atmosphere, which may be considered as a vacuum, this investigation may be made by the methods which I have applied to the theory of the aurora.[1] In the following we neglect the action of the general magnetic field of the sun.

Let us first consider the motion along the axis of the whirl. This axis being a line of magnetic force, the magnetism has no influence on the motion; we have only to take into account the electrostatic field. In order to consider the most general case, we will take into account the variation of mass with velocity, a hypothesis well founded when the velocity is so great that it is comparable with the velocity of light.

We will follow the equations of Schott.[2] Let ε' be the electric charge of the corpuscle in electrostatic units; then the force acting on it will, with the same notation as before, be $\varepsilon' E_z$ and we obtain

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[1] See my memoirs in the *Archives des Sciences Physiques et Naturelles*, Geneva, 1907 and 1911–1912.

[2] See *Electromagnetic Radiation and the Mechanical Reactions Arising from It*, Cambridge University Press, 1912, Appendix F.

$$\frac{d}{dt}\left(\frac{cmv}{\sqrt{c^2-v^2}}\right)=\varepsilon' E_z=-\varepsilon'\frac{\delta W}{\delta z}\,,$$

where m is the mass of the corpuscle for zero velocity; c is here the velocity of light, and v the velocity of the corpuscle.

From this equation we find, as Schott has shown,

$$\frac{c^3m}{\sqrt{c^2-v^2}}=-\varepsilon' W+C'\,,$$

C' being a constant of integration, which can be determined if we suppose the velocity given for a given value of z. Let us for instance suppose $v=0$ for $z=z_0$, and let W_0 be the value of W for $z=z_0$. We get

$$c^2m=-\varepsilon' W_0+C'\,,$$

that is,

$$\frac{c^3m}{\sqrt{c^2-v^2}}=c^2m+\varepsilon'(W-W_0)\,.$$

Here

$$W=2\pi\varepsilon h[\sqrt{\rho_2^2+z^2}-\sqrt{\rho_1^2+z^2}]$$
$$W_0=2\pi\varepsilon h[\sqrt{\rho_2^2+z_0^2}-\sqrt{\rho_1^2+z_0^2}]$$

If we consider the motion over the whirl in general, the mathematical treatment is especially interesting for the case in which the current-lines are circles, as they are when $\omega=0$. If we do not then take into account the general magnetic field of the sun but consider only the solar vortex, we have a field that is symmetrical around an axis, which affords very important simplifications in the mathematical treatment.

Let us first suppose the velocity of the corpuscle negligible as compared with the velocity of light. The corpuscle being in motion *in vacuo* and under the action of a superposed electric and magnetic field, we can apply the researches which I have published in the *Comptes Rendus* for March 11, 1911.

The equations of motion are

$$m\frac{d^2x}{dt^2}=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{dt}-H_z\frac{dy}{dt}\right]+\varepsilon' E_x\,,$$

$$m\frac{d^2y}{dt^2}=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{dt}-H_x\frac{dz}{dt}\right]+\varepsilon' E_y\,,$$

$$m\frac{d^2z}{dt^2}=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{dt}-H_y\frac{dx}{dt}\right]+\varepsilon' E_z\,.$$

There the z-axis is the axis of the whirl, H_x, H_y, and H_z are the components of the magnetic force parallel to the X-, Y-, and Z-axes, respectively, and E_x, E_y, and E_z are the components of the electric force. m and ε' are the mass and charge of the corpuscle and $c=3\times 10^{10}$ the velocity of light. ε' is measured in electrostatic units.

Now in our case the components H_R, H_ϕ, and H_z, previously introduced, have the values

$$H_R=\frac{1}{R}\frac{\delta\Phi}{\delta z}\,,\qquad H_\phi=0\,,\qquad H_z=-\frac{1}{R}\frac{\delta\Phi}{\delta R}\,,$$

and

$$H_x=\frac{x}{R}H_R\,,\qquad H_y=\frac{y}{R}H_R\,,$$

where

$$\Phi=\frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi\frac{\varepsilon\rho v\cos\theta}{D}\,d\theta d\rho d\zeta$$

is the previously introduced function of R and z.

In the same manner

$$E_x=\frac{x}{R}E_R\,,\qquad E_y=\frac{y}{R}E_R$$

and

$$E_R=-\frac{\delta W}{\delta R}\,,\qquad E_z=-\frac{\delta W}{\delta z}\,,$$

where W is the electric potential, also a function of R and z only.

By introducing semipolar co-ordinates, R and ϕ, we get, as in my paper,

$$\frac{1}{2}mv^2=C-\varepsilon' W$$

$$mR^2\frac{d\phi}{dt}=C_1-\frac{\varepsilon'}{c}\Phi$$

which give further

$$\frac{d^2R}{dt^2}=\frac{1}{2}\frac{\delta Q}{\delta R}, \qquad \frac{d^2z}{dt^2}=\frac{1}{2}\frac{\delta Q}{\delta z},$$

$$\left(\frac{dR}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2=Q,$$

where

$$Q=\frac{2C}{m}-\left(\frac{C_1-\frac{\varepsilon'}{c}\Phi}{mR}\right)^2-2\frac{\varepsilon'}{m}W$$

Here v is the velocity and C and C' are two constants of integration. If s denotes the arc along the orbit and θ the angle between the tangent and the plane through the point of contact and the Z-axis, we have

$$\sin\theta=R\frac{d\phi}{ds}$$

and as

$$v=\frac{ds}{dt}$$

we get, as in my paper,

$$\sin\theta=\frac{C_1-\frac{\varepsilon'}{c}\Phi}{R\sqrt{2mC-2m\varepsilon' W}}.$$

Hence the orbit must lie within the space defined by

$$-1\leqq\frac{C_1-\frac{\varepsilon'}{c}\Phi}{R\sqrt{2mC-2m\varepsilon' W}}\leqq 1.$$

The detailed study of these regions is of fundamental importance in the discussion of the orbits, as in the mathematical theory of the aurora, where corresponding inequalities have been found.

As has already been said, the equations of motion take another form if we suppose the velocity of the corpuscle not to be negligible as compared with the velocity of light.

We have then to use the fundamental equations due to Schott,[1] and we get

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dx}{dt}\right)=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{dt}-H_z\frac{dy}{dt}\right]+\varepsilon' E_x\,,$$

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dy}{dt}\right)=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{dt}-H_x\frac{dz}{dt}\right]+\varepsilon' E_y\,,$$

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dz}{dt}\right)=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{dt}-H_y\frac{dx}{dt}\right]+\varepsilon' E_z\,,$$

where m is the mass of the corpuscle for zero velocity, and where c is the velocity of light.

From this, as Schott has shown, we get

$$\frac{mc^3}{\sqrt{c^2-v^2}}=C'-\varepsilon' W\,,$$

C' being a constant of integration and W the electric potential.

If now we introduce instead of the time t, a new variable τ defined by

$$\frac{dt}{d\tau}=\frac{c}{\sqrt{c^2-v^2}}\,,$$

the equations of motion can be written

$$m\frac{d^2x}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{d\tau}-H_z\frac{dy}{d\tau}\right]+\varepsilon' E'_x\,,$$

$$m\frac{d^2y}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{d\tau}-H_x\frac{dz}{d\tau}\right]+\varepsilon' E_y\,,$$

$$m\frac{d^2z}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{d\tau}-H_y\frac{dx}{d\tau}\right]+\varepsilon' E'_z\,,$$

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[1] *Loc. cit.*

where

$$E'_x = \frac{x}{R} E'_R , \qquad E'_y = \frac{y}{R} E'_R$$

and

$$E'_R = -\frac{\delta W'}{\delta R} , \qquad E'_z = -\frac{\delta W'}{\delta z}$$

with

$$W' = -\frac{(C' - \varepsilon' W)^2}{2m\varepsilon' c^2} .$$

This remarkable transformation is due to Schott. Further, we find as before

$$mR^2 \frac{d\phi}{d\tau} = C_1 - \frac{\varepsilon'}{c}\Phi$$

and, as Schott has shown in his book (published in 1912),

$$\frac{d^2R}{d\tau^2} = \frac{1}{2}\frac{\delta P}{\delta R} , \qquad \frac{d^2z}{d\tau^2} = \frac{1}{2}\frac{\delta P}{\delta z} ,$$

$$\left(\frac{dR}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2 = P ,$$

where

$$P = -c^2 - \left(\frac{C_1 - \frac{\varepsilon'}{c}\Phi}{mR}\right)^2 + \left(\frac{C' - \varepsilon' W}{mc}\right)^2 .$$

Further

$$\frac{d\tau}{dt} = \frac{\sqrt{c^2 - v^2}}{c} = \frac{mc^2}{C' - \varepsilon' W} ,$$

which gives

$$R\frac{d\phi}{dt} = \frac{c^2}{C' - \varepsilon' W} \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R} ,$$

and since

$$v = \frac{ds}{dt} = c\frac{\sqrt{(C' - \varepsilon' W)^2 - m^2c^4}}{C' - \varepsilon' W}$$

we find

$$R\frac{d\phi}{ds} = \frac{c}{\sqrt{(C' - \varepsilon' W)^2 - m^2c^4}} \cdot \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R}$$

As $R\frac{d\phi}{ds}=\sin\theta$, we have the conditions

$$-1\leqq\frac{c}{\sqrt{(C'-\varepsilon' W)^2-m^2c^4}}\,,\qquad \frac{C_1-\frac{\varepsilon'}{c}\Phi}{R}\leqq 1\,,$$

which define the regions in space to which the orbits are restricted.

We have here a wide field for research which can be carried out by the same methods as those employed for the aurora.

As I have shown in the *Comptes Rendus* for March 6, 1911, November 25, 1912, and February 10 and 17, 1913, and in *Videnskapsselskapets Skrifter*, Christiania, 1913, these considerations may also be applied to other cosmical problems, e.g., to the theory of the solar corona and to cosmogony.

XIV. CONCLUDING REMARKS. IMPORTANCE OF SIMULTANEOUS INVESTIGATIONS OF THE AURORA BOREALIS AND SOLAR PHENOMENA

As we have seen in Section XII, the difficulties in explaining the absence of the Stark effect, supposing the Zeeman effect due to convection currents, lead to the hypothesis of galvanic currents around sun-spots. In the first approximation we can apply the theory of currents in plates and try to determine the current lines and the lines of magnetic force in space over the plate for comparison with the configuration of *Hα* flocculi.

The conclusions based on the hypothesis of convection currents have then to be modified. This will be done in another paper, which is in preparation. We shall return to these hypotheses when more data regarding sun-spot fields have been collected.

In this connection a detailed study of the aurora, its situation, and its altitude will be of considerable interest, especially if we have auroras connected with definite sun-spot phenomena as have sometimes been observed, for it is most probable that the electric corpuscles that cause the aurora are the same as those thrown out from the sun's atmosphere; and having studied their physical properties in the aurora, we can draw corresponding conclusions for the phenomena in the sun. This connection between solar phenomena and auroras, which has been elaborated by physical methods by Professor Birkeland[1] and mathematically by myself,[2] will probably be of importance in future solar work. Furthermore, the photographic method of measuring the altitude of the aurora by means of two stations connected by telephone which I have employed on my expeditions[3] in 1910 and 1913 is capable of giving very exact information regarding the auroral phenomena, thus laying a firm foundation for the conclusions we may draw from them. During the coming period of solar activity, special attention will be paid to these points of view.

CHRISTIANIA
January 1914

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[1] *The Norwegian Aurora Polaris Expedition*, **1**, sections 1 and 2.

[2] *Archives des Sciences Physiques et Naturelles*, Geneva, 1907 and 1911–1912.

[3] "Bericht über eine Expedition nach Bossekop zwecks photographischer Aufnahmen und Höhenmessungen von Nordlichtern," *Videnskapsselskapets Skrifter. I. Math.-Naturw. Klasse*, 1911, No. 17, Christiania; and "On an Auroral Expedition to Bossekop in the Spring of 1913," *Astrophysical Journal*, **38**, 311, 1913.

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THE CHANGE OF COLOR WITH DISTANCE AND APPARENT MAGNITUDE TOGETHER WITH A NEW DETERMINATION OF THE MEAN PARALLAXES OF THE STARS OF GIVEN MAGNITUDE AND PROPER MOTION

By P. J. VAN RHIJN

The present paper deals with the question of a possible relation between the color of the stars and their distances, apparent magnitude and spectral type being the same. It is found that the stars of the *Yerkes Actinometry*[1] show a relation between distance and color such that, *ceteris paribus*, the distant stars are redder than the nearer ones. Whether this phenomenon is to be ascribed to a loss of light in space or whether it is due to an influence of absolute magnitude cannot certainly be decided from the material available at present. Apart from its explanation, however, the question mentioned has an importance of its own, because its solution will afford a basis for further investigations; moreover, independently of the question as to how it is to be explained, a well-established relation between color and distance, apparent magnitudes and spectral types being the same, furnishes a valuable method for the determination of the distances of very remote objects.

For an investigation of the kind considered the *Yerkes Actinometry* contains very valuable material, viz., the visual magnitude, the color-index, and the spectral type of all the stars of the *Potsdam Photometric Durchmusterung* in the zone from 73° north declination to the pole. If the distances of these stars were also known, it would be a comparatively easy matter to derive the relation between color-index and distance. As, however, the parallaxes of only a few stars of the *Yerkes Actinometry* have been directly measured, we are obliged to use proper motion and apparent magnitude as a criterion of distance. I therefore derived the mean distance of stars of determined proper motion and magnitude, for which purpose I have made the following assumptions, in accordance with Professor Kapteyn[1] and Professor Schwarzschild:[2]

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[1] *Astrophysical Journal*, **36**, 169, 1912.

a) That the mean parallax of stars of determined proper motion and magnitude can be represented by the formula

$$\bar{\pi} = a\mu_1^b \epsilon^{m_1} \tag{1}$$

where $\mu_1 = 100\,\mu$ = total angular motion per century; $m_1 = m - 5.0$, m being the visual magnitude on the Harvard scale; a, b, ϵ are constants to be determined from the data of observation.

b) That the quantities

$$z = \log \frac{\pi}{\pi_0} \tag{2}$$

are distributed in accordance with the law of errors, π being the true parallax of a star and π_0 the most probable parallax of stars of the same magnitude and proper motion.

If the constants a, b, ϵ and the probable error ρ of the error-curve (2) have been determined from the data of observation, the mean distance of stars of given magnitude and proper motion can be derived by the formula

$$\bar{R} = \frac{1}{\bar{\pi}} e^{+\frac{\rho^2}{(0.6745)^2 \,\mathrm{mod.}^2}} \tag{3}$$

This is easily proved by means of supposition (*b*).

The data from which the constants a, b, ϵ, and ρ must be determined are of two kinds: (1) the individual parallaxes; (2) the mean parallaxes of certain groups of stars, which are derived from the parallactic motions and radial velocities. As these constants may be different for stars of different spectral types, they have been derived separately for the B, A, and the second-type (F, G, K) stars.

Without entering into details as to the determination of the constants, the following may be stated:[3] The coefficients a and b were computed by means of the known individual parallaxes. For the helium stars I used those determined by Professor Kapteyn;[4]

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[1] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

[2] *Astronomische Nachrichten*, **190**, 361, 1912.

[3] The details of the whole investigation have been published in my thesis under the same title as the present note.

[4] *Mt. Wilson Contr.*, No. 82; *Astrophysical Journal*, **40**, 43, 1914.

for the F, G, and K stars, the parallaxes compiled by Professor Kapteyn and Dr. Weersma in 1910,[1] and those afterward published by the Yale Observatory,[2] by Flint,[3] Slocum and Mitchell,[4] and Abetti.[5] The parallaxes of stars of the same proper motion were combined into mean values which gave the data required for the determination of a and b. For the A stars this procedure was impossible, because the number of directly measured parallaxes of this type is too small to give trustworthy results.

The constant ϵ was determined from the mean parallaxes for stars of different magnitudes, derived from the parallactic motion and radial velocity. Only those stars were used whose radial velocities have been determined by Campbell.[6] For the second-type stars a second value of ϵ was derived from the directly measured parallaxes.

Finally the values of a and b derived from the individual parallaxes were corrected by means of the final mean parallax for stars of different magnitudes and proper motions, which had been computed from the radial velocities and parallactic motions. The final mean parallax of the A stars was used to determine the constant a, the coefficients b and ϵ being supposed equal to the average of the values found for the B and F, G, K stars. The constant ρ was determined from the differences between the logarithm of the observed individual parallaxes and the logarithm of the mean parallax computed by the formula (1). As the errors of observation in the parallaxes have a large influence on the value of ρ, I used only stars whose parallaxes are well determined.

Table I gives a summary of the results for the constants appearing in the formulae (1) and (3). The results are, in general, in agreement with those derived by Professor Kapteyn,[7] but the mean parallaxes of stars of different magnitudes found by equation (1), which are given in Tables II, III, and IV, are entirely at variance with those derived by Campbell.[1] The Lick Observatory results, however, cannot be considered to be representative of the whole stellar system, as the stars with large angular motion were omitted in deriving the mean parallaxes. It can be shown that the greater part of the difference between Dr. Campbell's parallaxes and those found here will disappear if the former are multiplied by certain factors which allow for the omission of the stars of large angular motion.

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[1] *Publications of the Astronomical Laboratory at Groningen*, No. 24, 1910.

[2] Chase, Smith, and Elkin, *Transactions of the Astronomical Observatory of Yale University*, **2**, 1912.

[3] *Astronomical Journal*, **27**, 49, 1912.

[4] *Astrophysical Journal*, **38**, 1, 1913.

[5] *Memorie del R. Osservatorio Astronomico al Collegio Romano*, Serie III, Vol. V Parte 2.

[6] *Lick Observatory Bulletins*, **6**, 108, 1911; **7**, 19, 1912; **7**, 113, 1913.

[7] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

TABLE I

Type	a	b	ϵ	ρ
B.	0.″0031	0.904	0.895	0.12
A.	0.0028	0.80	0.895	0.15
F, G, K.	0.0038	0.695	0.895	0.17

TABLE II

Mean Parallax, Type B

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.″00. . . .	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000	0.″0000
0.01. . . .	0.0054	0.0048	0.0043	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018
0.02. . . .	0.0101	0.0090	0.0081	0.0072	0.0065	0.0058	0.0052	0.0047	0.0042	0.0037	0.0033
0.03. . . .	0.0145	0.0131	0.0116	0.0105	0.0093	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048
0.04. . . .	0.0189	0.0170	0.0151	0.0136	0.0121	0.0109	0.0097	0.0087	0.0078	0.0070	0.0062
0.05. . . .	0.0232	0.0207	0.0186	0.0166	0.0148	0.0133	0.0119	0.0107	0.0096	0.0085	0.0076
0.06. . . .	0.0273	0.0244	0.0219	0.0195	0.0174	0.0157	0.0140	0.0126	0.0113	0.0100	0.0090
0.07. . . .	0.0314	0.0281	0.0251	0.0225	0.0201	0.0180	0.0161	0.0144	0.0130	0.0115	0.0104
0.08. . . .	0.0354	0.0317	0.0284	0.0254	0.0227	0.0203	0.0182	0.0163	0.0146	0.0130	0.0117
0.09. . . .	0.0394	0.0353	0.0316	0.0282	0.0253	0.0226	0.0203	0.0181	0.0163	0.0145	0.0130
0.10. . . .	0.0433	0.0388	0.0347	0.0310	0.0277	0.0248	0.0223	0.0199	0.0179	0.0159	0.0142
0.2.	0.081	0.073	0.065	0.058	0.052	0.047	0.042	0.037	0.033	0.030	0.027
0.3.	0.116	0.105	0.094	0.084	0.075	0.067	0.060	0.054	0.048	0.043	0.039

Having computed the mean distances of stars of given magnitude and proper motion, we pass on to the determination of the relation connecting color and distance. The color-index may also depend on the apparent magnitude, for we are not sure that the range of one magnitude is the same for both the visual and photographic scales. Therefore the equation for the color-index is

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[1] *Lick Observatory Bulletin*, **6**, 131, 1911.

$$\sigma = a + bm + cR, \tag{4}$$

where

σ = color-index.
R = distance.
m = apparent visual magnitude.
b = change of color-index for a change of one unit in apparent magnitude.
c = increase of color-index per unit of distance (parsec).

TABLE III

Mean Parallax, Type A

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″.00....	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000
0.01....	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018	0.0016
0.02....	0.0085	0.0076	0.0068	0.0061	0.0054	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028
0.03....	0.0118	0.0105	0.0094	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048	0.0043	0.0039
0.04....	0.0148	0.0132	0.0119	0.0106	0.0095	0.0085	0.0076	0.0068	0.0061	0.0055	0.0049
0.05....	0.0177	0.0158	0.0142	0.0126	0.0113	0.0101	0.0091	0.0081	0.0073	0.0065	0.0058
0.06....	0.0204	0.0183	0.0164	0.0146	0.0131	0.0117	0.0105	0.0094	0.0084	0.0075	0.0067
0.07....	0.0231	0.0207	0.0185	0.0166	0.0148	0.0133	0.0119	0.0106	0.0095	0.0085	0.0076
0.08....	0.0257	0.0230	0.0206	0.0184	0.0165	0.0148	0.0132	0.0118	0.0106	0.0095	0.0085
0.09....	0.0283	0.0253	0.0226	0.0202	0.0181	0.0163	0.0145	0.0130	0.0117	0.0104	0.0093
0.10....	0.0308	0.0275	0.0247	0.0220	0.0197	0.0177	0.0158	0.0142	0.0127	0.0114	0.0101
0.2.....	0.054	0.048	0.043	0.038	0.034	0.031	0.028	0.025	0.022	0.020	0.018
0.3.....	0.074	0.066	0.059	0.053	0.047	0.043	0.038	0.034	0.031	0.027	0.024
0.4.....	0.093	0.083	0.075	0.067	0.060	0.054	0.048	0.043	0.039	0.034	0.031
0.5.....	0.111	0.100	0.089	0.080	0.071	0.064	0.057	0.051	0.046	0.041	0.037
0.6.....	0.129	0.115	0.103	0.092	0.083	0.074	0.066	0.059	0.053	0.047	0.042
0.7.....	0.146	0.130	0.117	0.104	0.094	0.084	0.075	0.067	0.060	0.054	0.048
0.8.....	0.162	0.145	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.053
0.9.....	0.178	0.160	0.143	0.128	0.114	0.102	0.092	0.082	0.074	0.066	0.059
1.0.....	0.194	0.174	0.155	0.139	0.124	0.111	0.100	0.089	0.080	0.072	0.064
2.0.....	0.338	0.303	0.271	0.242	0.217	0.194	0.174	0.156	0.140	0.125	0.111

The constants a, b, and c must be determined separately for each spectral type from a number of equations of the form (4). The color-index and visual magnitude were taken from the *Yerkes Actinometry;* the distance was computed for each star by means of formulae (1) and (3).

The solution involved two steps: First, the stars of each spectral class were divided into two or more groups according to their magnitudes. Forming the mean distance, color-index, and visual magnitude for all the stars of a group, a number of equations of the form

$$a + bm = \sigma + cR$$

were obtained, from which b was derived as a function of c. As the distances of the different groups were approximately equal, the constant c entered with a small coefficient. Secondly, the stars of each spectral class were arranged according to the magnitude of the proper motion. By means of these groups c was derived as a function of b, in which b entered with a small coefficient. From these two results, the values of b and c expressed in magnitudes were easily derived.

TABLE IV

MEAN PARALLAX, TYPES F, G, AND K

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″.00....	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000
0.01....	0.0066	0.0059	0.0053	0.0047	0.0042	0.0038	0.0034	0.0030	0.0027	0.0024	0.0022
0.02....	0.0107	0.0096	0.0086	0.0077	0.0069	0.0062	0.0055	0.0049	0.0044	0.0040	0.0035
0.03....	0.0142	0.0127	0.0114	0.0102	0.0091	0.0082	0.0073	0.0065	0.0059	0.0052	0.0047
0.04....	0.0174	0.0156	0.0139	0.0124	0.0111	0.0099	0.0089	0.0080	0.0072	0.0064	0.0057
0.05....	0.0203	0.0181	0.0163	0.0145	0.0130	0.0116	0.0104	0.0093	0.0084	0.0075	0.0067
0.06....	0.0230	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0105	0.0095	0.0085	0.0076
0.07....	0.0256	0.0229	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0106	0.0094	0.0084
0.08....	0.0281	0.0251	0.0225	0.0201	0.0180	0.0162	0.0144	0.0130	0.0116	0.0103	0.0093
0.09....	0.0305	0.0273	0.0244	0.0218	0.0196	0.0175	0.0157	0.0140	0.0126	0.0112	0.0101
0.10....	0.0328	0.0294	0.0263	0.0235	0.0210	0.0188	0.0169	0.0151	0.0135	0.0121	0.0108
0.2.....	0.053	0.048	0.043	0.038	0.034	0.030	0.027	0.024	0.022	0.020	0.018
0.3.....	0.070	0.063	0.056	0.050	0.045	0.040	0.036	0.032	0.029	0.026	0.023
0.4.....	0.086	0.077	0.069	0.061	0.055	0.049	0.044	0.040	0.035	0.032	0.028
0.5.....	0.100	0.090	0.081	0.072	0.064	0.058	0.052	0.046	0.041	0.037	0.033
0.6.....	0.114	0.102	0.091	0.082	0.073	0.065	0.059	0.052	0.047	0.042	0.038
0.7.....	0.127	0.114	0.102	0.091	0.081	0.073	0.065	0.058	0.052	0.047	0.042
0.8.....	0.139	0.125	0.112	0.100	0.089	0.080	0.072	0.064	0.057	0.051	0.046
0.9.....	0.151	0.135	0.121	0.108	0.097	0.087	0.078	0.069	0.062	0.055	0.050
1.0.....	0.163	0.146	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.054
2.0.....	0.263	0.236	0.211	0.188	0.169	0.151	0.135	0.121	0.109	0.097	0.087
3.0.....	0.348	0.312	0.279	0.249	0.223	0.200	0.179	0.161	0.144	0.129	0.115
4.0.....	0.426	0.381	0.341	0.305	0.273	0.244	0.219	0.196	0.176	0.157	0.140
5.0.....	0.498	0.446	0.399	0.356	0.319	0.285	0.256	0.229	0.206	0.184	0.164
6.0.....	0.565	0.506	0.453	0.404	0.362	0.324	0.290	0.260	0.233	0.208	0.186
7.0.....	0.628	0.562	0.503	0.450	0.403	0.360	0.322	0.289	0.259	0.232	0.207
8.0.....	0.690	0.618	0.553	0.494	0.442	0.396	0.355	0.318	0.285	0.255	0.227
9.0.....	0.748	0.670	0.600	0.536	0.479	0.430	0.384	0.344	0.309	0.276	0.246

Before proceeding to the actual solution of c as a function of b, it was necessary to correct the second members of the equations of condition for a systematic error, which tends to diminish the value of c for the early-type stars and to increase its value for the later types. The character of this error has been fully explained by Professor Kapteyn.[1]

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[1] *Mt. Wilson Contr.*, No. 42; *Astrophysical Journal*, **30,** 293, 1909.

The value of b proved to be independent of the spectral type. We may therefore adopt for b the mean of the values found for the different spectral classes, viz.,

$$b=+0^{\mathrm{m}}.025\pm0.004. \text{ (p. e.)}$$

The values found for the constant c are given in Table V.

TABLE V

Spectrum	c	Probable Error
	Mag.	Mag.
B5 to A0	0.00000	±0.00006
A1 to A9	+0.00021	0.00006
F0 to G2	+0.00033	0.00007
G3 to K5	+0.00011	±0.00005

The weighted mean of these values is

$$c=+0^{\mathrm{m}}.00015\pm0.00003.$$

The residuals with respect to the mean of the separate values in Table V show scarcely any systematic change with the spectral type. It may be that the B and A0 stars show the effect to a smaller degree than the later types, as is indicated by the value $c=0$ for the B5 to A0 stars. Mr. Adams' results,[1] derived by an entirely different method, give some indications in the same direction. Omitting the value for the B5 to A0 group, the mean for the other types is

$$c=+0.000195\pm0.00003,$$

a value 6.5 times its probable error.

CONCLUSION

The distant stars of the *Yerkes Actinometry* are, *ceteris paribus*, redder than the nearer ones. The increase of color-index per parsec is:

$$c=+0.00015\pm0.00003.$$

There are some indications that this effect does not exist for the B and early A stars. Supposing this to be the case, the amount of c for the other spectral types increases to

$$c=+0.000195\pm0.00003.$$

GRONINGEN
October 1915

[1] *Mt. Wilson Contrs.*, Nos. 78, 89; *Astrophysical Journal*, **39**, 89; **40**, 385, 1914.

Contributions from the Mount Wilson Solar Observatory, No. 111

THE PHOTOGRAPHIC DETERMINATION OF STELLAR PARALLAXES WITH THE 60-INCH REFLECTOR

By ADRIAAN VAN MAANEN

The determination of stellar parallaxes by means of photography dates back as far as 1886, when Pritchard made a first attempt in this direction.[1] Since that time the method has been applied by a number of astronomers, all of whom used the ordinary refractor for taking the plates, with the exception of Russell,[2] who employed the Sheepshanks equatorial of the Cambridge Observatory, a coudé telescope of the polar siderostat type, in which a mirror is inserted in the optical train. The question arose whether reflectors might not give results as good as or even better than those obtained with refractors, and the long-focus arrangement (80-foot) of the 60-inch reflector of the Mount Wilson Solar Observatory seemed a satisfactory instrument for an investigation of the question.

OBSERVATIONAL DETAILS

The very good definition with the equivalent 80-foot focus arrangement of the 60-inch reflector noted during observations with the Cassegrain spectrograph, suggested that good images might be secured by direct photography if a plate-holder were attached at this focus. The results have proved this to be true. With good seeing we obtain photographs showing very good definition over nearly the entire area of 8×10 plates (204×254 mm$=27'\times32'$).

From November 1913 until April 1914 the plate-carrier was attached to the instrument by four studs at the corners of a square with sides of 11 inches (230 mm). The two longer studs (at the north end of the plate-carrier) were 58 mm long, while the others were only a few millimeters in length, and were made so that the photographic plate would be nearly normal to the optical axis of the telescope. A test plate showed the deviation from 90° to be less than 5′, the north end of the plate being in shorter focus.

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[1] *Monthly Notices*, **47**, 87, 444, 1887.

[2] *Carnegie Institution of Washington Publication*, No. 147, 1911.

Were the relative position of the plate-holder and the optical axis constantly the same, this small inclination would have been of no disadvantage for the final parallaxes. But as the studs had to be removed every time the instrument was used for any other kind of work, there was a chance for a change in the inclination. The deviation from normal, however, could never exceed 5′, which, for the most distant comparison stars used in the work, would produce a maximum error of 0″.0024, and for the parallax star a still smaller error. Although such an error probably would be accidental rather than systematic for the 10–18 exposures used for each field, and therefore negligible in the resulting parallax, it seemed better to avoid it entirely. In March 1914 a cast-iron ring with inclined surfaces was therefore constructed for the attachment of the plate-carrier to the telescope. The ring which supports the plate-carrier is bolted to the tube of the telescope when the attachment is made. The orientation is fixed by two tapered pins in such a way that the relative inclination of two plates taken at different times can scarcely exceed 0′.5. The errors in the later plates arising from inclination of the plate-holder will therefore always be quite negligible.

But even though the relative position of plate-holder and the telescope tube remains the same, errors might be introduced by changes between the optical axis and the tube. Owing to the construction of the instrument such changes are hardly to be expected, especially as the plates of the same field are taken in practically the same position of the instrument. In replacing the mirrors after silvering, changes might occasionally occur, but all observers with the 60-inch reflector feel sure that changes in the inclination of the optical axis are very small, not exceeding one minute of arc; this means that the final parallaxes cannot be vitiated by errors from this source larger than 0″.001, a quantity which can be wholly neglected. Moreover, an actual comparison of the deviations for the pairs of exposures arranged according to different combinations of epochs of silvering the mirrors has shown that there is no appreciable effect. The large scale at the 80-foot focus, viz., 1 mm = 8″.245, makes it a very good instrument for accurate measures.

The plates were focused by the knife-edge method as many times during a night as the change in temperature required. The adjustment was made by moving the convex mirror at the upper end of the tube inward or outward by means of a motor operated from the station of the observer. Great accuracy can be reached in this way, while the operation requires only two or three minutes.

As the parallax star is in nearly all cases much brighter than the comparison stars, the rotating sector device, used by Schlesinger[1] in his parallax determinations with the Yerkes refractor, was employed. During the first three months a sector with a diameter of 78 mm was used, but this prevented the registration of the fainter stars within a radius of 5′ which otherwise could have been used as comparison stars. In November 1913 Mr. Abbot kindly made a sector of much smaller diameter for me, viz., 13 mm. The field now lost on account of the sector, its support, and the rubber belt is very small and the number of good comparison stars near the parallax star is accordingly greatly increased. The sector can be replaced by others with different openings from 0°.5 to 24°.0. The reduction constant can thus be varied from 3 to 7 magnitudes, approximately; for our kind of work, however, it is not necessary to know the constant with any great accuracy.

The use of a rotating sector or any other device to cut down the magnitude of the parallax star is necessitated by the requirement that the images of the parallax star and those of the comparison stars shall be nearly the same. A neglect of this precaution might introduce a serious magnitude error, due principally to a "guiding error"; the vitiating influence of such an error in parallax work was pointed out by Kapteyn in 1900.[2]

It will be shown later that it is advisable to use comparison stars as close to the parallax star as possible. For this reason a small sector is preferable to one of larger diameter. The difficulty of finding a suitable guiding star when the parallax star is placed behind the sector, which might be serious in using a small sector, is largely overcome by making the guiding microscopes (of which there is one on each of the longer sides of the plate) movable in two directions at right angles to each other. The total area which can be covered by the two eyepieces is in this way twice $4' \times 13'.5$. Moreover, the plate-holder can be rotated, but in practice it has never been necessary to use it in any position but 0° or 90°.

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[1] *Astrophysical Journal*, **32**, 384, 1910.

[2] *Publications of the Astronomical Laboratory at Groningen*, No. 1, p. 67, 1900.

Another serious systematic error, and one even more dangerous than a guiding error, is what Kapteyn has called the "hour-angle error."[1] The earlier parallax observer tried to take his plates when the parallax factor was a maximum or minimum. But in the available hours of darkness this can be done for stars of all right ascensions only by taking the plates at all possible hour-angles. As, however, a large difference in hour-angle has been proved to be a source of serious systematic error, it was thought better to avoid such differences as far as possible, even when the magnitude of the parallax factors might suffer thereby. I therefore adopted the rule never to take plates when the hour-angle at the middle of the exposure would be larger than 15°. Moreover, in combining morning and evening exposures I took care that the difference in their hour-angles should be small. In two cases only has this difference exceeded 5°; usually it was less than 2°, as can be seen from the tables for each field.

As a rule the exposure time was 15 minutes; in case of clouds or bad seeing this was lengthened up to 25 minutes. In most cases two exposures were made on a plate, but with moonlight only one, to avoid unnecessary fogging of the plates.

Seed "23" plates were used throughout; they were developed in a 4 per cent solution of Rodinal. The fine grain of these plates and the greater sharpness of image as compared with the Seed "27" fully counterbalance the disadvantage of the longer exposures. To facilitate the orientation of the plates in the measuring machine a star trail extending across the whole length of the field, except the center, was added to one plate of each field.

THE MEASURING MACHINE

The measuring apparatus used was the monocular arrangement of a Zeiss stereocomparator, which is fully described by Pulfrich.[2]

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[1] *Publications of the Astronomical Laboratory at Groningen*, No. 1, p. 67, 1900.

[2] *Zeitschrift für Instrumentenkunde*, **24**, 161, 1904.

It was shown in a previous article[1] that this instrument can be used successfully for accurate measurement, even when the completion of a single pair of plates requires several hours or even days, provided only that care be taken that the temperature does not change too much.

With this precaution in view a few improvements were made in the instrument. The lamps for illuminating the plates were provided with shields to prevent heat reaching the instrument. Moreover, heavy pieces of plate glass (7 mm thick) were inserted between the lamps and the plates. To reduce further the heating effect other parts of the instrument were covered with asbestos. During the time needed for measuring a pair of plates (about one hour) no appreciable change in temperature was found. As the room is maintained at a nearly constant temperature by thermostatic control, it is unlikely that any effect would be observed even if the measures consumed much more time. As a final precaution the parallax star was measured at the beginning as well as at the end of each series; by taking the mean of these measures we eliminate a great deal of any possible effect due to a change of temperature during the measures.

A few other changes in the instrument may be mentioned here: the right-hand plate-holder was provided with a slow motion for rotation, which is of great help in the orientation of the plates. The plate-carriers were provided with attachments such that plates of all thicknesses have their films in the same position.

Only a small part of the micrometer screw (less than one revolution, or 0.5 mm) was used in the measures. This was tested for periodic and progressive errors, both of which are exceedingly small, as was evident from three series of measures made for this purpose.

The manipulation of the plates in the measuring machine will be briefly described. The two plates to be compared may for the sake of brevity be called *m* (morning) and *e* (evening). They are placed in the carriers with the film toward the measurer, *m* in the left-hand, *e* in the right-hand, carrier. By means of the occulting shutter we can look either at *m* or at *e*. Focus the eyepiece on the wires; look at m and move the whole optical system inward or outward by means of the rack-and-pinion arrangement until a star image on m is in the same focus as the wires. Rotate the micrometer until the horizontal wires are parallel to the horizontal motion of the plates; the vertical wire will then give the readings in the direction of the horizontal motion. As a star trail has been impressed on one plate of each field, we may assume m to be this plate. Rotate m so that at equal distances from the center of the plate the trail is seen half-way between the two horizontal wires. Now bring the central star on the vertical wire and half-way between the horizontal wires and change the shutter. We are now looking at e; focus this plate by its separate focusing arrangement; by means of the horizontal and vertical slow motion of the e-plate-holder and the newly provided slow motion in rotation we make the star images on e coincide as closely as possible with the corresponding images on m. The orientation of later plates is accomplished by means of those previously measured, one plate of any given pair being left in position until one of the next succeeding pair has been adjusted.

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[1] *Astronomical Journal*, **27**, 140, 1912.

We are now ready for the measurement of the pair m–e. Bring a star on m half-way between the horizontal wires and nearly on the vertical wire, bisect it with the vertical wire of the micrometer, and read the head; repeat this as many times as is thought necessary (5 times in the present case); turn the handle which occults m and simultaneously opens the shutter for e. The same star will be seen at nearly the same place in the field. Bisect this the necessary number of times, reading the head after each. Then move both plate-holders horizontally and vertically until the next star appears in the center of the field, and proceed as before. After finishing a pair of plates in one position, both are rotated 180° and the measures repeated.

Before beginning the discussion of the reduction of the measured quantities I wish to point out the advantages of the stereo-comparator over the usual measuring machine. With the latter, one plate at a time is measured and the positions of the stars are either compared with a scale or determined by means of a screw as long as the part of the plate to be measured. In using a scale, settings on both the star images and the scale are required, and the errors of the latter must be known or they will enter into the measures. In using a long screw (unless the changes of scale from plate to plate are negligible) we must know its errors over the whole length to be used, which in our case is as much as 114 mm. With the stereocomparator, on the other hand, we use only a very small part of the micrometer screw, in the present case less than 0.5 mm. With the scale measuring-machine the relative position of the images of a star on a pair of plates would require four settings, viz., star, scale, star, scale; with the long-screw machine, two settings. With the stereocomparator as here used two settings and two readings are also required, but by introducing an extra slide in the micrometer, with a wire whose slow motion would leave the distance between itself and the original micrometer wire unchanged, the distance separating the images would be given by two settings and a single reading. A further advantage of the stereocomparator is that the orientation is greatly simplified. From this it will be clear that with the stereocomparator the measures can be made in a shorter time and, on account of the errors in a scale or a long screw, with greater accuracy; and if we can avoid the use of a scale, there is also a gain in the reductions.

THE REDUCTIONS

For practical reasons it is advisable to measure parallaxes in right ascension. The decrease in the parallax factor as compared with that for measures made in longitude is small, while the gain in convenience and time is rather large. I have therefore made it a rule to measure only in right ascension. As stated above, the measures were made in two positions of the plates, differing in orientation by 180°. The measures give for each star on each pair of plates a distance D, from which, by the reductions now to be described, we derive the parallax of the parallax star as well as those of the comparison stars. The means of the D's from the two positions of each pair of plates were used in the computations. As we may subtract a constant from all the values of D for each pair of plates, and as it is easier to deal with small numbers, all the D's of each pair were diminished by the D of the parallax star; the resulting values for the parallax star were then always equal to zero.

Boss 96 $\alpha=0^h26^m13^s$ $\delta=+52^\circ17'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
187 ...	1914 Aug. 13	+0°.5	VM	g	+0.676	+0.62	
196_1...	14	−8.2	H	g	+0.665	+0.62	
196_2...	14	−0.5	VM	g	+0.664	+0.62	
197 ...	14	+4.8	H	g	+0.664	+0.62	
368 ...	Nov. 10	+5.2	Ca	fg	−0.593	+0.86	
369 ...	10	+11.7	Ca	fg	−0.593	+0.86	
384_1...	11	−15.0	VM	fg	−0.602	+0.87	
384_2...	11	−10.7	VM	fg	−0.602	+0.87	Trail
385 ...	11	−5.0	H	fg	−0.602	+0.87	
401 ...	Dec. 8	+13.2	H	fg	−0.842	+0.94	Clouds 20m exp.
406_1...	9	−2.5	H	g	−0.847	+0.94	
406_2...	9	+2.0	VM	g	−0.847	+0.94	
616 ...	1915 July 31	−10.5	VM	g	+0.790	+1.58	
617 ...	31	−4.7	H	g	+0.790	+1.58	
667 ...	Aug. 31	+9.5	VM	g	+0.443	+1.67	Clouds 20m exp.
682 ...	Sept. 1	+10.5	VM	g	+0.429	+1.67	Clouds 25m exp.

Mor.–Ev.	Δp	Δm	M	v	Δt
616 −384_1....	+1.39	+0.71	−14	+12	+4°.5
196_1−384_2....	+1.27	−0.25	+26	+10	+2.5
617 −385	+1.39	+0.71	−27	−1	+0.3
196_2−406_1....	+1.51	−0.32	+4	−15	+2.0
187 −406_2....	+1.52	−0.32	+22	+3	−1.5
197 −368	+1.26	−0.24	+22	+6	−0.4
667 −369	+1.04	+0.81	−20	+12	−2.2
682 −401	+1.27	+0.73	−52	−25	−2.7

$$+2.51\,\mu_\alpha+2.16\,\pi=-103 \qquad \pi=+3.6=+0''.006\pm0''.005^*$$
$$+2.16\,\mu_\alpha+14.34\,\pi=-44 \qquad \mu_\alpha=-44.2=-0''.073\pm0''.010$$

* In a former paper (*Proceedings of the National Academy of Sciences*, **1**, 189, 1915) the parallax was given as +0″.026±0″.007. This, however, was based on some plates of inferior quality. The value given here is therefore the more trustworthy.

Comp. Star	Br.	x	y	π	P.E.
1........	f	−0′.5	+1′.1	−0″.004	±0″.009
2........	f	+1.6	+2.9	−0.010	0.006
3........	f	+4.0	+3.6	+0.006	0.004
4........	bf	+2.7	0.0	+0.005	0.007
5........	f	+0.4	−3.0	−0.001	0.006
6........	bf	+1.6	−5.8	−0.006	0.004
7........	b	−5.5	−0.5	+0.007	0.004
8........	f	−4.3	+2.7	+0.005	±0.004

Boss 420 $\alpha = 1^h 47^m 17^s$ $\delta = +40^\circ 14'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
64_1 . . .	1913 Nov. 23	− 5°7	VM	fas	−0.491	−0.10	Some astigmatism of 60-in. mirror
64_2 . . .	23	− 1.0	C	fas	−0.491	−0.10	
66 . . .	Dec. 20	− 8.5	VM	pf	−0.792	−0.03	
198_1 . . .	1914 Aug. 14	− 9.5	VM	g	+0.876	+0.62	
198_2 . . .	14	− 5.0	H	g	+0.876	+0.62	
205 . . .	Sept. 11	−15.0	VM	f	+0.611	+0.70	
206 . . .	11	− 8.7	H	f	+0.611	+0.70	20^m exp.
230 . . .	13	− 2.7	H	f	+0.586	+0.70	20^m exp.
246 . . .	14	+ 0.2	VM	g	+0.572	+0.71	
247 . . .	14	+ 5.7	H	fas	+0.572	+0.71	
386 . . .	Nov. 11	−13.5	H	g	−0.312	+0.87	
387_1 . . .	11	− 7.0	VM	fg	−0.313	+0.87	
387_2 . . .	11	− 2.5	H	fg	−0.313	+0.87	
402 . . .	Dec. 8	+ 7.5	VM	g	−0.674	+0.94	Clouds 20^m exp.

Mor.–Ev.	Δp	Δm	M	v	Δt
205 −386	+0.92	−0.17	+11	+10	−1°5
198_1− 66	+1.67	+0.65	0	− 8	−1.0
206 −387_1	+0.92	−0.17	−16	−17	−1.7
198_2− 64_1	+1.37	+0.72	+26	+18	+0.7
230 −387_2	+0.90	−0.17	+25	+24	−0.2
246 − 64_2	+1.06	+0.81	− 3	−11	+1.2
247 −402	+1.25	−0.23	−11	−12	−1.8

$$+1.74\,\mu_\alpha + 2.18\,\pi = +17 \qquad \pi = +2.2 = +0''.004 \pm 0''.007$$
$$+2.18\,\mu_\alpha + 9.86\,\pi = +37 \qquad \mu_\alpha = +6.9 = +0''.011 \pm 0''.018$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	−1′.9	+4′.0	+0″.007	±0″.008
2.	f	−0.1	+1.0	+0.003	0.008
3.	f	+0.3	+4.0	+0.007	0.006
4.	f	+4.0	+1.4	+0.007	0.004
5.	f	+1.5	−2.7	−0.030	0.010
6.	bf	−1.0	−5.6	+0.013	0.004
7.	ff	−1.2	−1.6	−0.015	0.003
8.	bf	−4.7	−0.4	+0.007	±0.004

The resulting differences for the comparison stars are partly due to the shift caused by parallax and proper motion, but also to corrections for zero point, scale, orientation, refraction, aberration, precession, nutation, and inclination. To derive the shifts due to parallax and proper motion the differences must be freed from the

Boss 592, 593 = Bu 1332 $\alpha = 2^h31^m12^s$ and $2^h31^m14^s$ $\delta = +24°13'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
280_1...	1914	Sept.	16	$-2°.0$	VM	g	+0.694	+0.71	
280_2...			16	+2.2	H	fg	+0.694	+0.71	Trail
294 ...		Oct.	11	+12.0	VM	g	+0.359	+0.78	
312 ...			12	+9.0	VM	g	+0.360	+0.78	
403_1...		Dec.	8	+5.5	VM	g	−0.547	+0.94	Clouds 20^m exp.
403_2...			8	+11.7	H	g	−0.547	+0.94	Clouds 20^m exp.
408_1...			9	−11.2	VM	fg	−0.559	+0.94	
408_2...			9	−6.7	H	fg	−0.561	+0.94	
409_1...			9	−0.7	VM	g	−0.561	+0.94	
409_2...			9	+3.7	H	fg	−0.561	+0.94	
410 ...			9	+10.0	VM	fg	−0.561	+0.94	
696_1...	1915	Sept.	2	−11.5	H	fg	+0.844	+1.67	
696_2...			2	−6.7	VM	fg	+0.844	+1.67	
697 ...			2	−0.2	H	fg	+0.844	+1.67	

Mor.–Ev.	Δp	Δm	M (592)	M (593)	v (592)	v (593)	Δt
696_1–408_1.....	+1.40	+0.73	+75	+113	+5	+15	$-0°.3$
696_2–408_2.....	+1.41	+0.73	+102	+124	+32	+25	0.0
280_1–409_1.....	+1.26	−0.23	−38	−43	−24	−40	−1.3
697 –409_2.....	+1.41	+0.73	+31	+57	−39	−42	−3.9
280_2–403_1.....	+1.24	−0.23	+19	−4	+33	−1	−3.3
312 –410	+0.92	−0.16	+28	+15	+37	+16	−1.0
294 –403_2.....	+0.91	−0.16	−56	+41	−47	+42	+0.3

$$\left.\begin{aligned} +1.75\,\mu_a + 2.20\,\pi &= +161 \\ +2.20\,\mu_a + 10.75\,\pi &= +244 \end{aligned}\right\}(592)\quad \left.\begin{aligned} +216 \\ +405 \end{aligned}\right\}(593)$$

$$(592)\begin{cases} \pi = +\ 5.1 = +0''.008 \pm 0''.015 \\ \mu_a = +\ 86.1 = +0''.142 \pm 0''.037 \end{cases} \qquad (593)\begin{cases} \pi = +\ 16.7 = +0''.028 \pm 0''.014 \\ \mu_a = +102.5 = +0''.169 \pm 0''.035 \end{cases}$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	+4'.8	+2'.7	−0''.010	±0''.010
2........	f	+1.1	+5.8	−0.015	0.007
3........	f	−4.9	+0.7	+0.005	0.006
4........	ff	−5.4	−4.7	−0.003	0.009
5........	f	−0.5	−5.9	−0.002	0.008
6........	f	+0.1	−4.4	+0.001	0.006
7........	bf	+6.5	−0.5	+0.014	0.007
8........	ff	−2.1	+5.4	+0.012	±0.007

displacements arising from all other sources, which are functions of the co-ordinates of the stars on the plate and take the form

$$a+bx+cy+dx^2+exy+fy^2+\text{terms of higher order in } x \text{ and } y.$$

Boss 672 $\alpha = 2^h 53^m 3^s$ $\delta = +46^\circ 49'$

Pl. No.	Date	*t*	Obs.	*Qy*	*p*	*m*	Remarks
54 ...	1913 Nov. 22	−13°5	VM	f	−0.234	−0.10	
68 ...	Dec. 22	+ 0.2	VM	pf	−0.665	−0.02	
80 ...	1914 Jan. 20	− 1.0	VM	f	−0.906	+0.06	Sky thick
232 ...	Sept. 13	− 4.0	H	fg	+0.791	+0.70	20^m exp.
233 ...	13	+ 3.5	VM	f	+0.789	+0.70	20^m exp.
234 ...	13	+10.2	H	fas	+0.789	+0.70	20^m exp. Trail
248 ...	14	− 4.0	VM	g	+0.781	+0.71	
249 ...	14	+ 6.7	H	vg	+0.779	+0.71	
263 ...	15	− 7.5	H	g	+0.771	+0.71	
264 ...	15	− 2.2	VM	g	+0.771	+0.71	Trail
423_1 ...	1915 Jan. 9	+ 7.5	VM	f	−0.840	+1.03	
423_2 ...	9	+12.0	H	fg	−0.840	+1.03	
435 ...	10	− 6.7	H	f	−0.847	+1.03	
436 ...	10	+ 4.5	H	f	−0.847	+1.03	

Mor.–Ev.	Δ*p*	Δ*m*	*M*	*v*	Δ*t*	Remarks
263–435	+1.62	−0.32	−24	0	−0°8	
248– 54	+1.02	+0.81	+48	+15	+9.5	Large difference in *t* caused by mistake in record book
264– 80	+1.68	+0.65	+19	− 3	−1.2	
232– 68	+1.46	+0.72	+13	−14	−4.2	
233–436	+1.64	−0.33	−17	+ 8	−1.0	
249–423_1	+1.62	−0.32	− 8	+16	−0.8	
234–423_2	+1.63	−0.33	−46	−21	−1.8	

$$+2.02\,\mu_a + 0.85\,\pi = +92 \qquad \pi = -\ 5.8 = -0''.009 \pm 0''.004$$

$$+0.85\,\mu_a + 16.55\,\pi = -55 \qquad \mu_a = +48.0 = +0''.079 \pm 0''.012$$

Comp. Star	Br.	*x*	*y*	*π*	P.E.
1	f	−0'.5	+1'.6	+0".001	±0".010
2	bf	+0.1	+3.4	+0.018	0.010
3	f	+1.1	+1.6	+0.003	0.009
4	bf	+4.0	+1.0	+0.001	0.006
5	bf	+1.9	−1.2	+0.012	0.011
6	ff	0.0	−2.5	−0.026	0.010
7	f	−1.0	−1.4	−0.017	0.015
8	f	−4.3	−1.2	+0.041	0.011
9	ff	−2.7	+1.1	−0.033	±0.009

If due precautions are taken in the choice of the comparison stars, the corrections corresponding to the terms of the second and higher orders of the co-ordinates will be negligible, and the correction to be applied takes the simple form

$$a + bx + cy$$

Boss 1549 $\alpha=6^h6^m15^s$ $\delta=+22°56'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
74 ...	1914	Jan.	19	− 2°.0	VM	g	−0.458	+0.06	
82_1...			20	− 5.0	VM	g	−0.473	+0.06	
82_2...			20	− 0.7	VM	g	−0.473	+0.06	Clouds, trail
93 ...		Feb.	27	−11.5	VM	fas	−0.913	+0.16	Some astigmatism of 60-in. mirror
316 ...		Oct.	12	−11.5	H	g	+0.950	+0.78	
317 ...			12	− 6.0	VM	g	+0.950	+0.78	
318 ...			12	− 0.2	VM	g	+0.950	+0.78	
333 ...			13	− 7.0	H	vg	+0.945	+0.79	
375 ...		Nov.	10	−11.2	Ca	g	+0.680	+0.86	
376 ...			10	− 4.7	Ca	g	+0.680	+0.86	
394_1...			11	− 6.2	VM	g	+0.668	+0.87	
394_2...			11	− 1.5	H	g	+0.668	+0.87	
453_1...	1915	Feb.	6	−10.0	VM	g	−0.707	+1.10	
453_2...			6	− 5.2	H	g	−0.707	+1.10	
454 ...			6	− 0.5	VM	g	−0.707	+1.10	
474 ...			7	− 6.5	H	fg	−0.714	+1.11	Trail

Mor.–Ev.	Δp	Δm	M	v	Δt
316 − 93	+1.86	+0.62	+12	+ 5	0°.0
375 −453_1....	+1.39	−0.24	− 9	− 8	−1.2
333 −474	+1.66	−0.32	+ 3	+ 5	−0.5
394_1−453_2....	+1.38	−0.23	− 1	0	−1.0
317 − 82_1....	+1.42	+0.72	+11	+ 3	−1.0
376 − 74	+1.14	+0.80	+ 9	+ 1	−2.7
394_2−454	+1.38	−0.23	− 2	− 1	−1.0
318 − 82_2....	+1.42	+0.72	+ 2	− 5	+0.5

$$+2.32\,\mu_\alpha+\ 2.60\,\pi=+24 \qquad \pi=+0.8=+0''.001\pm0''.001$$
$$+2.60\,\mu_\alpha+17.29\,\pi=+39 \qquad \mu_\alpha=+9.4=+0''.016\pm0''.004$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	+0′.1	+0′.4	−0″.007	±0″.003
2........	ff	−1.2	+0.7	+0.007	0.007
3........	b	−0.5	+1.2	−0.007	0.007
4........	bf	−2.2	+1.9	−0.005	0.007
5........	ff	−0.8	−1.5	−0.017	0.007
6........	ff	+0.3	−2.2	+0.017	0.008
7........	b	+0.5	−1.6	+0.003	0.004
8........	ff	+1.9	−0.5	0.000	0.007
9........	f	+3.4	−0.8	+0.007	±0.006

where x and y are the co-ordinates of the stars, and a, b, and c constants to be derived for each pair of plates from the values for D. The necessary condition is that x and y of the comparison stars

Boss 1868 $\alpha = 7^h 9^m 42^s$ $\delta = +28^\circ 4'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
90_1...	1914	Feb.	26	$-11^\circ.7$	C	pf	-0.770	-0.84	
90_2...			26	-8.0	VM	pf	-0.770	-0.84	
94_1...			27	-13.5	C	fas	-0.781	-0.84	
94_2...			27	-7.2	VM	fas	-0.781	-0.84	
334 ...		Oct.	13	-15.0	VM	fg	$+0.989$	-0.21	
365 ...			15	-14.5	VM	g	$+0.985$	-0.21	Trail
378 ...		Nov.	10	-3.5	Ca	pf	$+0.835$	-0.14	
379 ...			10	$+2.7$	H	fg	$+0.835$	-0.14	
380 ...			10	$+12.0$	Ca	g	$+0.835$	-0.14	
395_1...			11	-11.0	VM	g	$+0.822$	-0.13	
395_2...			11	-6.2	H	g	$+0.822$	-0.13	
396 ...			11	-0.2	VM	g	$+0.822$	-0.13	
455_1...	1915	Feb.	6	-9.7	H	g	-0.506	$+0.10$	
455_2...			6	-5.2	VM	g	-0.507	$+0.10$	
456 ...			6	$+0.2$	H	g	-0.507	$+0.10$	
492 ...		Mar.	8	$+7.5$	VM	g	-0.864	$+0.19$	

Mor.–Ev.	Δp	Δm	M	v	Δt
$334-94_1$....	$+1.77$	$+0.63$	0	-2	$-1^\circ.5$
$265-90_1$....	$+1.76$	$+0.63$	-3	-5	-2.8
395_1-455_1....	$+1.33$	-0.23	-13	-19	-1.3
395_2-90_2....	$+1.59$	$+0.71$	$+18$	$+16$	$+1.8$
$378-94_2$....	$+1.62$	$+0.70$	-8	-10	$+3.7$
$396-455_2$....	$+1.33$	-0.23	$+14$	$+8$	$+5.0$
$379-456$	$+1.34$	-0.24	$+14$	$+7$	$+2.5$
$380-492$	$+1.70$	-0.33	$+11$	$+3$	$+4.5$

$$+2.06\,\mu_a + 2.99\,\pi = -2 \qquad \pi = +3.5 = +0''.006 \pm 0''.003$$
$$+2.99\,\mu_a + 19.61\,\pi = +51 \qquad \mu_a = -6.1 = -0''.010 \pm 0''.010$$

Comp. Star	Br.	x	y	π	P.E.
1........	ff	$+1'.6$	$+1'.8$	$-0''.013$	$\pm 0''.008$
2........	f	$+2.6$	$+2.7$	$+0.020$	0.006
3........	f	$+5.5$	$+1.0$	$+0.002$	0.006
4........	ff	$+5.5$	-2.2	-0.012	0.011
5........	ff	$+1.6$	-2.9	$+0.030$	0.015
6........	ff	-1.9	-3.2	-0.033	0.011
7........	bf	-5.5	-1.0	$+0.026$	0.006
8........	f	-4.3	$+3.3$	$+0.008$	0.005
9........	b	-2.5	$+2.5$	-0.025	± 0.007

should be small. In this investigation the distance of every comparison star from the central star has been less than 8′.

Let us see what the corresponding values of the second-order terms may be. The differential refraction would cause a maximum

Boss 2020 $\alpha=7^h36^m25^s$ $\delta=+14°27'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
88 . . .	1914	Feb.	21	+2°0	VM	f	−0.631	−0.86	Hazy
91_1. . .			26	−6.5	C	pf	−0.695	−0.84	
91_2. . .			26	−1.0	VM	f	−0.695	−0.84	
95 . . .			27	−1.5	VM	fas	−0.706	−0.84	Some astigmatism of 60-in. mirror
381 . . .		Nov.	10	+13.5	H	g	+0.880	−0.14	
397 . . .			11	+0.5	H	g	+0.869	−0.13	
398 . . .			11	+6.0	VM	g	+0.868	−0.13	
399 . . .			11	+11.7	H	fg	+0.868	−0.13	
415 . . .		Dec.	9	−7.0	VM	g	+0.555	−0.06	Clouds 20^m exp.
416 . . .			9	+0.5	H	g	+0.555	−0.06	Clouds 20^m exp.
417 . . .			9	+7.5	VM	g	+0.555	−0.06	
488_1. . .	1915	Mar.	6	+5.2	VM	g	−0.781	+0.18	Clouds 25^m exp.
488_2. . .			6	+12.7	VM	g	−0.781	+0.18	Clouds 25^m exp.
493 . . .			8	+8.0	VM	g	−0.802	+0.19	Trail

Mor.–Ev.	Δp	Δm	M	v	Δt
415– 91_1.	+1.25	+0.78	+21	+24	−0°5
397– 95	+1.58	+0.71	−16	−15	+2.0
416– 91_2.	+1.25	+0.78	+8	+11	+1.5
398– 88	+1.50	+0.73	−20	−19	+4.0
417–488_1.	+1.34	−0.24	0	−5	+2.3
399–493	+1.67	−0.32	+8	+2	+3.7
381–488_2.	+1.66	−0.32	+14	+8	+0.8

$$+2.51\mu_\alpha+2.80\pi=-11 \qquad \pi=+2.4=+0\overset{''}{.}004\pm0\overset{''}{.}005$$

$$+2.80\mu_\alpha+15.22\pi=+17 \qquad \mu_\alpha=-7.1=-0\overset{''}{.}012\pm0\overset{''}{.}013$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	+1′9	+0′5	+0″005	±0″010
2.	f	+1.6	+2.9	−0.020	0.004
3.	f	+4.3	−0.7	−0.002	0.007
4.	f	+4.4	−4.8	+0.007	0.004
5.	bf	−0.8	−4.4	+0.003	0.002
6.	ff	−2.9	−2.2	−0.007	0.003
7.	f	−3.0	+2.9	+0.007	0.007
8.	bf	−4.3	+4.3	+0.003	±0.006

error of 0″001 if the difference in hour-angle were 15°. In no case has the difference been as large as this, and the correction can therefore be neglected. The differential aberration can never exceed 0″0002 for x and y smaller than 8′. The second-order terms due to precession and nutation are rigorously zero, since these changes

Boss 2921 $\alpha = 10^h 53^m 58^s$ $\delta = +36°38'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
77 ...	1914 Jan. 19	+ 5°.5	C	g	+0.617	−0.95	
85 ...	20	− 1.2	C	g	+0.603	−0.94	
106 ...	April 17	+ 0.5	VM	f	−0.657	−0.70	
115_1 ...	18	0.0	VM	fas	−0.668	−0.70	Some astigmatism of 60-in. mirror
115_2 ...	18	+ 4.2	VM	gas	−0.668	−0.70	
124_1 ...	19	+ 2.2	VM	gas	−0.690	−0.70	
124_2 ...	19	+ 6.5	VM	gas	−0.690	−0.70	
429 ...	1915 Jan. 9	+ 0.2	H	fg	+0.727	+0.03	
447_1 ...	10	− 5.0	H	vg	+0.716	+0.03	
447_2 ...	10	− 1.0	VM	vg	+0.716	+0.03	Trail

Mor.–Ev.	Δp	Δm	M	v	Δt	Remarks
447_1–115_1....	+1.38	+0.73	+108	+17	−5°.0	
447_2–106	+1.37	+0.73	+ 77	−13	−1.5	Star 5 not measured
429 –124_1....	+1.42	+0.73	+ 90	− 3	−2.0	
85 –115_2....	+1.27	−0.24	+ 60	+ 9	−5.4	
77 –124_2....	+1.31	−0.25	+ 46	− 7	−1.0	

$$+1.71\,\mu_\alpha + 2.42\,\pi = +176 \qquad \pi = +47.1 = +0''.078 \pm 0''.006$$
$$+2.42\,\mu_\alpha + 9.13\,\pi = +518 \qquad \mu_\alpha = +36.3 = +0''.059 \pm 0''.014$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	+2'.9	−1'.0	+0''.086	±0''.015
2........	f	+5.8	+0.3	−0.053	0.011
3........	ff	−2.7	+3.6	−0.007	0.015
4........	f	−5.5	+2.5	0.000	0.014
5........	ff	−1.5	−1.4	−0.026:	±0.007

in the co-ordinates affect only the orientation of the plates. As to the relative inclination of the plates, it has already been pointed out (p. 2) that this cannot surpass 5′, in which case the maximum correction for a star may be as high as 0''.0024. It was also pointed out that such an error probably would be accidental rather than systematic for the 10–18 exposures used for each field, and could therefore introduce no serious error into the resulting parallaxes. For the later plates, taken after the introduction in March 1914 of the cast-iron ring for attaching the plate-carrier to the instrument, any error of this kind is entirely negligible.

Boss 3233 $\alpha = 12^h 20^m 13^s$ $\delta = +24^\circ 29'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
78_1 . . .	1914	Jan.	19	$-6^\circ.5$	VM	f	$+0.824$	-0.95	
78_2 . . .			19	-2.5	VM	g	$+0.824$	-0.95	
86 . . .			20	-9.2	VM	f	$+0.819$	-0.94	
116_1 . . .		April	18	-10.0	VM	g	-0.355	-0.70	
116_2 . . .			18	-6.5	VM	g	-0.355	-0.70	
130_1 . . .		May	17	$+0.7$	P	fas	-0.721	-0.62	
130_2 . . .			17	$+7.2$	P	fas	-0.721	-0.62	
133_1 . . .			18	-5.2	P	f	-0.731	-0.62	
133_2 . . .			18	$+1.5$	P	f	-0.731	-0.62	
431 . . .	1915	Jan.	9	-8.2	H	fg	$+0.877$	$+0.03$	
433 . . .			9	$+3.0$	H	g	$+0.877$	$+0.03$	
466 . . .		Feb.	6	$+6.0$	VM	g	$+0.677$	$+0.10$	Trail

Mor.–Ev.	Δp	Δm	M	v	Δt
$86-116_1$	$+1.17$	-0.24	-2	$+4$	$+0^\circ.8$
$431-116_2$	$+1.23$	$+0.73$	-22	-47	-1.7
78_2-130_1	$+1.55$	-0.33	-14	-5	-3.2
78_1-133_1	$+1.56$	-0.33	-11	-2	-1.3
$433-133_2$	$+1.61$	$+0.65$	$+38$	$+14$	$+1.5$
$466-130_2$	$+1.40$	$+0.72$	$+57$	$+32$	-1.2

$$+1.75\mu_a + 1.66\pi = +59 \qquad \pi = +1.6 = +0''.003 \pm 0''.010$$
$$+1.66\mu_a + 12.26\pi = +73 \qquad \mu_a = +32.0 = +0''.053 \pm 0''.026$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	$-2'.3$	$-4'.0$	$-0''.008$	$\pm 0''.009$
2.	f	-3.8	-1.5	-0.007	0.015
3.	f	-3.6	$+2.2$	$+0.015$	0.012
4.	ff	-0.8	$+1.2$	-0.038	0.017
5.	f	$+1.0$	$+3.4$	-0.010	0.007
6.	bf	$+2.1$	$+3.2$	$+0.030$	0.006
7.	f	$+3.4$	-1.1	$+0.045$	0.014
8.	f	$+3.2$	-2.7	-0.025	± 0.007

Having shown that we may neglect terms of the second and higher orders of the co-ordinates I shall now outline the reduction process. On each pair of plates every star measured gave an equation of condition of the form

$$a + bx + cy + \Delta p\pi + \Delta m\mu_a = D$$

Boss 3650 $\alpha = 14^h 9^m 17^s$ $\delta = +13^\circ 26'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
136_1...	1914 June 15	$+6^\circ.2$	Sh	fg	-0.729	$+0.46$	
136_2...	15	$+12.7$	Sh	fg	-0.729	$+0.46$	
140 ...	16	$+2.2$	Sh	g	-0.738	$+0.46$	
470 ...	1915 Feb. 6	$+6.5$	VM	fg	$+0.906$	$+1.10$	11^m exp. Sector stopped
484 ...	7	-7.7	VM	g	$+0.900$	$+1.11$	
485 ...	7	-2.2	H	f	$+0.900$	$+1.11$	
486 ...	7	$+3.5$	VM	fg	$+0.900$	$+1.11$	Trail
503_1...	Mar. 8	$+3.5$	VM	g	$+0.683$	$+1.19$	
503_2...	8	$+8.0$	VM	g	$+0.683$	$+1.19$	
554_1...	June 4	-6.7	VM	g	-0.599	$+1.43$	
554_2...	4	-2.5	H	g	-0.599	$+1.43$	
555 ...	4	$+2.0$	VM	fg	-0.599	$+1.43$	

Mor.–Ev.	Δp	Δm	M	v	Δt
484 – 554_1....	$+1.50$	-0.32	$+153$	$+7$	$-1^\circ.0$
485 – 554_2....	$+1.50$	-0.32	$+134$	-12	$+0.3$
486 – 555	$+1.50$	-0.32	$+150$	$+4$	$+1.5$
503_1 – 140	$+1.42$	$+0.73$	-48	$+4$	$+1.3$
470 – 136_1....	$+1.64$	$+0.64$	-31	-8	$+0.3$
503_2 – 136_2....	$+1.41$	$+0.73$	-45	$+7$	-4.7

$$+1.77\,\mu_a + 1.68\,\pi = -228 \qquad \pi = +58.2 = +0''.096 \pm 0''.003$$

$$+1.68\,\mu_a + 13.45\,\pi = +474 \qquad \mu_a = -184.0 = -0''.302 \pm 0''.008$$

Comp. Star	Br.	x	y	π	P.E.
1........	ff	$+3'.4$	$+0'.8$	$-0''.002$	$\pm 0''.011$
2........	f	$+4.1$	$+2.1$	$+0.018$	0.011
3........	b	-5.6	$+5.2$	$+0.019$	0.012
4........	ff	-2.9	$+0.7$	-0.045	0.031
5........	ff	-3.4	-2.1	-0.015	0.014
6........	bf	-3.0	-3.4	$+0.011$	0.009
7........	f	-0.5	-6.6	$+0.036$	0.004
8........	f	$+3.6$	-4.7	$+0.005$	0.009
9........	ff	$+4.3$	-4.0	-0.026	± 0.008

where Δp is the difference in parallax factors and Δm the difference in years between the two exposures. On the supposition that the parallaxes and the proper motions of the comparison stars are zero, as is necessary in all differential parallax work, each comparison star gave an equation of condition of the form

$$a + bx + cy = D$$

Boss 4048 $\alpha=15^h50^m10^s$ $\delta=+20°36'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
110_1...	1914 April 17	+4°5	VM	fg	+0.526	+0.30	
110_2...	17	+9.2	VM	fg	+0.526	+0.30	
137 ...	June 15	+3.7	H	f	−0.410	+0.46	
144_1...	July 25	+7.0	VM	g	−0.881	+0.57	
144_2...	25	+12.0	VM	g	−0.881	+0.57	
157_1...	27	+10.0	VM	g	−0.891	+0.57	
157_2...	27	+14.2	VM	g	−0.891	+0.57	
522_1...	1915 May 5	+4.5	VM	fg	+0.259	+1.35	
522_2...	5	+9.0	H	g	+0.259	+1.35	
523 ...	5	+14.5	VM	g	+0.258	+1.35	Trail

Mor.-Ev.	Δp	Δm	M	v	Δt
110_1–137	+0.94	−0.16	+5	−25	+0°8
522_1–144_1....	+1.14	+0.78	−18	−6	−2.5
522_2–157_1....	+1.15	+0.78	−1	+11	−1.0
110_2–144_2....	+1.41	−0.27	+61	+16	−2.8
523 –157_2....	+1.15	+0.78	−16	−4	+0.3

$$+1.93\,\mu_\alpha+2.16\,\pi=-44 \qquad \pi=+22.8=+0''.038\pm0''.010$$
$$+2.16\,\mu_\alpha+6.81\,\pi=+51 \qquad \mu_\alpha=-48.7=-0''.081\pm0''.019$$

Comp. Star	Br.	x	y	π	P.E.
1........	ff	−0′.1	+1′.8	+0″005	±0″063
2........	ff	+1.9	+3.0	−0.030	0.052
3........	bf	+1.2	+5.6	+0.021	0.020
4........	f	−1.9	+5.4	+0.002	0.007
5........	f	−1.0	−1.6	−0.028	0.026
6........	f	−3.7	−1.2	−0.025	0.038
7........	f	−4.8	−3.0	+0.037	0.011
8........	f	+2.9	−5.5	+0.017	0.018
9........	ff	+5.6	−0.4	−0.001	±0.024

from which for each pair of plates the constants *a*, *b*, and *c* were derived by a least-squares solution. It will be noticed that in using the same comparison stars for the different pairs of plates of the same field, the first members of the equations of condition are exactly the same for each pair. This simplifies the work greatly, as we compute these, and also the first members of the normal equations, but once for each field. Only in two cases, in which one of the comparison stars could not be measured on all the plates, had

Boss 4393 $\alpha = 17^h 14^m 53^s$ $\delta = +28°56'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
111_1 . . .	1914 April 17	−4°2	VM	g	+0.785	+0.30	
111_2 . . .	17	+1.2	VM	g	+0.785	+0.30	
120 . . .	18	−6.0	VM	g	+0.775	+0.30	
145_1 . . .	July 25	−1.5	VM	g	−0.692	+0.57	
145_2 . . .	25	+3.7	H	g	−0.692	+0.57	
152 . . .	26	+0.2	VM	g	−0.705	+0.57	Clouds 25^m exp.
158_1 . . .	27	−0.2	VM	g	−0.716	+0.57	
158_2 . . .	27	+4.5	H	g	−0.716	+0.57	
165 . . .	Aug. 12	−0.2	VM	g	−0.879	+0.62	
524 . . .	1915 May 5	+6.7	H	g	+0.568	+1.35	
531_1 . . .	6	−1.0	VM	fg	+0.554	+1.35	
531_2 . . .	6	+3.7	H	g	+0.554	+1.35	

Mor.–Ev.	Δp	Δm	M	v	Δt
120 −145_1	+1.47	−0.27	− 4	− 5	−4°5
111_1−158_1	+1.50	−0.27	−11	−12	−4.0
531_1−165	+1.43	+0.73	+31	− 7	−0.8
111_2−152	+1.49	−0.27	+20	+19	+1.0
531_2−145_2	+1.25	+0.78	+30	− 9	0.0
524 −158_2	+1.28	+0.78	+52	+13	+2.2

$$+1.96\,\mu_a + 1.81\,\pi = +86 \qquad \pi = +7.6 = +0''\!.013 \pm 0''\!.005$$
$$+1.81\,\mu_a + 11.87\,\pi = +156 \qquad \mu_a = +37.7 = +0''\!.062 \pm 0''\!.013$$

Comp. Star	Br.	x	y	π	P.E.
1.	b	−0'.5	+2'.6	+0''.012	±0''.008
2.	ff	−0.3	+3.4	−0.007	0.006
3.	f	+0.1	+5.6	−0.002	0.008
4.	bf	+5.5	+0.7	+0.007	0.006
5.	bb	+4.7	−5.1	−0.012	0.004
6.	f	+0.8	−4.5	−0.011	0.008
7.	f	−0.1	−4.1	+0.012	0.007
8.	ff	−1.1	−1.0	−0.023	0.009
9.	f	−5.1	−0.7	+0.020	±0.004

we to compute a second set of equations. The plates for which this was the case are mentioned in the tables giving the details for the individual stars.

The zero point for every field was made to coincide with the parallax star (an exception was made in the case of the double star Boss 592, 593, where the zero point of the plates was taken halfway between 592 and 593); in this way x_π and y_π are zero, and we

Boss 4555 $\alpha=17^h57^m5^s$ $\delta=+45°30'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
112 ...	1914 April 17	$-1°.5$	VM	fg	$+0.885$	$+0.30$	
121 ...	18	-3.7	VM	fg	$+0.877$	$+0.30$	
132 ...	May 17	$+1.0$	H	g	$+0.548$	$+0.38$	Clouds
135_1...	18	$+3.2$	Sh	g	$+0.533$	$+0.38$	
135_2...	18	$+9.5$	Sh	f	$+0.532$	$+0.38$	
146_1...	July 25	$+0.5$	VM	g	-0.553	$+0.57$	
146_2...	25	$+5.0$	H	g	-0.553	$+0.57$	
159_1...	27	$+1.0$	VM	g	-0.582	$+0.57$	
159_2...	27	$+5.5$	VM	g	-0.582	$+0.57$	
166_1...	Aug. 12	$+1.5$	VM	fg	-0.780	$+0.62$	
166_2...	12	$+6.2$	H	g	-0.780	$+0.62$	Trail
190 ...	14	-3.5	VM	fg	-0.798	$+0.62$	Clouds
191_1...	14	$+6.2$	VM	fg	-0.798	$+0.62$	
191_2...	14	$+11.0$	H	fg	-0.798	$+0.62$	
525_1...	1915 May 5	$+3.0$	VM	g	$+0.708$	$+1.35$	
525_2...	5	$+7.5$	H	g	$+0.708$	$+1.35$	
532_1...	6	0.0	VM	fg	$+0.695$	$+1.35$	
532_2...	6	$+5.7$	H	g	$+0.695$	$+1.35$	

Mor.–Ev.	Δp	Δm	M	v	Δt
121 − 190	$+1.68$	-0.32	$+26$	$+11$	$-0°.2$
112 − 146_1....	$+1.44$	-0.27	$+10$	-3	-2.0
132 − 159_1....	$+1.13$	-0.19	$+19$	$+9$	0.0
532_1 − 166_1....	$+1.48$	$+0.73$	$+17$	-25	-1.5
525_1 − 146_2....	$+1.26$	$+0.78$	$+40$	-1	-2.0
135_1 − 159_2....	$+1.12$	-0.19	-3	-13	-2.3
532_2 − 191_1....	$+1.49$	$+0.73$	$+66$	$+23$	-0.5
525_2 − 166_2....	$+1.49$	$+0.73$	$+47$	$+4$	$+1.3$
135_2 − 191_2....	$+1.33$	-0.24	$+6$	-6	-1.5

$$+2.51\,\mu_a + 2.57\,\pi = +110 \qquad \pi = +14.5 = +0''.024 \pm 0''.004$$

$$+2.57\,\mu_a + 17.41\,\pi = +327 \qquad \mu_a = +29.0 = +0''.048 \pm 0''.011$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	$+1'.1$	$+1'.8$	$-0''.010$	$\pm 0''.011$
2........	b	$+2.5$	$+2.9$	0.000	0.013
3........	f	-0.7	$+4.3$	-0.012	0.008
4........	f	-2.1	$+3.4$	$+0.003$	0.004
5........	ff	-1.5	$+2.5$	-0.002	0.007
6........	f	-1.5	-0.8	$+0.019$	0.008
7........	ff	-1.1	-1.4	$+0.020$	0.010
8........	f	-4.3	-4.0	-0.020	0.006
9........	ff	$+2.7$	-4.5	-0.012	0.003
10........	f	$+1.6$	-0.7	$+0.012$	± 0.009

Boss 4860 $\alpha=19^h1^m9^s$ $\delta=+31°36'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
219_1...	1914 Sept. 13	$-4°.7$	VM	fg	-0.916	-0.30	
219_2...	13	$+0.2$	H	f	-0.916	-0.30	
220_1...	13	$+6.0$	VM	fg	-0.916	-0.30	
220_2...	13	$+10.0$	VM	fg	-0.916	-0.30	Trail
526 ...	1915 May 5	-1.0	VM	g	$+0.862$	$+0.35$	
533 ...	6	-4.0	VM	g	$+0.854$	$+0.35$	
534 ...	6	$+0.7$	H	fg	$+0.854$	$+0.35$	
547_1...	June 3	-13.0	H	f	$+0.526$	$+0.42$	Some astigmatism of 60-in. mirror
547_2...	3	-8.5	VM	f	$+0.525$	$+0.42$	
548_1...	3	-3.0	H	f	$+0.525$	$+0.42$	
548_2...	3	$+1.5$	VM	f	$+0.525$	$+0.42$	
549 ...	3	$+7.2$	H	f	$+0.525$	$+0.42$	
607_1...	July 31	-14.5	VM	g	-0.405	$+0.58$	
607_2...	31	-9.7	H	g	-0.406	$+0.58$	
608_1...	31	-4.7	VM	vg	-0.406	$+0.58$	
608_2...	31	-0.5	H	vg	-0.406	$+0.58$	

Mor.–Ev.	Δp	Δm	M	v	Δt
547_1–607_1....	$+0.93$	-0.16	$+23$	$+11$	$+1°.5$
547_2–607_2....	$+0.93$	-0.16	$+4$	-8	$+1.2$
533 –608_1....	$+1.26$	-0.23	$+15$	-1	-0.7
548_1–219_1....	$+1.44$	$+0.72$	$+9$	-20	$+1.7$
526 –608_2....	$+1.27$	-0.23	$+8$	-8	-0.5
534 –219_2....	$+1.77$	$+0.65$	$+50$	$+16$	$+0.5$
548_2–220_1....	$+1.44$	$+0.72$	$+14$	-15	-4.5
549 –220_2....	$+1.44$	$+0.72$	$+50$	$+21$	-2.8

$$+2.14\mu_a+3.39\pi=+75 \qquad \pi=+14.7=+0''.024\pm0''.006$$
$$+3.39\mu_a+14.16\pi=+248 \qquad \mu_a=+11.8=+0''.019\pm0''.015$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	$+4'.1$	$+0'.1$	$+0''.027$	$\pm0''.010$
2........	f	$+1.8$	$+1.4$	-0.004	0.011
3........	bf	-1.1	$+2.1$	-0.008	0.009
4........	f	-2.5	$+1.8$	$+0.020$	0.008
5........	bf	-2.9	$+0.3$	-0.006	0.011
6........	f	-2.6	-0.7	-0.028	0.010
7........	b	0.0	-2.3	$+0.047$	0.011
8........	f	$+2.9$	-1.8	-0.045	±0.017

need for the computation of the parallax of the principal star only the constant a. Substituting its numerical value $(-a=M)$ into the equation of condition for the parallax star we have, since D for this object is zero,

$$\Delta p\pi+\Delta m\mu_a=M.$$

Boss 5043 $\alpha = 19^h 40^m 25^s$ $\delta = +41°32'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
192 . . .	1914 Aug. 14	− 1°.2	H	fg	−0.471	+0.62	Hazy 20^m exp.
199$_1$ . . .	Sept. 11	−14.2	VM	g	−0.814	+0.70	
199$_2$ . . .	11	− 9.0	H	g	−0.814	+0.70	Trail
564$_1$ . . .	1915 June 4	−11.5	VM	g	+0.639	+1.43	
564$_2$ . . .	4	− 7.2	H	g	+0.639	+1.43	
565$_1$ . . .	4	− 2.0	VM	g	+0.639	+1.43	
565$_2$ . . .	4	+ 2.2	H	g	+0.639	+1.43	
566$_1$ . . .	4	+ 7.7	VM	vg	+0.639	+1.43	
566$_2$ . . .	4	+11.7	H	g	+0.639	+1.43	
659$_1$ . . .	Aug. 31	− 3.5	VM	g	−0.695	+1.67	Hazy 25^m exp.
659$_2$ . . .	31	+ 3.2	H	g	−0.695	+1.67	Hazy 25^m exp.
677 . . .	Sept. 1	+ 8.0	VM	fg	−0.707	+1.67	

Mor.–Ev.	Δp	Δm	M	v	Δt
564$_1$–199$_1$	+1.45	+0.73	0	−10	+2°.7
564$_2$–199$_2$	+1.45	+0.73	+12	+ 2	+1.8
565$_1$–659$_1$	+1.33	−0.24	+12	+10	+1.5
565$_2$–192	+1.11	+0.81	+16	+ 7	+3.4
566$_1$–659$_2$	+1.33	−0.24	+ 7	+ 5	+4.5
566$_2$–677	+1.35	−0.24	−11	−13	+3.7

$$+1.90\,\mu_a + 2.06\,\pi = +20 \qquad \pi = +2.7 = +0''.004 \pm 0''.004$$

$$+2.06\,\mu_a + 10.79\,\pi = +45 \qquad \mu_a = +7.6 = +0''.012 \pm 0''.009$$

Comp. Star	Br.	x	y	π	P.E.
1.	b	+5′.1	0′.0	−0″.040	±0″.002
2.	f	+0.4	+1.2	+0.012	0.005
3.	bf	+1.8	+3.0	+0.018	0.003
4.	bf	−2.2	+2.1	−0.015	0.008
5.	bf	−3.3	+1.1	+0.001	0.006
6.	f	−3.7	−2.1	−0.019	0.003
7.	f	+1.0	−3.0	+0.021	0.006
8.	f	+1.8	−2.6	+0.023	±0.007

Each pair of plates for a field gave an equation of this form, from which π and μ_a were computed by a least-squares solution. The parallax factor p was calculated by the rigorous formula, and checked by the tables published by Schlesinger;[1] m was expressed in years; both are given in the tables for the individual fields.

For the computation of the parallaxes and proper motions of the comparison stars we need also the plate constants b and c. The

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[1] *Publications of the Allegheny Observatory*, **3**, 163, 1915.

Boss 5184 $\alpha=20^h9^m57^s$ $\delta=+61°47'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
269 . . .	1914	Sept.	16	−1°.0	VM	g	−0.785	+0.71	
283_1. . .		Oct.	11	−1.0	VM	fg	−0.952	+0.78	Some astigmatism of 60-in. mirror
283_2. . .			11	+3.7	VM	fg	−0.952	+0.78	
575_1. . .	1915	July	2	−3.5	VM	g	+0.340	+1.50	
575_2. . .			2	+0.5	H	g	+0.340	+1.50	
591_1. . .			3	−4.7	VM	g	+0.324	+1.51	
591_2. . .			3	−0.7	VM	g	+0.324	+1.51	
592 . . .			3	+3.7	H	g	+0.324	+1.51	
600_1. . .			4	−5.7	VM	f	+0.309	+1.51	
600_2. . .			4	−1.2	VM	f	+0.309	+1.51	
676 . . .		Sept.	1	−10.0	VM	f	−0.609	+1.67	
686 . . .			2	−8.0	VM	f	−0.623	+1.67	
687_1. . .			2	+1.2	H	fg	−0.623	+1.67	
687_2. . .			2	+5.5	VM	fg	−0.623	+1.67	

Mor.–Ev.	Δp	Δm	M	v	Δt
600_1–676	+0.92	−0.16	+10	+3	+4°.3
591_1–686	+0.95	−0.16	+14	+7	+3.3
575_1–269	+1.13	+0.79	+88	+1	−2.5
600_2–283_1. . . .	+1.26	+0.73	+86	+1	−0.2
591_2–687_1. . . .	+0.95	−0.16	+14	+7	−1.9
575_2–283_2. . . .	+1.29	+0.72	+85	−1	−3.2
592 –687_2. . . .	+0.95	−0.16	−8	−15	−1.8

$$+1.79\,\mu_a+2.14\,\pi=+179 \qquad \pi=+21.4=+0''.035\pm0''.004$$
$$+2.14\,\mu_a+8.08\,\pi=+344 \qquad \mu_a=+80.0=+0''.132\pm0''.007$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	+1′.1	+0′.7	−0″.046	±0″.014
2.	f	+2.6	+1.2	0.000	0.015
3.	f	−0.1	+4.8	0.000	0.005
4.	bf	−6.2	+2.5	+0.010	0.006
5.	f	−2.7	−0.4	+0.001	0.011
6.	bf	−1.2	−5.4	+0.008	0.005
7.	bf	+2.1	−3.3	−0.010	0.010
8.	f	+5.8	−0.4	+0.035	±0.006

first members of the equations are the same as for the parallax star itself. For several reasons, indicated later in the discussion, it seemed worth while to do the relatively small amount of extra work necessary for the derivation of the parallaxes of the comparison stars.

Boss 5210 = Bu 10100 $\alpha=20^h14^m34^s$ $\delta=+40°25'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
138_1...	1914 June 15	$-11°.2$	H	fg	+0.599	+0.46	
138_2...	15	− 6.2	H	fg	+0.599	+0.46	
142 ...	16	− 1.2	Sh	f	+0.586	+0.46	
200 ...	Sept. 11	−11.0	VM	fas	−0.721	+0.70	
201_1...	11	− 0.7	VM	f	−0.721	+0.70	
201_2...	11	+ 4.2	VM	f	−0.721	+0.70	
236 ...	14	− 5.2	H	fg	−0.750	+0.71	
350_1...	Oct. 15	0.0	VM	fg	−0.961	+0.79	
350_2...	15	+ 4.5	VM	fg	−0.961	+0.79	
550_1...	1915 June 3	− 4.0	VM	f	+0.744	+1.42	
550_2...	3	0.0	H	f	+0.744	+1.42	
551 ...	3	+ 5.0	VM	f	+0.742	+1.42	

Mor.-Ev.	Δp	Δm	M	v	Δt
138_1–200	+1.32	−0.24	+ 7	0	$-0°.2$
138_2–236	+1.35	−0.25	0	− 8	−1.0
550_1–201_1....	+1.47	+0.72	− 8	− 5	−3.3
142 –350_1....	+1.55	−0.33	+14	+ 5	−1.2
550_2–201_2....	+1.47	+0.72	+ 2	+ 5	−4.2
551 –350_2....	+1.70	+0.63	+ 1	+ 2	+0.5

$$+1.67\mu_a+2.02\pi=-11 \qquad \pi=+3.4=+0''.006\pm0''.002$$
$$+2.02\mu_a+13.17\pi=+24 \qquad \mu_a=-10.7=-0''.018\pm0''.006$$

As on nearly all plates the eighth magnitude companion is not clearly separated from the principal star, the measures are too uncertain to derive a satisfactory parallax for it.

Comp. Star	Br.	x	y	π	P.E.
1.......	b	$+1'.1$	$+1'.2$	$+0''.007$	$\pm0''.009$
2.......	bf	+1.2	+1.5	−0.002	0.003
3.......	bf	−1.0	+2.1	−0.004	0.007
4.......	f	−2.6	+1.8	+0.002	0.006
5.......	bf	−3.3	−1.0	−0.011	0.002
6.......	bb	−1.6	−1.8	+0.025	0.012
7.......	f	−0.4	−1.6	−0.020	0.005
8.......	f	−0.1	−1.9	−0.010	0.007
9.......	bf	+0.5	−2.6	+0.012	0.002
10.......	f	+2.3	−0.4	+0.001	±0.007

To all pairs of plates of a field the same weight has been given. Schlesinger, and after him many other parallax observers, have used a different method for the computation, which may be called "the method of dependences."[1] While this may be somewhat

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[1] *Astrophysical Journal*, **33**, 163, 1911.

Boss 5519 $\alpha = 21^h 23^m 52^s$ $\delta = +31^\circ 47'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
139_1...	1914	June	15	$-16^\circ.0$	Sh	g	+0.785	+0.46	
139_2...			15	−11.0	Sh	g	+0.785	+0.46	
143 ...			16	− 6.2	Sh	g	+0.776	+0.46	
149 ...		July	25	+ 7.5	VM	g	+0.270	+0.57	
202_1...		Sept.	11	− 6.0	H	fg	−0.477	+0.70	
202_2...			11	− 1.5	VM	fg	−0.477	+0.70	
211 ...			12	−13.0	VM	pf	−0.491	+0.70	
222_1...			13	− 6.2	H	fg	−0.503	+0.70	
222_2...			13	− 1.5	VM	f	−0.504	+0.70	Trail
238 ...			14	+ 4.0	VM	fg	−0.518	+0.70	
284 ...		Oct.	11	− 7.5	H	fg	−0.818	+0.78	
285 ...			11	+ 3.0	VM	g	−0.818	+0.78	
302 ...			12	− 7.2	H	fg	−0.824	+0.78	
593_1...	1915	July	3	− 8.7	VM	g	+0.587	+1.51	
593_2...			3	− 4.2	H	g	+0.587	+1.51	
594 ...			3	+ 0.2	VM	g	+0.587	+1.51	
602_1...			4	− 1.0	H	f	+0.575	+1.51	
602_2...			4	+ 3.7	VM	f	+0.575	+1.51	

Mor.-Ev.	Δp	Δm	M	v	Δt
139_1–211	+1.28	−0.24	+12	+ 5	$-3^\circ.0$
139_2–284	+1.60	−0.32	+13	+ 5	−3.5
143 –302	+1.60	−0.32	+ 5	− 3	+1.0
593_1–222_1....	+1.09	+0.81	+78	+ 5	−2.5
593_2–202_1....	+1.06	+0.81	+51	−21	+1.8
602_1–222_2....	+1.08	+0.81	+77	+ 5	+0.5
594 –202_2....	+1.06	+0.81	+88	+16	+1.7
602_2–285	+1.39	+0.73	+68	− 4	+0.7
149 –238	+0.79	−0.13	−10	−16	+3.5

$$+3.45\mu_a + 3.05\pi = +279 \qquad \pi = +17.9 = +0''.030 \pm 0''.004$$

$$+3.05\mu_a + 13.91\pi = +446 \qquad \mu_a = +65.0 = +0''.107 \pm 0''.008$$

Comp. Star	Br.	x	y	π	P.E.
1........	b	$+4'.9$	$+1'.9$	$-0''.017$	$\pm 0''.005$
2........	f	+1.1	+3.6	+0.013	0.005
3........	bf	−1.1	+4.0	+0.027	0.008
4........	f	−1.6	+3.0	−0.009	0.005
5........	ff	−2.2	−1.6	−0.041	0.014
6........	f	−3.0	−2.6	−0.011	0.005
7........	f	+0.3	−2.7	+0.001	0.007
8........	bb	+2.2	−5.6	+0.040	±0.007

Boss 5868 = Bu 11936 $\alpha = 22^h 39^m 34^s$ $\delta = +38°56'$

Pl. No.	Date			*t*	Obs.	*Qy*	*p*	*m*	Remarks
150 ...	1914	July	25	+ 7°5	H	vg	+0.553	+0.57	
155 ...			26	+ 7.2	VM	g	+0.553	+0.57	Sky thick 25^m exp.
170 ...		Aug.	12	+11.0	VM	fg	+0.302	+0.62	
286_1 ...		Oct.	11	− 3.7	VM	g	−0.590	+0.78	
286_2 ...			11	+ 0.7	H	g	−0.590	+0.78	
287 ...			11	+ 7.2	VM	g	−0.590	+0.78	
305_1 ...			12	+ 8.2	VM	g	−0.602	+0.78	
305_2 ...			12	+12.5	VM	f	−0.602	+0.78	
322 ...			13	− 1.7	H	f	−0.614	+0.79	
580 ...	1915	July	2	− 4.0	H	g	+0.801	+1.50	
596 ...			3	− 2.0	H	fg	+0.791	+1.51	
630 ...		Aug.	1	+ 1.5	H	g	+0.462	+1.59	

Mor.–Ev.	Δ*p*	Δ*m*	*M* (5868)	*M* (Comp.)	*v* (5868)	*v* (Comp.)	Δ*t*	Remarks
580–286_1 ..	+1.39	+0.72	+ 1	−27	− 3	−15	−0°3	
596–322 ..	+1.41	+0.72	+19	− 3	+15	+ 9	−0.3	
630–286_2 ..	+1.05	+0.81	−12	− 5	−13	+ 7	+0.8	
155–287 ..	+1.14	−0.21	−28	−22	−35	−23	0.0	
150–305_1 ..	+1.16	−0.21	+32	+17	+25	+16	−0.7	
170–305_2 ..	+0.90	−0.16	+14	+11	+ 8	+11	−1.5	Star 2 not measured

$$\left.\begin{aligned} +1.81\,\mu_a + 2.25\,\pi &= +\ 2 \\ +2.25\,\mu_a + 8.46\,\pi &= +33 \end{aligned}\right\}(5868) \qquad \left.\begin{aligned} -26 \\ -42 \end{aligned}\right\}(\text{Comp.})$$

$$(5868)\begin{cases} \pi = +\ 5.5 = \mathbf{+0''.009 \pm 0''.011} \\ \mu_a = -\ 5.7 = -0''.009 \pm 0''.024 \end{cases} (\text{Comp.})\begin{cases} \pi = -\ 1.8 = \mathbf{-0''.003 \pm 0''.008} \\ \mu_a = -12.1 = -0''.020 \pm 0''.018 \end{cases}$$

Comp. Star	Br.	*x*	*y*	π	P.E.
1........	f	+3′.7	+1′.8	−0″.001	±0″.008
2........	bf	+1.4	+3.0	+0.009:	0.009
3........	bf	−2.1	+3.4	−0.033	0.007
4........	bb	−4.4	+0.8	+0.088	0.016
5........	bf	−1.6	−1.0	−0.032	0.005
6........	f	−1.6	−1.1	−0.025	0.010
7........	f	−0.3	−2.3	−0.023	0.007
8........	bf	+1.1	−3.3	+0.023	0.005
9........	b	+7.7	−0.5	−0.002	±0.005

shorter than that followed here, it has the disadvantage of not allowing the calculation of the parallaxes of the comparison stars. Schlesinger states: "Even when the computer decides not to employ the dependences to effect the solutions, they should nevertheless be

Boss 6097 $\alpha=23^{h}39^{m}56^{s}$ $\delta=+55°15'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
62 ...	1913 Nov. 23	−1°.7	VM	f	−0.832	−0.10	
151 ...	1914 July 25	−0.7	VM	fg	+0.737	+0.57	
156 ...	26	+2.2	VM	g	+0.726	+0.57	Sky thick 25^{m} exp.
163 ...	27	+3.0	H&VM	fg	+0.717	+0.57	2 exp. superposed
186 ...	Aug. 13	+6.0	H	g	+0.518	+0.62	
323_1...	Oct. 13	−10.2	VM	fg	−0.394	+0.79	
323_2...	13	−5.7	H	fg	−0.395	+0.79	
339_1...	14	−5.2	VM	g	−0.409	+0.79	
339_2...	14	−1.0	H	g	−0.409	+0.79	
367_1...	Nov. 10	+1.2	Ca	g	−0.729	+0.86	
367_2...	10	+6.5	H	g	−0.729	+0.86	
615 ...	1915 July 31	−5.2	H	g	+0.675	+1.58	
631_1...	Aug. 1	−7.0	VM	g	+0.666	+1.59	
631_2...	1	−2.7	H	g	+0.664	+1.59	

Mor.–Ev.	Δp	Δm	M	v	Δt
151 − 62	+1.57	+0.67	+31	+9	+1°.0
156 − 367_1....	+1.46	−0.29	+12	+14	+1.0
631_1 − 323_1....	+1.06	+0.80	+31	+8	+3.2
615 − 323_2....	+1.07	+0.79	+14	−8	+0.5
631_2 − 339_1....	+1.07	+0.80	+11	−12	+2.5
163 − 339_2....	+1.13	−0.22	−25	−24	+4.0
186 − 367_2....	+1.25	−0.24	+2	+3	−0.5

$$+2.54\,\mu_a+2.64\,\pi=+69 \qquad \pi=+3.7=+0''.006\pm0''.006$$
$$+2.64\,\mu_a+10.83\,\pi=+102 \qquad \mu_a=+23.3=+0''.039\pm0''.012$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	−0′.1	+0′.5	+0″.003	±0″.009
2........	bf	+0.5	+1.8	+0.002	0.005
3........	f	+2.6	+2.6	−0.004	0.007
4........	bf	−2.1	+5.4	−0.001	0.006
5........	bf	−3.3	−1.6	−0.003	0.003
6........	bf	−1.5	−3.6	−0.017	0.006
7........	f	−0.5	−1.4	+0.050	0.013
8........	f	+2.5	−2.9	−0.029	±0.008

computed, since a knowledge of them is indispensable for an intelligent discussion of the measures." This, however, is true only when the distance of the parallax star from the "center of gravity" of the comparison stars is not small. When, as in our case, by the use of more and fainter comparison stars we can nearly always make this difference very small, the dependences become practically the same (namely, $1/n$ where n is the number of these stars) and the knowledge of their values is unnecessary.

As follows from the foregoing discussion, the comparison stars must fulfil several conditions: their mean magnitude should be nearly the same as that of the reduced parallax star; they should be as near the parallax star as possible; their center of gravity should be as near as possible to the parallax star. Moreover, they should be faint; in the computations we have assumed that their parallaxes and proper motions are zero, and the fainter the comparison stars the greater is the chance that these quantities will not be large.

The regular exposure time of 15 minutes gives us on Seed "23" plates stars as faint as magnitude 13.7. This limit, like all others in the discussion, was derived from countings of the stars in 16 fields by means of Kapteyn's tables which give the logarithm of the number of stars per square degree.[1] In the beginning stars from 11.2 to 13.7 were used for comparison purposes. In the discussion of the results it will be shown that it is better to avoid the very faint stars on the plates, and later the range of magnitudes was accordingly restricted from 11.2 to 13.2. For the sky as a whole there will be an average of 13 stars within a circle of 8′ radius available for comparison purposes, although near the poles of the Milky Way the number will be considerably smaller. Except for these regions it is clear that it will always be possible to fulfil very closely the conditions mentioned above.

By extrapolation from Kapteyn's tables[2] we find that the mean parallax of stars of the magnitudes used for comparison objects will range from $0''.003$ to $0''.0015$; to change our final relative parallaxes into absolute values we must therefore correct them by about $+0''.002$.

The number of comparison stars used, except in one case, has been 8 or more. A number larger than the necessary 3 is advisable, not only because it facilitates the fulfilment of the condition that the center of gravity of the comparison stars shall be near the parallax star, but also because it enables us to see if any of the stars used for comparison purposes has a large parallax or proper motion, which if necessary can then be rejected. Such cases will be more easily detected and at the same time be less vitiating to our final parallax, the larger the number of comparison stars used.

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[1] *Publications of the Astronomical Laboratory at Groningen,* No. 18, p. 54, 1908.

[2] *Ibid.,* No. 8, p. 31, 1901.

The method followed here, in which a morning and an evening exposure are combined into one pair, has several advantages over that in which all plates are measured and reduced separately. In the first place, it saves time at the measuring machine and some also in the reductions, unless we use the method of dependences. In the second place, with plates taken from 15° east to 15° west of the meridian, there will be an hour-angle error in each, although its effect may be made small by selecting comparison stars near the parallax star. In combining plates whose hour-angles are nearly the same, on the other hand, this effect enters only differentially.

THE DETAILED RESULTS

The detailed results for each star are collected in the tables. The right ascension and declination for 1900 are from Boss's *Preliminary General Catalogue*. The first table for each star contains the necessary data for the plates, viz., the current number; the date of exposure; the hour-angle in degrees, *t;* the initial of the observer, *Obs.;* the quality of the plate, *Qy;* the parallax factor, *p;* and the factor for annual proper motion, *m*. Under the heading *Obs.*, H indicates Hoge; VM, Van Maanen; Ca, Capon; C, Campbell; Sh, Shapley; P, Pease. For the quality we have used *p* for poor, *f* for fair, *g* for good, and *as* for plates showing asymmetrical images.

The second table for each star shows the combinations of morning and evening plates as measured; it contains the difference in the parallax factors, Δp, and that in the proper motion factors, Δm. Under *M* is given the value of $\Delta p\pi+\Delta m\mu_a$ for each pair of plates; under v the residuals; under Δt the difference in hour-angle; then follow the normal equations and the results. Thus far every measured quantity is expressed in 1/2000 of a revolution of the micrometer screw as a unit. To convert the final π and μ_a in seconds of arc we must multiply by 0.001649.

The third table contains the data for the comparison stars used. The values of the brightness in the second column were estimated from one of the plates of each field. The countings mentioned on p. 28 give as approximate values for the corresponding magnitudes:

bb = 11.2–11.6
b = 11.7–12.1
bf = 12.2–12.5
f = 12.6–13.2
ff = 13.3–13.7

The co-ordinates in right ascension and declination are given in minutes of arc, positive values indicating that the comparison stars are east and north of the origin of the co-ordinates. This table also contains the parallax and its probable error for each of the comparison stars. The sign : is used to mark a doubtful value.

In all least-squares solutions the usual checks were applied.

SUMMARY OF RESULTS AND DISCUSSION

The results are summarized in Table I, which contains in the successive columns, (1) the number from Boss's *Preliminary General*

TABLE I

SUMMARY OF RESULTS

Number in Boss *P.G.C.*	Mag. Harvard	Spectrum	P.M. Boss	Relative Parallax	P.E.	No. of Exp.	μ_α Boss	μ_α VM	Boss−VM
96....	5.69	K0p	0″.056	+0″.006	0″.005	16	−0″.049	−0″.073	+0″.024
420....	5.63	K1	0.009	+0.004	0.007	14	0.000	+0.011	−0.011
592....	7.37	F4	0.152	+0.008	0.015	14	+0.152	+0.142	+0.010
593....	6.57	F3	0.140	+0.028	0.014	14	+0.140	+0.169	−0.029
672....	5.61	G2	0.035	−0.009	0.004	14	+0.028	+0.079	−0.051
1549....	6.30	Map	0.027	+0.001	0.001	16	+0.023	+0.016	+0.007
1868....	5.87	K1p	0.020	+0.006	0.003	16	−0.018	−0.010	−0.008
2020....	5.81	Map	0.015	+0.004	0.005	14	−0.014	−0.012	−0.002
2921....	6.22	Map	0.095	+0.078	0.006	10	+0.077	+0.059	+0.018
3233....	6.08	G9	0.072	+0.003	0.010	12	+0.052	+0.053	−0.001
3650....	5.54	F*	0.267	+0.096	0.003	12	−0.259	−0.302	+0.043
4048....	5.76	K5p	0.083	+0.038	0.010	10	−0.073	−0.081	+0.008
4393....	5.78	G7	0.045	+0.013	0.005	12	+0.042	+0.062	−0.020
4555....	5.92	K5	0.044	+0.024	0.004	18	−0.006	+0.048	−0.054
4860....	5.80	K0	0.085	+0.024	0.006	16	+0.070	+0.019	+0.051
5043....	6.04	K2	0.017	+0.004	0.004	12	+0.016	+0.012	+0.004
5184....	5.72	F0	0.155	+0.035	0.004	14	+0.133	+0.132	+0.001
5210....	5.82	B7	0.011	+0.006	0.002	12	+0.003	−0.018	+0.021
5519....	5.74	A8	0.134	+0.030	0.004	18	+0.120	+0.107	+0.013
5868....	6.12	K2	0.016	+0.009	0.011	12	−0.007	−0.009	+0.002
6097....	6.62	G5	0.029	+0.006	0.006	14	+0.026	+0.039	−0.013

Catalogue; (2) the magnitude on the Harvard system; (3) the spectrum as determined by Adams, or by Miss Cannon in which case the spectrum is marked by an asterisk; (4) the total proper motion, derived from Boss's *Catalogue;* (5) the relative parallax; (6) its probable error; (7) the number of exposures; (8 and 9) the proper motion in right ascension as given by Boss and in this paper, respectively; and finally (10) the difference between these proper motions.

The mean relative parallax of these 21 stars of mean magnitude 6.0 and of mean proper motion 0."072 is +0."020. With the number of exposures ranging from 10 to 18 the mean probable error is 0."006.

In seeking for evidence of systematic errors we may first examine the results for the proper motions as compared with those determined by Boss. Their mean difference is less than 0."001, while there are 12 positive and 9 negative signs. This is as good as can be expected.

A more promising point for the discussion of systematic errors with the material now available is revealed by an examination of the parallaxes of the comparison stars. These might show that the parallaxes need a correction for magnitude or one arising from the neglect in the reductions of the quadratic terms of the co-ordinates of the comparison stars. The mean parallax of the comparison stars arranged according to their magnitude is given in the following table, where n is the number of stars used:

	bb	b	bf	f	ff
Mean π . .	+0.035	0.000	+0.003	+0.001	−0.011
n.	4	11	37	85	30

The value for the brightest stars is very uncertain, their number being only 4 while one of these stars has a parallax of +0."088. The next three classes do not show any sign of a magnitude error, while the faintest stars have a mean parallax of −0."011; this value can be explained by the larger uncertainty in the measures of the faintest stars, for which the probable error of the parallaxes is 0."014, while for the other stars it is only 0."008. When this became evident, it seemed best to avoid the use of the very faintest stars for comparison purposes, which has been done for the later fields measured. This fact, however, may have vitiated the final parallaxes to some extent. If we suppose the parallaxes of the brightest stars to be due also to a magnitude error, the necessary correction for the parallax stars would be, in the mean, $-0\overset{''}{.}0012$. If, on the other hand, we suppose them to be due to accidental causes, this correction might be as great as $-0\overset{''}{.}0020$.

For the investigation of a possible error in the parallaxes of the principal stars due to a neglect of the quadratic terms in x and y, the parallaxes of the comparison stars were divided into 37 groups according to their positions on the plates, each group containing from 1 to 13 stars. The weighted mean of each group was used as the second member of an equation of condition of the form:

$$+a+bx+cy+dx^2+exy+fy^2=\text{mean }\pi.$$

These equations were solved by least squares. The result is a mean correction for the central stars of $+0\overset{''}{.}0008 \pm 0\overset{''}{.}0020$.

From this it will be clear that the systematic errors due to the causes mentioned above cannot have vitiated the final results by more than $0\overset{''}{.}001$ or $0\overset{''}{.}002$. The exact effect of sytematic errors can be shown only after the lapse of some time, when there will be enough material available for a comparison with the results obtained at other observatories.

As stated before,[1] the small amount of extra work involved in computing the parallaxes of the comparison stars not only helps to give an idea of systematic errors, but may also give information as to the mean parallax of the faint stars, and as to the law which expresses the probability of a star's parallax being a certain multiple of the mean parallax. The material available (168 stars) is insufficient to afford any definite conclusions upon this point, but with the results for a thousand comparison stars it may prove to be of some value.

[1] Reference to this point was made in a paper read at the August meeting of the American Astronomical Society in 1915.

It has been stated on p. 24 that the same weight has been assigned to all the pairs of plates. An examination of the residuals, v, given in the tables of detailed results should show whether this is justified. Using for the sake of brevity figures 1 to 6 for the quality of a plate from poor to very good, we have the following table showing the relation of v to the mean quality of the plates, in which n again is the number of deviations used:

	Mean Quality						
	2.5	3	3.5	4	4.5	5	5.5
Mean v......	......	6	13	14	13	11	9
n..........	......	7	19	24	35	41	4
Mean v......	10	......	11	12	10	8	......
n..........	1	......	2	8	2	2	......

The last two lines refer to the plates marked asymmetric.

It thus seems entirely justifiable to assign the same weight to every pair of plates, as has been done throughout the work.

I wish to express my thanks to Mr. Hale and several members of the staff for the interest they have continuously shown in my work and for several suggestions they have made. I am greatly indebted to Mr. W. P. Hoge, night assistant, who aided very efficiently in securing a great number of the exposures; and to Messrs. F. G. Pease, H. Shapley, and R. S. Capon, who kindly took several exposures during my absence from the Observatory. I also wish to express my thanks to Miss Helen N. Davis of the Computing Division, who made all the duplicate computations needed in the work.

Mount Wilson Solar Observatory
October 1915

LIST OF STARS WITH PROPER MOTION EXCEEDING 0″50 ANNUALLY—ADDENDA AND ERRATA

BY ADRIAN VAN MAANEN

The following numbers of the list in *Mt. Wilson Contr.*, No. 96[1] (several of which were marked doubtful) should be excluded: Nos. 6, 7, 63, 69, 130, 167, 196, 226, 267,[2] 330, 331, 364, 384, 389, 390, 441, 442, 480, 514, and 526.

Further, in the case of

No. 25 read $\phi = 66°6$
“ 62 “ “$= 110.4$
“ 159 “ “$= 206.3$
“ 162 “ “$= 209.3$
“ 166 “ “$= 179.0$
“ 187 delete Sp. Bin.
“ 247 read $\phi = 291°6$

No. 269 read $\phi = 268°0$
“ 325 add β G.C. 7166
“ 365 read $\phi = 304°7$
“ 440 “ “$= 11.7$
“ 499 “ “$= 61.9$
“ 508 add β G.C. 12274

Attention may be drawn to Porter's note in *Astronomical Journal*, No. 678, which gives several stars whose proper motions, as indicated by later observations, are a little below the limit of half a second.

Besides the additions to the original list which appear in the accompanying table, 12 stars with proper motions in excess of 0″50 have just been given by Wolf in *Astronomische Nachrichten*, **201**, 345, 1915.

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[1] *Astrophysical Journal*, **41**, 187, 1915.

[2] *Astronomische Nachrichten*, **201**, 271, 1915.

STARS TO BE ADDED TO THE LIST IN *Mt. Wilson Contr.*, NO. 96; *Astrophysical Journal*, 41, 187, 1915

Name	B.D.	Mag.	α 1900	δ 1900	μ	φ	Authority	Remarks
Comp. to Lal. 248		10.7*	$0^h12^m43^s$	+43°28′	2″.83	82°.9	O. Struve	
Anonymous		13 *	2 8 14	+15 31	1.1–	90.0	Wolf	
A. Oe. 2526	+64°312	7.5*	2 9 12	+64 30	0.50	228.5	Po.	
Comp. to Pi 2^h123		11.5†	2 30 46	+ 6 24	2.63	48.9	Van Maanen	
A. Oe. 3303	+51°658	9.1	2 50 18	+52 5	0.50	107.9	Po.	
A. Oe. 4961	+52°857	8.7	4 29 47	+52 42	0.53	149.7	Po.	
B.D.+17°1320	+17°1320	9.5	6 31 26	+17 38	0.88	293.–	Wolf	
B.D.+33°1505	+33°1505	9.3	7 12 58	+33 2	0.57	127.–	Hertzsprung	
B.D.+33°1694	+33°1694	9.2	8 18 54	+32 58	0.69	173.–	Hertzsprung	
Schroeter 899	+67°552	9.3	8 27 23	+67 38	1.09	270.0	Schroeter	
B.D.+77°361	+77°361	9.2	9 6 24	+77 40	1.02	268.0	Jones	
Anonymous		9.5*	10 56 56	+ 1 19	0.7–	38.–	Wolf	
Anonymous		13 *	11 2 9	+ 2 9	0.67	146.–	Wolf	
Anonymous		10.5*	11 4 56	– 2 14	0.49	166.–	Wolf	
Anonymous		12.5*	11 10 58	+ 8 34	0.81	261.5	Wolf	
Anonymous		10 *	11 18 39	+ 9 7	1.05	278.7	Wolf	
Comp. to Lal. 27744		7.5	15 8 50	– 0 58	1.38	247.4	β	β G.C. 7166
Comp. to W.B. 23^h175		8.4	23 11 54	–14 22	1.31	202.9	β	β G.C. 12274

† Visual magnitude. The photographic magnitude is 13.2. These values were kindly determined by Mr. Shapley.

MOUNT WILSON SOLAR OBSERVATORY
November 24, 1915

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ON THE CHANGES IN THE SPECTRUM, PERIOD, AND LIGHT-CURVE OF THE CEPHEID VARIABLE RR LYRAE

By HARLOW SHAPLEY

I. INTRODUCTION

The seventh-magnitude star RR Lyrae is the brightest representative of the cluster-type subdivision of the Cepheid variables.[1] Because of its brightness and its period of 13.5 hours the star has appropriately received more consideration of various kinds than other members of the class. Its study, therefore, may contribute more than that of the ordinary cluster-type star to our knowledge of the nature of Cepheid variation.

The variability of the light was first noted at the Harvard College Observatory by Mrs. Fleming prior to July 1899, and was announced in 1901.[2] The discovery was effected in a unique manner. An exposure of 6½ hours on the field of the variable was interrupted for a few seconds every half-hour by means of an automatic device that stopped the driving clock and thus permitted the stars to drift to new places on the plate. The resulting rows of thirteen images furnished for each star a means of detecting any conspicuous short-period variation.[3] The new variable was immediately placed upon the observing-list of the polarizing photometer at Harvard, and the long series of measures by Professor Wendell are of highest value in studies of the star's peculiarities.[1]

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[1] We must here except β Cephei, for which the amplitude of light-variation is too small for ordinary photometric investigation. It is very likely that in the course of time many other naked-eye stars will be included among the cluster-type variables of small light-variation; for instance, β Canis Majoris for which the spectrum and the spectroscopic orbit are nearly identical with those of β Cephei (*Lick Observatory Bulletins*, **6**, 22, 1910), and 12 Lacertae (*Journal of the Royal Astronomical Society of Canada*, **9**, 423, 1915). Similarly other bright stars may be found to be Cepheids of longer periods and of small variation, such as is suggested very recently by Guthnick and Prager for α Cygni (*Astronomische Nachrichten*, **201**, 443, 1915). For 12 Lacertae, observations of the spectrum by the writer, using the 10-inch portrait lens and objective prism, have shown no conspicuous change throughout the four-hour period either in stellar magnitude or in spectrum lines.

[2] *Harvard Circulars*, No. 54, 1901; *Astronomische Nachrichten*, **154**, 423, 1901.

[3] *Harvard Circulars*, No. 29, 1898; *Astronomische Nachrichten*, **147**, 93, 1898.

That RR Lyrae is a spectroscopic variable of the typical Cepheid variety was discovered at the Lick Observatory by Kiess, who also published photometric observations of the star and some considerations of changes in the intensity of the continuous spectrum.[2] As is usual for Cepheid variables the spectroscopic study discloses an abnormally small mass function and a very small apparent orbit, while no trace of a secondary spectrum is found. The periodic shift of the spectral lines suggests the explanation that the variable moves in an orbit about an obscure companion, but the improbability of the double-star interpretation has been pointed out in a previous article.[3] It was found, in fact, from Wendell's photometric observations of RR Lyrae that the time of the rise to maximum light varied greatly from that required by a uniform period,[4] and that to a certain extent the variation was erratic. More recently Martin and Plummer, in a photographic study of the light-curve, have considered the possible interpretation of this type of variation, concluding that the evidence is strongly against the double-star interpretation, and that a hypothesis involving pulsations is much more reasonable.[5]

The present communication contains a further discussion of the anomalies of the light-variation, as far as they are shown in the several available series of measures, as well as a report on some observations of the star's spectrum. It is hoped that through investigations of this kind the true explanation of Cepheid variation can be definitely determined, and this in turn should throw light on the method of stellar evolution.

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[1] *Harvard Annals*, **69**, Part I, 45, 1909; Part II, 124, 1914.

[2] *Lick Observatory Bulletins*, **7**, 140, 1913.

[3] "On the Nature and Cause of Cepheid Variation," *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[4] *Publications of the American Astronomical Society*, Report of the 16th Meeting, p. 16, 1914; *Popular Astronomy*, **22**, 144, 1914.

[5] *Monthly Notices*, **75**, 566, 1915.

II. VARIATION OF THE SPECTRUM

Harvard classifies the spectrum of RR Lyrae as F.[1] Kiess observes that the hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ are of equal intensity and that the calcium line K is about 0.8 as intense as $H+H\epsilon$.[2] On a photograph of the spectrum, made by the writer with a 15-degree prism mounted in connection with the 10-inch Cooke photographic triplet, the line K was seen to be less than 0.1 the intensity of $H+H\epsilon$. A series of observations was accordingly undertaken for the purpose of determining the class of spectrum at different phases of the light-variation. The results are collected in Table I.

Most of the plates listed in the first column contain multiple images, and the corresponding exposures are designated by postscript letters. The phases in the fifth column will be discussed in a later section of the paper. The brightness in the seventh column increases with the number designating it.

TABLE I

THE SPECTRUM OF RR LYRAE

No. Plate	Date	G.M.T.	Length of Exposure	Phase	Spectrum	Brightness	Remarks
28*a*	1915 Nov. 27	$16^h 55$	10^m	$-0^d.031$	A0	5	
28*b*		17 22	15	− .013	A0	5	Low
32*a*	Dec. 8	14 22	10	+ .093	A6	2	
32*b*		14 33	10	+ .100	A8	2	
33		15 23	15	+ .135	A5	2	
34*a*		16 9	11	+ .167	A8	1	Diffuse
34*b*		16 23	11	+ .177	A8	1	
42*a*	Dec. 9	14 51	10	− .021	B9	5	
42*b*		15 4	10	− .012	A0	4	
42*c*		15 19	15	− .001	A2	4	
44		17 20	10	+ .083	A4:	3	Haze
52*a*	Dec. 10	14 29	16	+ .397	A8:	0	Faint
52*b*		14 49	24	+ .411	F2	0	
53		16 8	16	+ .466	A7:	1	Clouds
55		17 16	11	+ .513	A2:	3	Drift
56*a*	Dec. 11	14 45	10	+ .275	>A2	2	Clouds
56*b*		15 4	26	+0.288	>A5	2	Clouds

A conspicuous change of spectrum with magnitude is at once apparent. The determinations of the class, except in the cases marked as uncertain, are fairly definite, though high accuracy is not to be expected because of the small dispersion. The classification was made without knowledge of the phase of variation or the relative brightness of the star, and similarly the estimate of the integrated brightness of the spectrum, relative to that of neighboring stars, was made without knowledge of the phase or the spectral type. The relation of the variation of spectrum to variation in light is shown in Fig. 1, the spectral classification being plotted along the mean light-curve derived from Wendell's observations.

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[1] *Harvard Annals*, **55**, 25, 1907; **56**, 194, 1912.

[2] *Op. cit.*, p. 144.

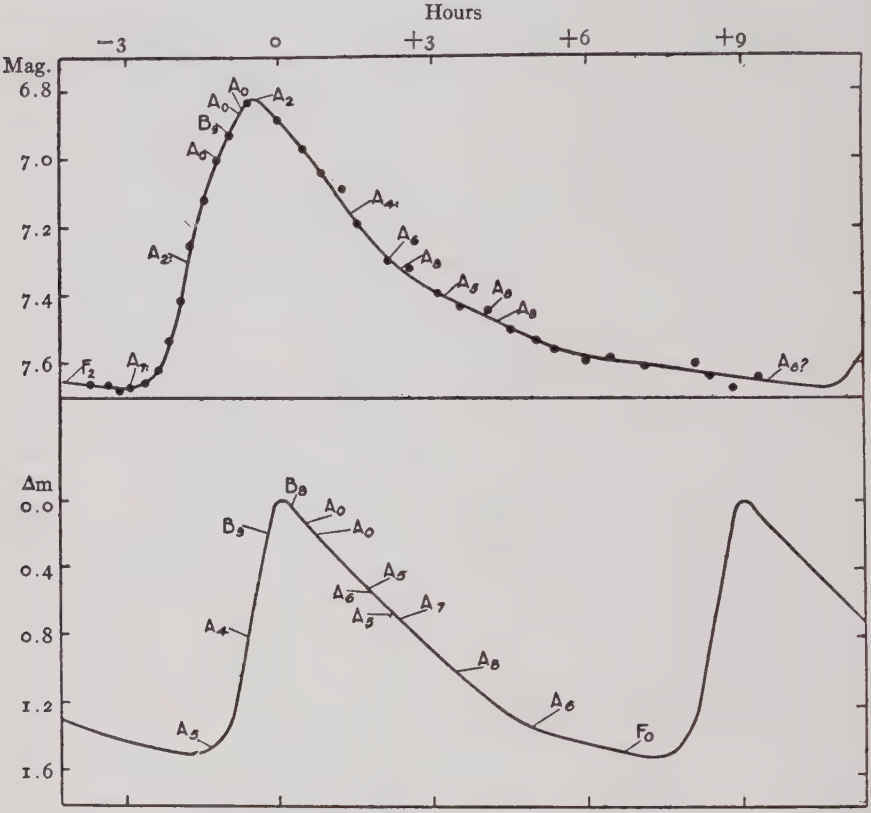

FIG. 1.—*Top:* Mean visual light-curve of RR Lyrae (observations by Wendell) and variations of spectrum (Shapley).

Bottom: Photographic light-curve of RS Boötis (Seares and Shapley) and variation of spectrum (Pease).

A notable feature of this change of spectrum with changing light is that the variation is from one recognizable spectral type to another. Moreover, it is a change from one spectral type to another that adjoins it in the usual conception of the evolutionary sequence of spectral classes. We might say that in this star we have visible proof of the evolution of stars along a spectral series—a phenomenon that is universally believed to exist, but for which our evidence heretofore (if we except the phenomena of novae) is altogether the indirect evidence of the continuous gradations from one star to the next of various constant characters (such as spectrum, motion, intrinsic luminosity), and not the direct fact of variation.[1]

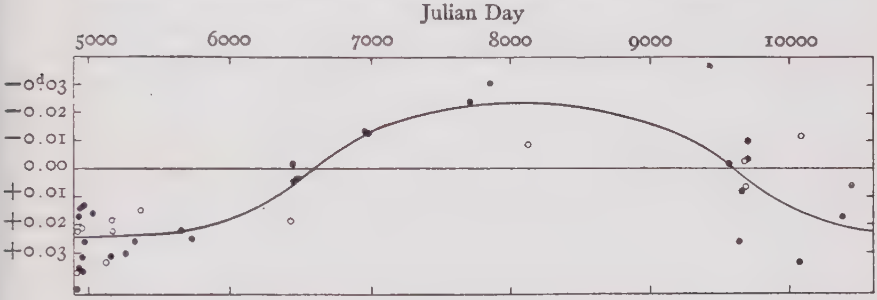

FIG. 2.—Long-period variation in light-elements of RR Lyrae. Open circles indicate observations of weight less than 3.

The data of Table I could be increased without trouble, but beyond establishing the qualitative result there is little more to be done to advantage with low dispersion. The change of spectrum with brightness is considered proved. This is in complete agreement with the results for SW Andromedae[2] and RS Boötis.[3] For all variables of the cluster type we should expect a similar variation of the spectrum, at least for those for which the photographic range exceeds the visual.

The photographic range of RR Lyrae, according to Martin and Plummer, does not exceed the visual, a result hardly to be expected if the change in spectrum is from a normal A type at maximum to the redder F type at minimum.[1] Their result, however, is not necessarily conclusive, for none of the photographic series of measures completely covers the maximum. Moreover, Fig. 3 indicates that the individual maxima may vary greatly in height. It may be, then, that simultaneous observations of the spectrum and of the photographic and visual light-curves, referring the light-curves to standard magnitude scales, would show either a visual range less than the photographic, or, when equal, no change in the spectrum.

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[1] See "A Short Period Cepheid with Variable Spectrum," *Proceedings of the National Academy of Sciences*, **2**, 132, March, 1916.

[2] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914.

[3] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914; *Publications of the Astronomical Society of the Pacific*, **26**, 256, 1914.

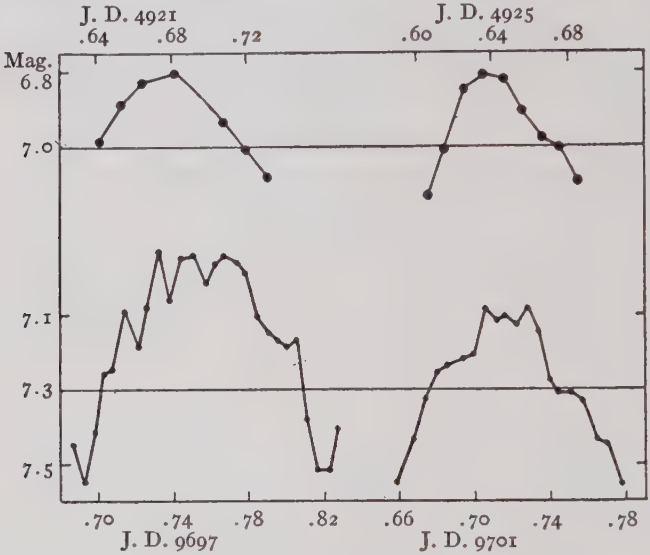

FIG. 3.—Variations in the shape of maxima of RR Lyrae.

III. THE MEAN PERIOD AND ITS SLOW VARIATION

From the first year's observation of RR Lyrae, Wendell found the approximate period $0^{d}.5668$, and chose as a convenient epoch of reference the round numbers J.D. 2414856.5000, G.M.T.[2] In this case, as in many others where light-elements have been determined at Harvard, the initial epoch is not intended to be coincident with a principal phase of the light-variation—maximum or minimum.[1] In the present case the chosen epoch comes nearly an hour after the observed maximum, but the two have been used as coincident in various discussions of the period. This has led to some error in determinations of the light-elements, and particularly in comparisons of photographic and visual variations; but the chief criticisms of the two or three recent discussions must be that the long and important series of Harvard observations have been almost completely neglected, and that the periodic irregularities in the form of the light-curve have been ignored.

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[1] The variation in photographic magnitude is 0.80, and visually (Wendell) is 0.84.

[2] *Harvard Annals*, **69**, Part I, 95, 1907; Part II, 165, 1914.

No attempt to improve the light-elements was made at Harvard after 1900, and the phases of all the observations by Wendell are computed from the foregoing approximate elements. Hertzsprung found that the mean period should be lengthened,[2] deriving the value $0^{\mathrm{d}}.56682$. Fontana[3] suggested, on the basis of data he considered weak, that the period should be still longer, $0^{\mathrm{d}}.5668267$; and Kiess derived at the same time the value $0^{\mathrm{d}}.566826$, finding difficulty in harmonizing photographic and visual observations. In the most recent discussion Martin and Plummer conclude that the period is increasing secularly from the value given by Wendell in 1899 to $0^{\mathrm{d}}.566863$ in 1914, giving the formula for maximum:

$$\text{J.D. } 2414856.470+0.566798E+(10^{-4}E)^2\div 3. \qquad (1)$$

That the period of RR Lyrae is not constant is shown clearly in the two Harvard series.[4] Plotting the observations, it is also immediately evident that Wendell's approximate period soon fails to represent his measures. A better uniform period for the observations of the first three years is the same one that, within the known errors and irregularities, represents the observations by Martin twelve years later. The period, then, is probably not longer now than it was fifteen years ago, but in the interval it has not remained constant.

A closer examination of this question, which may have an important bearing in the interpretation of Cepheid variation, is afforded by the material collected in the following tables. The various sources of light-observations, so far as they are known to the writer, are enumerated in Table II.

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[1] See a case cited in *Popular Astronomy*, **20**, 656, December 1912.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **46**, 284, 1911.

[3] Fontana was the first to suspect a variation in the mean period, *Memorie della Società degli Spettroscopisti Italiani*, Serie II, **2**, 188, 1913.

[4] We are not yet considering the short-period oscillations.

In all these series of measures very few continuous sets cover light-maximum, and but little could be determined of the variations in the period if we depended only on actual measures at the brightest phase. On the other hand, a great many determinations are possible of some selected point on the steep rising branch of the curve, for much of Wendell's work was evidently planned with an investigation such as the present in view. His series of nightly measures usually cover only the steepest part of the ascending branch, and, though they serve but little in showing directly the changes in the shape of the curve, they are of prime importance in studying the period, and especially in demonstrating the oscillations in the time of the rise to maximum light.

TABLE II

SOURCES OF OBSERVATIONS OF RR LYRAE

Observer	Place	Publication	Observing Method	Interval	Number
Wendell I......	Harvard	*H.A.*, **69**, 45	Polarizing	1899.6–1902.1	241
Wendell II......	Harvard	*H.A.*, **69**, 124	Polarizing	1903.8–1907.7	61
Von Zeipel......	Upsala	*A.N.*, **177**, 372	Zöllner	1906.9–1908.0	38
Haynes........	Laws Obs.	Unpublished	Zöllner	1908.4–1908.5	24
Hertzsprung.....	Potsdam	Unpublished*	Photogr.	....	160
Fontana........	Catania	*Spett. It.*, **2**, 183	Wedge	1912.4–1912.5	44
Townley........	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.7	40
Kiess..........	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.9	130
Shapley........	Princeton	This paper	Polarizing	1913.8–1913.9	28
Martin.........	Dunsink	*M.N.*, **75**, 566	Photogr.	1913.8–1914.9	108

Vierteljahrsschrift der Astronomischen Gesellschaft*, **49, 192, 1914. For Hertzsprung's notes on the proper motion, see *Astronomische Nachrichten*, **196**, 205, 208, 1913.

The time on the ascending branch when the magnitude is halfway between the minimum and maximum values has been chosen as the epoch of reference, and for convenience will be referred to as the time of *median magnitude*. The method of determining these times is as follows: Neglecting the computed and published phases, the observations from all sources are reduced to Greenwich heliocentric mean time and plotted on a uniform scale, furnishing fragmentary light-curves for each Julian Day on which the star was observed.[1] A mean light-curve, plotted to the same scale on tracing paper, is superposed on the plot of each night's measures and adjusted in the time co-ordinate so as to satisfy all the observations as closely as possible. The time of the median magnitude is then read off, with an error of not more than two or three minutes in a good series of observations.

For the photographic observations the mean curve by Plummer and Martin was used. To determine the time of median magnitude for the visual observations a mean curve was derived from Wendell's first series. Allowance was made in each observer's work for the difference in range of variation, a factor depending on the choice of comparison stars as well as on the scale of the photometer. To form the mean visual light-curve the phases in *Harvard Annals*, **69,** Part I, are corrected to conform to the period $0\overset{d}{.}566826$, and the initial epoch adopted by Wendell is retained. The resulting normal magnitudes are given in Table III and the curve is plotted in Fig. 1.[2]

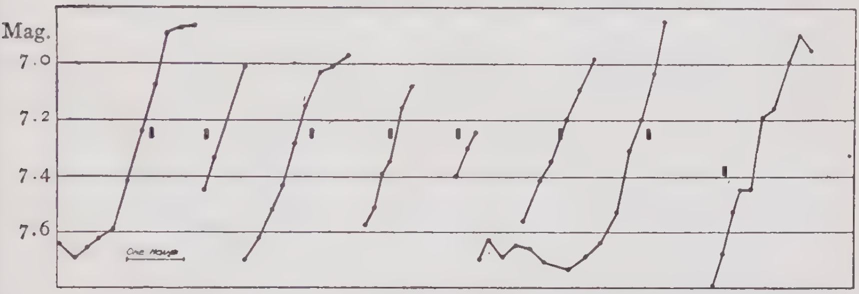

FIG. 4.—Observed ascending branches and computed times of median magnitude for epochs 144, 195, 225, 765, 2755, 3766, 5271, 9210. The existence of the short-period oscillation is illustrated.

It should be noted that the determination of mean curves for stars of this type probably has no value beyond occasional use, such as the present. The great deviation in the form at different epochs[3]

[1] Such as are plotted in Fig. 4.

[2] It was found after the completion of this work that the correction for the equation of light applied at Harvard to the phases in this series of measures was made with the wrong sign. As the correction is not large, the error has little effect on the mean curve. The times of observation recorded at Harvard are geocentric.

[3] See Fig. 3.

makes a mean curve representative of only those possibly widely unlike maxima which chance to compose it, and at most but a rough model of the curve at an unobserved maximum.[1] With this idea in mind no attempt is made to derive a definitive mean curve from all the observations, or even from the homogeneous Harvard series alone.

TABLE III

NORMAL MAGNITUDE CURVE OF RR LYRAE

No.	Phase	No. Observations	Magnitude	No.	Phase	No. Observations	Magnitude
1......	$0^{d}018$	8	6.97	18......	$0^{d}369$	4	7.67
2......	.034	5	7.04	19......	.388	7	7.64
3......	.051	7	7.09	20......	.412	3	7.66
4......	.065	3	7.19	21......	.426	7	7.66
5......	.087	3	7.30	22......	.436	10	7.68
6......	.108	4	7.32	23......	.444	6	7.67
7......	.130	3	7.39	24......	.456	9	7.66
8......	.148	3	7.43	25......	.466	8	7.62
9......	.169	7	7.44	26......	.475	9	7.53
10......	.190	5	7.50	27......	.486	15	7.41
11......	.209	6	7.53	28......	.495	17	7.25
12......	.230	8	7.56	29......	.505	22	7.12
13......	.248	4	7.59	30......	.515	18	7.00
14......	.268	6	7.58	31......	.525	12	6.93
15......	.296	4	7.61	32......	.540	5	6.83
16......	.336	4	7.60	33......	0.564	5	6.88
17......	0.348	4	7.64				

The short series of observations by von Zeipel, Haynes, and Fontana contain no sets of successive observations sufficient to give the time of the rise to single maxima, but in each case a mean curve of more or less weight can be obtained which gives a determination of the time of median magnitude for the mean date. These observations cannot be used, however, to discuss short-period oscillations.

The observations by the writer, made with the polarizing photometer and 23-inch refractor of the Princeton Observatory, have been reported upon briefly in an earlier note.[2] The individual observations are now given in Table IV. The comparison star is

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[1] Harmonic analyses of the light-variations of cluster-type stars, if they are to have any meaning, must make allowance for these wide oscillations. See, for example, the diagrams of maxima of XX Cygni, *Astrophysical Journal*, **42**, 159, 160, 1915.

[2] *Popular Astronomy*, **22**, 144, 1914.

B.D.$+42^\circ 3328$, for which the Harvard[1] visual magnitude is 8.96. My observations show the same range as Wendell's, but the whole curve is $0^{M}.15$ below his, evidently because the comparison stars are different and their adopted magnitudes may have errors of that amount. Only one measure of the time of the median magnitude is obtainable from this series, but that determination is of considerable weight (Fig. 4).

TABLE IV

PRINCETON OBSERVATIONS OF RR LYRAE

No.	E.S.T.	G.H.M.T.	Julian Day	Mag. Diff.	Mag.	Remarks
	1913		2420000+			
1	Oct. 23 $6^h 16^m$	$11^h 16^m$	64.469	1.15	7.81	
2	27.5	28	478	1.15	7.81	
3	38	38	485	1.17	7.79	
4	49	49	499	1.18	7.78	Clouds
5	7 4.5	12 5	503	1.26	7.70	Thick
6	15	15	510	1.24	7.72	
7	30	30	521	1.22	7.74	Clouds
8	45	45	531	1.23	7.73	
9	Nov. 5 5 56	10 56	77.456	1.24	7.72	
10	6 5.5	11 5	462	1.11	7.85	
11	16.5	16	469	1.10	7.86	
12	26	26	476	1.11	7.85	Haze
13	43	43	488	1.14	7.82	
14	54.5	54	496	1.28	7.68	
15	7 4.5	12 4	503	1.43	7.53	
16	13	13	509	1.51	7.45	
17	23.5	23	516	1.51	7.45	
18	35	35	524	1.77	7.19	
19	45	45	531	1.80	7.16	
20	8 0	13 0	542	1.97	6.99	
21	12	12	550	2.07	6.89	
22	23	23	558	2.01	6.95	
23	Nov. 17 5 35	10 34	89.440	1.98	6.98	
24	46	45	448	1.97	6.99	
25	57	56	456	1.96	7.00	
26	6 7.5	11 6	462	1.98	6.98	
27	20	19	472	1.90	7.06	
28	30	29	478	1.85	7.11	

The epoch of the one maximum (photographic) available from Hertzsprung's observations is taken from the statement by Kiess.[2] It is corrected to the time of the median magnitude by means of the Dunsink mean photographic curve—a doubtful procedure, hence the low weight assigned it in Table V.[3]

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[1] *Harvard Annals*, **63**, 178, 1913.

[2] *Op. cit.*, p. 140.

[3] The observation is not plotted in Fig. 2.

The data for the discussion of the period are summarized in Table V. The heliocentric times in the second column refer to the median magnitude. The number of the epoch in the third column is counted from the initial epoch used by Wendell. Weights depend on the number and arrangement of the observations and on the general reliability of the photometric method involved.

TABLE V

Observations of Median Magnitude

No.	J.D. and G.H.M.T.	Epoch	Observer	Weight	$O-C_1$	$O-C_2$	$O-C_3$
	2400000+						
1.....	14913.690	100	Wendell I	2	$+0^{d}.015$	$+0^{d}.012$	$+0^{d}.014$
2.....	14921.624	114	"	7	+ .013	+ .010	+ .012
3.....	14925.599	121	"	6	+ .021	+ .018	+ .020
4.....	14933.514	135	"	2	.000	− .004	− .002
5.....	14938.611	144	"	9	− .005	− .009	− .007
6.....	14947.684	160	"	3	− .001	− .005	− .003
7.....	14967.538	195	"	4	+ .014	+ .010	+ .012
8.....	14968.667	197	"	4	+ .009	+ .005	+ .007
9.....	14980.554	218	"	9	− .007	− .011	− .009
10.....	14984.521	225	"	8	− .008	− .012	− .010
11.....	14993.602	241	"	6	+ .004	.000	+ .002
12.....	15018.533	285	"	8	− .006	− .010	− .008
13.....	15150.621	518	"	2	+ .011	+ .007	+ .009
14.....	15171.592	555	"	4	+ .009	+ .005	+ .007
15.....	15175.550	562	"	2	.000	− .004	− .002
16.....	15184.616	578	"	3	− .004	− .008	− .006
17.....	15290.625	765	"	7	+ .008	+ .004	+ .005
18.....	15315.561	809	"	7	+ .004	+ .001	+ .002
19.....	15387.538	936	"	3	− .007	− .010	− .009
20.....	15663.591	1423	"	5	.000	− .001	.000
21.....	15743.517	1564	"	6	+ .003	+ .002	+ .003
22.....	16418.608	2755	Wendell II	3	− .001	+ .012	+ .012
23.....	16443.528	2799	"	8	− .021	− .008	− .008
24.....	16456.571	2822	"	5	− .015	− .001	− .001
25.....	16477.543	2859	"	6	− .016	− .001	− .001
26.....	16991.641	3766	"	8	− .033	.000	− .001
27.....	17000.711	3782	"	9	− .032	+ .001	.000
28.....	17703.002	5021	Von Zeipel	9	− .043	− .001	− .003
29.....	17844.704	5271	Wendell II	10	− .049	− .006	− .008
30.....	18122.474	5761	Haynes	1	− .026	+ .018	+ .016
31.....	18919.393	7167	Hertzsprung	2	− .070	− .032	− .035
32.....	19570.165	8315	Fontana	9	− .018	+ .004	.000
33.....	19635.946	8431	Townley	7	+ .010	+ .029	+ .025
34.....	19659.735	8473	"	6	− .008	+ .010	+ .006
35.....	19692.600	8531	Kiess	2	− .019	− .003	− .007
36.....	19693.743	8533	"	3	− .009	+ .007	+ .003
37.....	19697.694	8540	"	6	− .026	− .010	− .014
38.....	19701.668	8547	"	6	− .020	− .004	− .008
39.....	20077.515	9210	Shapley	10	+ .019	+ .024	+ .019
40.....	20093.341	9238	Martin	2	− .027	− .022	− .027
41.....	20390.388	9762	"	6	+ .002	+ .002	− .003
42.....	20453.296	9873	"	4	−0.009	−0.009	−0.014

The residuals in the sixth column, $O-C_1$, show the representation of the observations by the linear formula:

$$\text{Med. Mag.}=\text{J.D. } 2414856.992+0^{\text{d}}.566830\,E. \tag{2}$$

The agreement is fair at the beginning and end of the series, but for an intervening interval of ten years all residuals are negative, and the probability of a long-period change is at once apparent. Plotting $O-C_1$ against the epochs, it is found by graphical methods that the following harmonic expression represents these residuals with considerable accuracy:

$$-0^{\text{d}}.020-0^{\text{d}}.024 \sin\,(0^{\circ}.0340\,E-104^{\circ}.5). \tag{3}$$

The period of this variation is 10600 E, or about 16.5 years, and the amplitude is more than an hour. The necessity of the correction is exhibited in Fig. 2; the available data are somewhat fragmentary, however, and it cannot be claimed that the formula will satisfy fully the future deviations from a uniform period. The recent rough estimates of magnitude, listed in Table I, were made a year later than any observations involved in the derivation of the revised light-elements, and they appear to be in very good agreement with prediction. The phases in that table are computed by means of formula (5) below.

The residuals in the seventh column of Table V, $O-C_2$, result from correcting those of the preceding column by means of (3). The new residuals were then discussed by least squares to determine corrections to the initial epoch, T, and the mean period, P. Forty-two equations of condition were involved in the solution, and the corrections in days are as follows:[1]

$$\left.\begin{aligned}\Delta T&=-0.0020\pm0.0011\\ \Delta P&=+0.0000074\pm0.0000218\end{aligned}\right\} \tag{4}$$

Applying these corrections, reducing from the median magnitude to maximum light, and adding the correction (3), the heliocentric light-elements that represent all the observations to date are:

$$\begin{gathered}\text{Max.}=\text{J.D. } 2414856.451+0^{\text{d}}.566831\,E\\ -0^{\text{d}}.024 \sin\,(0^{\circ}.0340\,E-104^{\circ}.5).\end{gathered} \tag{5}$$

[1] The large probable error of the period is accounted for in Section V.

The representation by this formula is shown by the residuals in the last column of Table V and by the deviations from the sine curve in Fig. 2. They will be discussed further in Section V.

IV. CHANGES IN THE FORM OF THE LIGHT-CURVE

That a wide variation exists in the form of the light-curve at different maxima has been shown directly for a few cluster-type variables,[1] and the oscillations in the time of the median magnitude all but prove the same thing for others of this class.[2]

Fig. 3 contains diagrams of maxima of RR Lyrae which show that for it also the curve changes its shape very conspicuously and rapidly. The two upper curves are taken from Wendell's earliest observations. The maxima are only seven periods apart, but the interval of time when the star was brighter than $7^{M}.0$ is in one case $1^{h}55^{m}$, and in the other nearly 40 per cent less. The lower pair of curves in Fig. 3 are from the Lick Observatory measures by Kiess. The interval when the brightness exceeded $7^{M}.3$ is about twice as great at the first maximum as at the second, which follows by seven periods. Moreover, the brightness at maximum differs by $0^{M}.15$.[3] If the photographic observations by Martin chanced to refer to maxima of the "faint" kind, the apparent equality of photographic and visual ranges may have no real significance.[4]

It is obvious from the diagrams that the determination of the time of median magnitude involves directly the shape of the curve. It suggests that oscillations in that time indicate variations in shape rather than actual oscillations in the epoch of maximum light. Apparently the recovery from a delayed outburst is more rapid than from an early one; that is, the factors that cause the late rise to maximum light generally allow an early decline, and vice versa. These peaked and round-topped curves are shown more completely in the figures accompanying the earlier paper relative to XX Cygni.

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[1] XX Cygni, *loc. cit.;* SW Draconis and SU Draconis, *Astronomische Nachrichten*, **184**, 241, 1910.

[2] ST Ophiuchi, SW Andromedae, RR Lyrae, *Mt. Wilson Contr.*, No. 92, p. 5; *Astrophysical Journal*, **40**, 452, 1914.

[3] If we should attribute the difference in maximum magnitude to a variation in the comparison star, the curves would still be conspicuously dissimilar.

[4] Similarly, perhaps, we should not accept finally the hump on the photographic curve three hours after maximum, for there remains the strong possibility that some of the measures near that phase refer to narrow maxima and others to the wider variety.

V. OSCILLATIONS IN THE TIME OF THE RISE TO MAXIMUM LIGHT

From the least-squares solution for the light-elements the probable error of a unit-weight determination of the time of median magnitude is $\pm 0^{d}.0167 = \pm 24$ minutes. For the average determination listed in Table V, therefore, the probable error is more than ten minutes, and for none is it less than seven minutes. Such large uncertainties in the observations, however, are clearly impossible. In many of Wendell's measures of the rise to maximum light the error in determining the time of median magnitude cannot well exceed two minutes, and from an inspection of all the plotted curves on which Table V is based the probable error of unit weight would be estimated at less than ± 8 minutes. The curves given in Fig. 4 illustrate this point fully. All the plotted measures are from Wendell except the last, which is taken from the writer's observations. The short heavy vertical line near each series indicates the time of the median magnitude for that maximum, computed from the adopted light-elements (5).

There can be no doubt that a real irregularity is present. An attempt was made to find a uniform period for the variation that would satisfy all the observations. This failed in part, perhaps because of insufficient data, but it seems that for the whole series the oscillation is roughly periodic with a varying amplitude. For the first 14 months of Wendell's measures, however, the variation is definitely periodic. Throughout that interval the following simple formula represents the residuals, $O-C_3$, with considerable accuracy:

$$(O-C_3)-0^{d}.004-0^{d}.013 \cos \frac{2\pi}{70}(E-45)=0. \qquad (6)$$

Table VI contains the data relative to this correction. The residuals from (6) are given in the fifth column, and the last column contains the difference between the weighted squared residuals before and after correcting. The existence of the periodic variation is completely established. The sum of the squares of the residuals is decreased from 0.007762 to 0.001034, which gives, in this interval, for the new probable error of an observation of weight unity ±9 minutes. The variation is shown diagrammatically in Fig. 5.

TABLE VI

The Application of Formula (6)

No.	Epoch	Weight	$O-C_3$	$O-C_4$	$(pvv)_3-(pvv)_4$
1	100	2	$+0^{d}.014$	$+0^{d}.007$	+0.000294
2	114	7	+ .012	− .005	+ .000833
3	121	6	+ .020	+ .005	+ .002250
4	135	2	− .002	− .003	− .000010
5	144	9	− .007	.000	+ .000441
6	160	3	− .003	+ .001	+ .000024
7	195	4	+ .012	.000	+ .000576
8	197	4	+ .007	− .003	+ .000160
9	218	9	− .009	.000	+ .000729
10	225	8	− .010	− .002	+ .000768
11	241	6	+ .002	− .006	− .000192
12	285	8	− .008	.000	+ .000512
13	518	2	+ .009	+ .004	+ .000130
14	555	4	+ .007	+ .006	+ .000052
15	562	2	− .002	+ .003	− .000010
16	578	3	− .006	.000	+ .000108
17	765	7	+0.005	+0.004	+0.000063

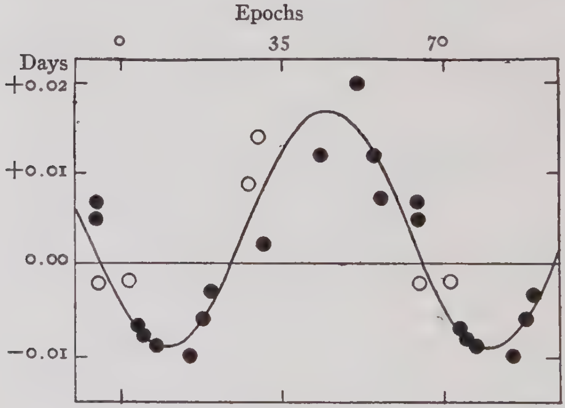


Fig. 5.—Short-period variation in time of median magnitude.

Eleven oscillations are involved; the period is 70 epochs or 40 days, and the amplitude is 37 minutes.

If we make the reasonable assumption that the errors of all the observations could be reduced proportionately as much as these earlier ones by analogous corrections for short-period irregularities in the median magnitude, then the probable errors from the least-squares solutions for T and P, given by (4), would be reduced from $\pm 0^{d}.0011$ and $\pm 0^{d}.000022$ to $\pm 0^{d}.0004$ and $\pm 0^{d}.000008$, respectively.

VI. SUMMARY

The present paper discusses in some detail the peculiarities of the light-variations of the Cepheid variable RR Lyrae, and considers the variations of its spectrum as observed at Mount Wilson. The star is the brightest of the sub-group with periods of a half-day. It is shown in Section II to be a variable in spectrum as well as in light and velocity. The computations of the third section demonstrate upon the basis of the work of nine observers throughout an interval of fifteen years that the mean period is affected by a slow variation which may complete its cycle in 16.5 years. The fourth section contains diagrams to show that the maxima of the light-curve are not constant in form, and in the fifth the closely analogous phenomenon of variations in the time of the rise to maximum light is proved to exist.

Mount Wilson Solar Observatory
December 1915

Contributions from the Mount Wilson Solar Observatory, No. 113
Reprinted from the *Astrophysical Journal*, Vol. XLIII, 1916

MONOCHROMATIC PHOTOGRAPHY OF JUPITER AND SATURN

By R. W. WOOD

In the *Astrophysical Journal*, **36**, 75, 1912, I gave an account of the results obtained by photographing the moon through ray-filters transmitting limited regions of the spectrum. Marked differences in the distribution of light and shade were observed in the different pictures, the most conspicuous being an extensive dark deposit around the crater Aristarchus, which appeared in the picture made by means of ultra-violet light but was quite absent in the one made by the yellow rays. Laboratory experiments on the appearance of various rocks and minerals, when photographed through these same ray-filters, made it appear probable that the dark spot was due to sulphur or some sulphur-bearing rock.

It seemed quite likely that results of even greater interest would be obtained if the method of monochromatic photography were applied in the case of planets, but instrumental facilities were not at my disposal at the time.

These earlier experiments were made with a 16-inch nickeled glass mirror of 26 feet focus, mounted on the equatorial of the Princeton Observatory, which unfortunately was not adapted to the very accurate guiding necessary in work of this nature. As it appeared probable that increased definition and magnification would bring out regions of smaller area showing selective reflection, and as there was every reason to suppose that the planets would show peculiarities worthy of study, I determined to repeat and improve upon the earlier experiments. In this work I have been aided by a grant of $200 from the Gould Fund of the National Academy. The preliminary experiments were made at East Hampton during the past summer.

The Naval Observatory placed at my disposal a very good Gaertner coelostat, which was used in conjunction with a horizontal reflecting telescope of 56 feet focal length and 16 inches aperture, figured by Mellish and nickeled by the electrolysis of a dilute solution of nickel fluoborate, as described in the *Astrophysical Journal* for November 1915 (**42**, 365). The nickel deposit was necessary, since silver reflects only about 4 per cent of the light in the ultra-violet region of the spectrum utilized in the work.

A new mirror cell was made for the coelostat, large enough to carry a 16-inch flat, which was also nickeled, and the instrument mounted on a very solid pier of brick and cement, sheltered by a small house with a hinged roof. A double-slide carrier for the plate-holder and ray-filters, with two slow motions and two eyepieces for following, was made of sheet brass and mounted on a heavy pedestal of iron.

The photographs made with this apparatus, while distinctly better than those made with the 26-foot mirror, failed to show the very fine details which were desired. This was due partly to bad seeing and partly to the difficulty of following a rather rapid periodic oscillation or drift of the image originating in some small error or errors in the gear wheels which could not be located or remedied.

On this account it appeared best to study the behavior of the various ray-filters, and the spectrum range of the light after reflection from two or more surfaces of silver or nickel and transmission through the filters, rather than to waste time in trying to secure a more accurate drive of the coelostat. These experiments showed that it would be feasible to complete the work with a large reflecting telescope with silvered mirrors, using as an ultra-violet screen a cell filled with dense bromine vapor. Such a cell transmits the entire ultra-violet region below λ 3500, while silver reflects the region λ 3300–3500 sufficiently well to make photography possible in this region with silver-on-glass mirrors.

Photographs were made of the moon and Jupiter with infra-red, yellow, violet, and ultra-violet light, and the times of exposure, with the screen, were determined for both silvered and nickeled mirrors. As a result of this preliminary work, I was able to get satisfactory results at once with the 60-inch reflector of the Mount Wilson Observatory, which was placed at my disposal for four nights in the latter part of October.

With this instrument a very complete set of photographs was made of the moon, Jupiter, and Saturn, by light of the four regions of the spectrum referred to above. Results of very great interest were obtained in the case of the two planets, for dark belts were shown by violet and ultra-violet light, of which no trace appeared visually or on the plates made by yellow light, while the infra-red pictures showed scarcely any trace of the belts ordinarily seen. By applying the methods of three-color printing to these plates we can produce a colored positive, in which the belts of selectively absorbing gases appear strongly colored; in other words, we produce an effect somewhat similar to what we should see if our eyes were sensitive to a wider spectral range.

In the earlier experiments, made several years ago, I used for the ultra-violet ray-filter a rather thick deposit of metallic silver on a thin plate of quartz or uviol glass. Silver films are, however, extremely opaque to the selectively transmitted region, which lies between λ 3000 and λ 3200, so much so that an exposure of two or three minutes is required for the full moon. With bromine vapor I found, however, that a fully exposed plate could be obtained in eight or ten seconds, and that if we use in addition a thin cell filled with a very dilute solution of potassium chromate we obtain a transmitted region practically the same as with a silver film, but of about tenfold intensity. The silver film was accordingly abandoned in the present work and the bromine cell used exclusively in its place.

The cell was made by cutting a section from a square glass bottle and grinding the edges flat. A small hole was then drilled through the wall for the introduction of the bromine. The uviol plates (1 mm in thickness) were cemented with beeswax, which was found by Ribaud, in the course of his investigations on the absorption spectrum of bromine, to withstand the action of the vapor fairly well. A piece of asbestos paper, impregnated with a saturated solution of calcium chloride and strongly heated to drive off all of the water, was placed in the cell before the plates were cemented in position. This was necessary owing to the dampness at East Hampton, for, even when the bromine was thoroughly dried, moisture deposited on the inner surface of the uviol plates when the cell cooled off at night. After several hours there appeared also a deposit which resembled a light dew but which did not dry off when the plates were removed. It appeared to be of a greasy nature, and I attributed it to some volatile compound formed by the action of the bromine on the wax. I subsequently found it on clean glass plates which had been standing on my desk for several days, and was quite at a loss for an explanation until Dr. Mees, of the Eastman research laboratory, called my attention to the fact that beeswax sublimes quite rapidly, a circumstance which I had not suspected, as it is one of the constituents of a supposedly reliable vacuum cement. The trouble was minimized by applying the wax to the outside of the cell only, after the glass plates were placed against the ends. The wax was rolled into thin cylinders between the fingers and melted against the seam with a hot glass rod 2 mm in diameter. Cells prepared in this way showed no trace of the deposit for fifteen or twenty hours, and, as they were cleaned every evening and freshly filled, no further trouble was experienced from this source. Four or five drops of liquid bromine were introduced with a capillary pipette, and after vaporization was complete and most of the air expelled, which required a minute or two, the hole was closed with a plug of wax, touched with a hot glass rod.

The cell was 5 cm in thickness and was clamped in position in front of the plate-holder by means of two brass set screws. Exposure with the quartz spectrograph showed that such a cell was transparent from λ 3500 to the end of the solar spectrum (λ = 2900), while it cut off completely all of the less refrangible rays capable of affecting an ordinary photographic plate.

If a more restricted region of the spectrum is required, we may add a thin cell, say a millimeter or two thick, containing a very dilute solution of chromate of potash; the yellow color should be barely perceptible by daylight. The correct dilution can be determined by trial only, the quickest way being to use sunlight filtered through the bromine cell, a quartz spectrograph, and a plate of uranium glass. If the solution is not extremely dilute, it removes all of the ultra-violet. If just right, it removes the region λ 3250–3500, the combination transmitting the same region as a thick silver film, but with a much greater intensity (about tenfold). If the chromate-of-potash cell is used, it is necessary to give three or four times the exposure required with the bromine cell alone.

The infra-red screen transmitted the region above λ 7000, the yellow screen everything above λ 5000, and the violet the region

PLATE V

1 Infra-red

2 Yellow

3 Violet

4 Ultra-violet

Monochromatic Photographs of Saturn

between λ 4000 and λ 4500. These screens were prepared from stained films of gelatine made by the Eastman company and mounted in balsam between thin sheets of plate glass. They were of such a size that they could be substituted for the thick bromine cell, and were held in place by the same set screws.

The work at Mount Wilson was commenced on October 22. The double-slide carrier with its attachments for the bromine cell was adapted to the 60-inch telescope, so that the work could be taken up without the construction of any new apparatus. The early hours of the evening were spent in making plates of the moon, which was full at the time, through the four ray-filters, while the rest of the night was given over to the planets.

The telescope was used at the 80-foot focus, and, as the mirrors were silvered, the region of the spectrum utilized for the ultra-violet photography was slightly less refrangible than in the earlier work with the silver film. Preliminary experiments had been made at East Hampton with a quartz spectrograph on the spectrum of sunlight after three reflections from silver and transmission through the bromine cell (the conditions which obtained with the 60-inch telescope).

The most interesting results were obtained in the case of Saturn. The picture taken through the infra-red screen showed the ball of the planet practically devoid of surface markings, there being only the merest trace of the belts ordinarily seen. Through the yellow screen the planet presented its usual visual appearance, the narrow belts showing distinctly. On the plates made with the violet ray-filter (transmission λ 4000–λ 4500) a very broad dark belt surrounded the planet's equator, occupying the region of the planet which was brightest in yellow light. In addition to this dark equatorial belt, a dark polar cap of considerable size appeared in the pictures. So different were the two pictures that, were it not for the ring, it would have been difficult to believe that they represented the same object. In ultra-violet light the appearance was much the same, but the dark belt was not quite as wide, the bright region between the polar cap and the belt being distinctly broader. Photographs made with the four monochromatic filters are reproduced in Plate V.

Two hypotheses suggest themselves in explanation of the dark belt. We may be dealing with a fine mist or dust which forms an extension of the crape ring down to the ball of the planet. This hypothesis appears to be favored by the circumstance that, on the negatives made by the violet and ultra-violet light, the sky between the ball of the planet and the ring is distinctly denser than the region just outside. This would indicate that the region inside of the ring is filled with some material which reflects the short wave-lengths to a slight degree. No trace of this darkening appears on any of the plates made with the yellow screen, even on one that was many times overexposed, which appears to show that the phenomenon is real. This peculiarity was, however, not detected until the work was finished, and I should prefer to verify it or have it verified before recording it as an established fact. The luminosity is much too feeble to show in the prints.

Measurements of the density of the plates between the ring and the ball of the planet and the sky just outside of the ring have been made for me by Mr. Harold D. Babcock, of the Mount Wilson staff, with the Hartmann photometer. His results showed greater density between the ring and ball on both plates (yellow and ultra-violet, but the density was greater in the latter). His values for the densities are as follows:

Yellow Negative		Ultra-Violet Negative	
Sky outside of ring	1.1	Sky outside of ring	1.05
Sky between ring and ball	1.17	Sky between ring and ball	1.27

One's first thought is that slight darkening of the region between the ring and ball may be the result of halation. It appears to me, however, that if this were the case we should expect the effect to be more marked with yellow light, which penetrates the film and the glass plate and is reflected back, while the ultra-violet light does not even get through the photographic film, as is shown by the circumstance that the image does not appear on the reverse side of the film with prolonged development, as is the case with ordinary light.

I do not, however, believe that the dark belt is in reality due to the absorption of a dust ring, for measurements made by Mr. Ellerman showed that the belt extended higher up on the ball of the planet than the line of intersection of the plane of the rings. Moreover, it seems highly probable that the belt and the dark polar cap are to be explained in the same way.

The second hypothesis assumes the existence in the planet's atmosphere of some substance capable of absorbing violet and ultra-violet light. This material might be a fine mist or dust, or some gas capable of absorbing the more refrangible part of the spectrum. Such a gas would be of a pale yellow color, and sulphur vapor and chlorine naturally occur to us. I have, however, examined the absorption spectrum of both of these gases in quartz bulbs, with the result that the absorption appears to be much stronger in the ultra-violet than in the violet, which is in disagreement with the circumstance that the band appears wider in violet than in ultra-violet light.

Two spectrograms of the planet have been made by Adams with the 60-inch reflector. In one the slit of the instrument was parallel to the major axis of the ring and cut through the dark belt shown on the violet and ultra-violet photographs. In the other the slit cut across the bright region above the belt. The spectra embraced the region between λ 5000 and λ 4000, and we should expect that between λ 4000 and λ 4500 the spectrum of the area occupied by the dark belt would be much darker (lighter in the negative) than the other. No difference could be detected, however, between the two negatives, except at the very end (λ 4000), where there was a very slight trace of the expected effect.

Of course it would be preferable to place the slit parallel to the short axis of the ellipse and show the spectra of the two regions juxtaposed on the same plate, but the spectroscope available at the time did not permit of rotation. A quartz spectrograph is being planned for a more complete study of the question. The spectra were made in February 1916, and it is of course possible, though hardly probable, that the ultra-violet belt had in the meantime disappeared.

Another point of interest is the decrease in contrast between the inner and outer ring as the wave-length is decreased. This suggests that the outer ring contains so much finely divided matter that it shines in part by diffusion. On the infra-red photograph the ball of the planet is much brighter in comparison to the brightest part of the ring than on the violet and ultra-violet pictures. This again suggests a mist or dust in the planet's atmosphere which scatters the shorter wave-lengths. My infra-red photographs of landscapes have clearly demonstrated that we can obtain clear photographs through a blue haze by means of the spectrum region above λ 7200. In these photographs it will be remembered that the blue sky comes out black and the grass and foliage show white. (See *Physical Optics*, 2d ed., p. 626.)

Photographs of Jupiter made with the four ray-filters showed differences almost as marked as those found in the case of Saturn, though not so striking by casual inspection. The rotation of the planet is so rapid that it was found necessary to make the series of pictures as quickly as possible, if comparable results were to be obtained. Following was accomplished by bringing one of the outer satellites on the cross-hair of the eyepiece in the movable plate-holder. The exposure times were two seconds for the violet and yellow negatives, seven seconds for the ultra-violet, and ninety seconds for the infra-red. The dark belts are scarcely visible on the infra-red plates, while the pictures made with violet light show them in the greatest contrast.

This circumstance made it appear probable that very marked color contrasts would appear if the planet were viewed through a screen which transmitted deep-red and violet light. The violet filter used in the work has this property, and through it the cloud belts appeared deep red on a violet background. Unfortunately the vivid color contrast obtained in this way is not of much help in the visual study of the planet, as the eye does not focus well for these two regions of the spectrum.

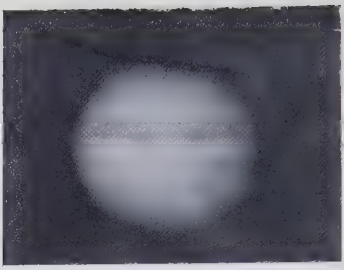
The appearance of the disk in the light of the four spectral regions specified is shown on Plate VI. They were all made on the evening of October 24, 1915, between 7:30 and 9:30 Pacific time. The details are given in Table I.

The direction of rotation of the planet is from right to left. On three of the plates slight blemishes have been noticed which have nothing to do with the surface markings: a light spot on the broad dark belt near the left hand edge, Fig. 8; a small dark spot near the lower edge, Fig. 11; and a small dark spot on the broad dark belt above the bright equatorial belt.

The chief differences between the views of the planet made by light of different wave-lengths may be summarized as follows: The infra-red pictures show less darkening of the disk as we approach

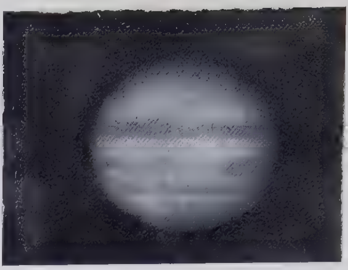
PLATE VI

5



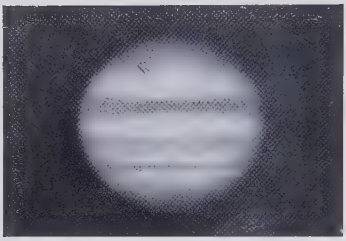
Infra-red

6



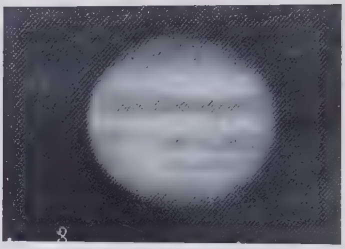
Yellow

7



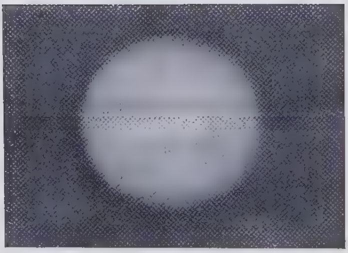
Violet

8



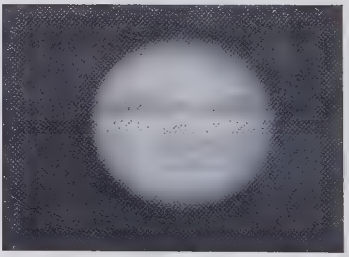
Ultra-violet

9



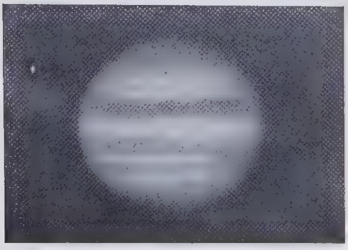
Infra-red

10



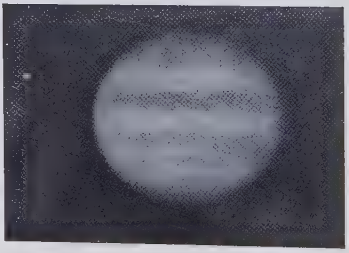
Yellow

11



Violet

12



Ultra-violet

Monochromatic Photographs of Jupiter

the limb, which is what would be expected on the theory that the darkening results from atmospheric absorption or scattering. The cloud belts, moreover, are very inconspicuous in these pictures. The two dark belts above the bright equatorial belt have a number of still darker spots on them which are most distinct in the yellow picture, less so in the violet, and almost invisible in the ultra-violet (Figs. 10, 11, 12). Above this dark belt a double bright belt appears in the violet and ultra-violet pictures, which cannot be seen in the infra-red and yellow pictures, or, to speak more accurately, the dark belt between them is visible only in the violet and ultra-violet. The upper dark polar cap shades off gradually in the yellow and infra-red views, but is sharply terminated in the violet and ultra-violet, its lower edge marking the upper boundary of the double bright belt. These are the chief peculiarities. Observers who have made a long study of the surface markings of Jupiter will doubtless find other points of interest, and I shall be glad to lend the original plates to anyone who wishes to study them in greater detail.

TABLE I

Fig.	Ray-Filter	Time	Exposure	Plate
5	Infra-red	7:30	90 sec.	Spect. Panchrom.
6	Yellow	7:33	2	Cramer Iso. Inst.
7	Violet	8:15	2	Process
8	Ultra-violet	7:50	7	Process
9	Infra-red	9:20	90	Spect. Panchrom.
10	Yellow	9:23	2	Cramer Iso.
11	Violet	9:25	2	Process
12	Ultra-violet	9:30	7	Process

TRICHROMATIC PHOTOGRAPHS OF THE PLANETS

By applying the methods of trichromatic color-photography to the negatives made through the ray-filters, very beautiful and striking transparencies have been made, which represent, in strong color contrasts, the differences which can be seen by comparison of the monochromatic photographs. As is well known, if three negatives are made of an object through red, green, and blue ray-filters, and the positives made from them on bichromatized gelatine are stained with blue, magenta, and yellow dyes, these positives when superposed in register yield a fairly faithful reproduction of the object in its natural colors.

If we apply this same method to pictures made by yellow, violet, and ultra-violet light, we shall obtain a colored picture of the object as it would appear if the region of the eye's sensitiveness were shifted slightly toward the ultra-violet, or, to put the matter in astrophysical language, as the object would appear if it were receding from us with a velocity sufficient to cause ultra-violet to appear violet! Color-photographs of Saturn made in this way show the ball of the planet yellowish in color with a broad orange-red band (the violet and ultra-violet dark belt) surrounding the equator and a dark-red polar cap. The narrow belts, which can be seen visually and which appear on the negative made with yellow light, come out blue in the picture, the lower one being in coincidence with the upper edge of the dark belt. Owing to the fact that the dark belt is wider on the violet negative than on the ultra-violet one, the upper edge lacks yellow in the finished picture, and the superposed narrow blue belt appears purple in consequence (magenta+blue).

Color-contrasts also appear in the ring, as a result of the circumstance that the outer or fainter ring is relatively more luminous in ultra-violet light. The inner ring appears white and the outer bluish green. Similar colored belts are shown in the pictures made of Jupiter. The photographs as a whole give us a vivid impression that we are really dealing with great belts of selectively absorbing, and hence colored, vapors or gases. They show us, moreover, that the absence of strong color-contrasts in the surface markings of the planets results merely from the limited range of the spectrum to which the eye is sensitive.

In view of the interesting results obtained with these two planets, it is my hope that similar observations will be made on Mars on the occasion of its next near approach to the earth. I made one photograph of the planet with ultra-violet light in October with the 60-inch reflector, but the disk was too small to show much of interest. Mr. Hoge was of very great assistance in helping me with the manipulation of the telescope.

JOHNS HOPKINS UNIVERSITY
February 1916

Contributions from the Mount Wilson Solar Observatory, No. 114
Reprinted from the *Astrophysical Journal*, Vol. XLIII, pp. 341–346, 1916

THE PRODUCTION IN THE ELECTRIC FURNACE OF THE BANDED SPECTRA ASCRIBED TO TITANIUM OXIDE, MAGNESIUM HYDRIDE, AND CALCIUM HYDRIDE

By ARTHUR S. KING

The purpose of this investigation was to produce the banded spectra of titanium oxide, magnesium hydride, and calcium hydride in the tube-resistance furnace, to compare the conditions required for their appearance with those observed in the arc and spark, and to note any phenomena resulting from the special control of conditions possible with the furnace.

All of these banded spectra have been identified in the spectra of sunspots, and the bands attributed to titanium oxide also occur in the spectra of third-type, or Antarian, stars, which fact has led Fowler to designate them as the "Antarian bands." The presence of such banded spectra is taken as evidence of a relatively low temperature, while the appearance of the titanium oxide bands furnishes direct evidence of the presence of oxygen in third-type stars and in the sun.

THE TITANIUM OXIDE BANDS

An extended investigation by Fowler[1] furnished concordant evidence that the Antarian bands in the arc and spark result from the presence of titanium oxide. The flutings occur in a series of groups, all degraded toward the red, throughout most of the visible spectrum. Fowler gives the following wave-lengths for the first head in each group:

4353.68
4462.34
4584.62
4761.08
4954.78
5167.00
5448.48
5597.92
5760.15
5954.66
6158.86
6357.9
6651.5
7054.5

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[1] *Proceedings of the Royal Society*, A, **79**, 509, 1907.

It was noted by the writer,[1] during an examination of the line spectrum of titanium in the furnace at different temperatures, that these bands did not appear with the furnace *in vacuo*. Experiments were recently undertaken to see whether the use of oxygen would bring them out. The photographs were made with a 15-foot concave grating spectrograph, this being set for the first observations to cover the region from λ 5500 to λ 7200 in the first order, with a dispersion of 1 mm=3.7 A. The furnace was usually operated at about 2300° C., which gives a strong titanium spectrum without excessive vaporization of carbon.

Admitting air to one-half atmospheric pressure gave no bands. They again failed to appear when the furnace was full of air, the outlet valve being left open. It seemed probable that a strong current of oxygen was needed, since, there being no circulation through the horizontal tube, the incandescent carbon probably used up the small supply of oxygen before the titanium was fully vaporized. The windows in the ends of the furnace chamber were therefore removed, a silica tube inserted in one end of the furnace tube, and a strong blast of air forced through by means of a bellows. The whole series of bands then came out with great brilliancy. Temperatures of 2300° and 2600° C. were used with no perceptible difference except in exposure time. A noteworthy effect was the almost complete suppression of the line spectrum of titanium when a strong current of air was used, though the lines of several impurities appeared as usual. Evidently a sufficient supply of oxygen resulted in turning all of the titanium vapor into the oxide, which seems to give only the band spectrum.

Pure oxygen in the furnace was next used, this being led directly into the tube from a tank of compressed gas. The spectrum could be changed at will from a mixture of lines and bands to a pure band spectrum by increasing the current of oxygen. When the bands were at their strongest, the image on the slit was a greenish yellow, similar to the envelope of a long carbon arc containing titanium, which is one of the best of the usual sources for obtaining the band spectrum.

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[1] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

There was no evidence of a material change in temperature caused by the introduction of oxygen. The same exposure time with and without oxygen gave the same general intensity of spectrum, this being composed of bands in the one case and lines in the other. Pyrometer measurements of the interior of the tube were practically the same in the two cases. Allowing for differences in clearness of the vapor, the change of temperature could scarcely have been greater than 50°.

Comparing this source with the usual flames employed in producing spectra, we have here the metallic vapor heated by the graphite tube instead of by a union of flame gases, and then the process of oxidation taking place. The results are probably essentially the same as in a flame of the same temperature, but the furnace with oxygen gives a method of obtaining a series of flame temperatures for which different combinations of gases are usually required. There is also the possibility, by the use of very high furnace temperatures combined with a stream of oxygen, of exceeding any of the flame temperatures regularly attainable.

It may be noted that a distinct difference in gradation of intensity with the wave-length appeared for the titanium band spectrum in the furnace as compared with the arc, the bands at the red end being relatively stronger in the furnace.

A supplementary set of photographs was made for the region as far as λ 4200 with the furnace in vacuum, in air at atmospheric pressure, and with varying amounts of oxygen. The results obtained for the yellow and red region were repeated. By the use of a strong current of oxygen, so that a flame extended several centimeters from the end of the furnace tube, a strong band spectrum appeared, while the titanium line spectrum was eliminated except for the faint appearance of a few of the lines strongest in the regular furnace. It was noteworthy, however, that the more prominent lines of iron and chromium, as well as λ 4227 of calcium and λ 4607 of strontium, appeared as strongly as could be expected from the slight impurities present. The evidence thus seems fairly conclusive that the bands of titanium oxide have been correctly ascribed to an actual compound, since otherwise, without a large reduction of temperature, there would seem to be no reason for the disappearance of the line spectrum.

Reproductions are given in Plate VII of the titanium spectrum from λ 4427 to λ 5065 in which the four strips represent respectively the arc in air; the furnace with a moderate stream of oxygen, both lines and bands appearing; the furnace with a strong stream of oxygen, the spectrum consisting almost wholly of bands; and the furnace in air without extra supply of oxygen, in which case the bands are absent. The wave-lengths are those given by Fowler for the principal heads.

THE BANDS OF MAGNESIUM IN HYDROGEN

In a paper giving detailed measurements of the fluted spectrum obtained when magnesium is burned in an atmosphere of hydrogen, Fowler[1] summarizes previous investigations as to the dependence of the spectrum on hydrogen and as to the probability of its corresponding to a chemical compound. The bands consist of three main groups, beginning at λλ 4844.9, 5211.1, 5621.6 and shading toward the violet. They appear best with the arc or spark in an atmosphere of hydrogen, but may often be seen when only a small residue of hydrogen is present. The furnace results are in close agreement with those for the arc, which is a fact of some significance, since the arc discharge is greatly modified by a hydrogen atmosphere, and this might account for the appearance of another spectrum.

The strongest of the bands, with its head at λ 5211, was situated most favorably for observation in the furnace photographs, having a portion of its length undisturbed by the carbon spectrum. The first experiments were made by simply admitting hydrogen to the furnace chamber. Photographs at 1900°, 2200°, and 2500° C. were made for each of three pressure conditions, viz., with a pressure below 1 cm of mercury, the residual gas being air, with hydrogen at a pressure of 10 cm, and with hydrogen at atmospheric pressure. The results were somewhat variable, the main effect being that a high temperature was unfavorable for the bands, which, if they appeared at 2500°, were usually in absorption. The band at

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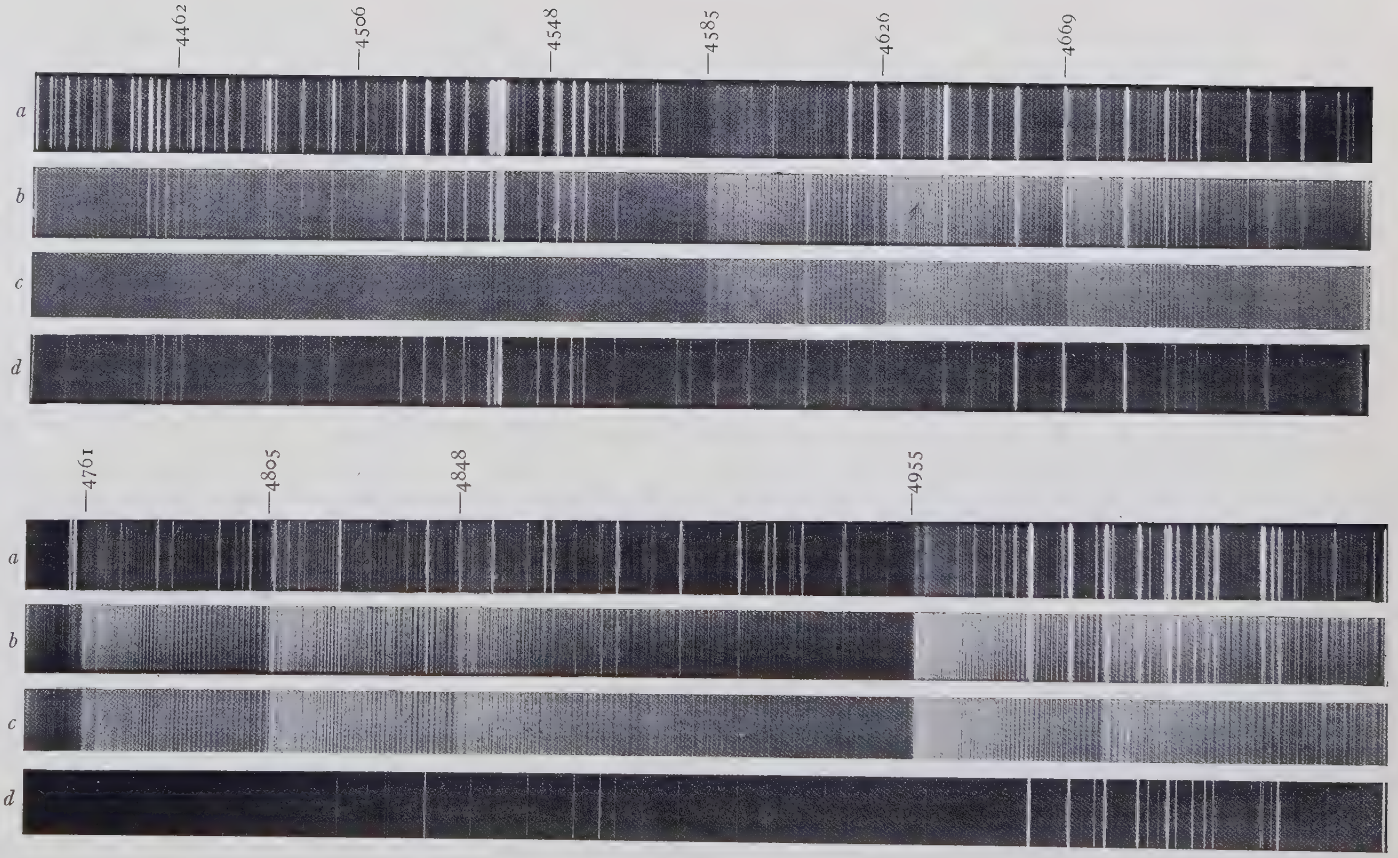
[1] *Philosophical Transactions of the Royal Society*, A, **209**, 447, 1909.

PLATE VII



BANDED SPECTRA OF TITANIUM OXIDE

a. Spectrum of the titanium arc in air.
b. Spectrum of titanium in the electric furnace with a moderate current of oxygen through the tube.
c. Same with a strong current of oxygen.

λ 5211 occasionally appeared distinctly with the furnace pumped out. The experiments with the tube-arc have shown, however, that considerable hydrogen is occluded by the graphite tube and probably by the metallic magnesium, since hydrogen lines appear strongly in the tube-arc under these conditions.

It appeared possible that the effect of hydrogen had not been fairly tested, since a certain degree of combination with the carbon would occur, probably with the formation of acetylene. Arrangements were therefore made for leading a stream of hydrogen directly into the furnace tube by means of a silica tube passing through a rubber stopper in one end of the furnace chamber. The hydrogen was generated chemically from zinc and sulphuric acid and dried by concentrated sulphuric acid and sodium hydroxide. Passing the gas through the tube in this way, the pressure being held at 10–20 cm, proved very effectual in producing the bands. They were obtained in emission at 2500°, but much better at 2200°, when the band at λ 5211 was stronger than the carbon head at λ 5165. The line spectrum of magnesium appeared with the bands, though the *b* group and λ 4571 are the only lines given strongly by the furnace in this region.

The experiments seemed to indicate a clear dependence of the bands on the presence of hydrogen, without any apparent change in the action of the source.

THE BANDS OF CALCIUM IN HYDROGEN

The calcium arc burning in hydrogen was observed by Olmsted[1] to show a banded spectrum in the red with well-marked heads at λλ 6382.2 and 6389.3 and a fainter group in the B region, in both cases shaded toward the violet. The furnace was supplied with a stream of hydrogen through the tube, as in the experiments with magnesium, and the calcium bands were found to be very similar in behavior to those of magnesium. They frequently appear, as was noted by the writer,[2] with the furnace pumped out, apparently from occluded hydrogen. A current of hydrogen through the tube brings them out strongly at temperatures from 1900° to 2200°.

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[1] *Mt. Wilson Contr.*, No. 21; *Astrophysical Journal*, **27**, 66, 1908.

[2] *Mt. Wilson Contr.*, No. 35; *Astrophysical Journal*, **29**, 190, 1909.

Higher temperatures are not so favorable, though the bands have been obtained at 2500°.

The component lines of the band to the red of λ 6700 can be measured to advantage when furnace photographs are made with higher dispersion. The denser portion shows distinct maxima at approximately λ 6902 and λ 6919, while a more scattered structure extends farther to the red. The calcium line spectrum appeared regularly with the bands.

SUMMARY

1. The bands ascribed to titanium oxide show a clear dependence on the presence of oxygen in the furnace, the spectrum consisting of bands alone when a sufficient amount of oxygen is supplied.

2. The bands attributed to magnesium hydride and to calcium hydride require in each case the presence of hydrogen in the furnace to give the bands strongly, although they frequently appear with moderate strength when only a small quantity of hydrogen is present.

3. While the bands appear through a considerable range of furnace temperatures, about 2300° C. seems to be the upper limit for their greatest strength.

Mt. Wilson Solar Observatory
April 5, 1916

Contributions from the Mount Wilson Solar Observatory, No. 115

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

FIRST PAPER: THE GENERAL PROBLEM OF CLUSTERS

By HARLOW SHAPLEY

An extensive investigation of the magnitudes and colors of individual stars in globular and open clusters has been undertaken at the Mount Wilson Observatory. The discussion of the results will be published in the form of a series of papers, each article dealing with a definite problem, or with the observations on a single cluster. In the present communication, which is intended to serve as a brief introduction, our knowledge of cluster systems will be reviewed, the value and general bearing of the cluster investigation in stellar astronomy will be considered, and an outline given of the methods and progress of the work at Mount Wilson. A bibliography of the more important papers dealing with the different phases of the past study of clusters appears in the footnotes[1] and in the list of catalogues on pp. 9 and 10.

I. THE CLASSIFICATION, DISTRIBUTION, AND NUMBER OF CLUSTERS

A stellar cluster is generally defined as any stellar group, the members of which are known to be physically connected, or which from their apparent positions may be assumed to form a distinct physical system. This broad category includes the Hercules cluster as well as the Ursa Major group (or even a system of groups such as our whole galactic domain).

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[1] The first and second papers of this series had been written when Bailey's valuable article on globular clusters was received (*Harvard Annals*, 76, No. 4, 1915). His paper includes a catalogue of globular clusters with individual descriptions, a discussion of the distribution of the brighter stars in ten systems, and an excellent bibliography, which is all but complete up to the beginning of the year 1914.

Various classifications have been proposed, but, because of the great variety of types and the gradual transition from one type to the next, none is much more than a working convenience. Clusters have been classed as loose, irregular, ragged, coarse, open, furrowed, globular, condensed, nebulous, etc., with many qualifying adjectives for each group, but practically all designations are based on superficial appearances.

It is proposed to adopt in the present series of papers the following working classification: (1) globular clusters; (2) open clusters; (3) moving clusters.

Although they are not mutually exclusive, there are some very good reasons which will be brought out in the later discussion for holding rather strictly to these three classes. The first class has for its type the Hercules cluster (M 13) or Messier 3, and for its distinguishing marks great condensation toward the center and numberless stars. The second contains clusters similar to Messier 11 or Messier 67 or even Praesepe, thus exhibiting a wide range in condensation and in number of stars. The third division includes the Taurus group, the Pleiades—in fact, any cluster of stars showing communal motion. Strictly speaking, all known members of the third class belong also in the second, and, further, all clusters are doubtless moving; but it seems best to distinguish between clusters of measured common motion and those in which such motion is as yet merely assumed. Also, roughly speaking, we may suppose a globular form for many of the open or moving clusters, as has already been done for the stars in Taurus.

The adopted classification has on its surface, therefore, the same lack of definite limits as other proposals; but in distribution we can discriminate sharply between open and globular clusters, and in probable distance we can temporarily, at least, differentiate moving groups from other open clusters. The apparent distribution in space is so striking that, taken in connection with other characteristics just coming to light, we are led to believe that there is a fundamental difference between the two main classes—the condensed globular objects and the open clusters.

Among others, Bailey,[1] Bohlin,[2] Hinks,[3] Perrine,[4] and more recently Melotte[5] have discussed the distribution of clusters with respect to the galaxy. The most conspicuous result is that the open clusters are almost exclusively in the Milky Way and symmetrically arranged with respect to it, while the globular clusters are widely distributed in galactic latitude and quite limited in galactic longitude. The globular clusters are mostly in one hemisphere, the pole of which, according to Melotte, is at galactic latitude −9°, galactic longitude 296°. It should be noted, moreover, that no other class of celestial objects shows so marked a dissymmetrical arrangement with respect to the galactic plane.[6]

Bailey's "Catalogue of Bright Clusters and Nebulae"[7] includes 66 globular clusters, and to this number Hinks[8] adds 41 from Dreyer's catalogues.[9] Several of these additions are doubtful because of the possibility of mistaking spiral nebulae for faint globular clusters, and also of confusing open and globular clusters when the scale is small. Bohlin[10] considered about 75 objects truly globular, while Miss Clerke states that approximately 500 clusters of all kinds are known,[1] of which possibly 120 have globular form.[2] This total obviously must include many chance aggregations of stars and a large number of very slight condensations in the Milky Way.[3]

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[1] "A Catalogue of Bright Clusters and Nebulae," *Harvard Annals*, **60**, 199, 1908.

[2] "On the Galactic System, with Regard to Its Structure, Origin, and Relations in Space," *Kungl. Svenska Vetenskapsakademiens Handlingar*, **43**, No. 10, 1909.

[3] *Monthly Notices*, **71**, 697, 1911. Hinks gives a bibliography of earlier studies of distribution, (*ibid.*, p. 588).

[4] *Lick Observatory Bulletins*, **5**, 102, 1909.

[5] *Journal of the British Astronomical Association*, **25**, 341, 1915. Melotte's catalogue of star clusters is to appear in the *Memoirs of the Royal Astronomical Society*, **60**, Part V.

[6] The concentration, noted by Hertzsprung and others, of the helium stars and stars of the fifth spectral type in the southern part of the galaxy is of a different order from that observed for the globular clusters. Moreover, these two types of stars are strongly condensed in the Milky Way for all longitudes (*Astronomische Nachrichten*, **192**, 265, 1912). The preliminary work by Hinks (*loc. cit.*, and *Monthly Notices*, **71**, 588, 1911) shows a possible asymmetry in the distribution of spiral nebulae and of the "somewhat extended" gaseous nebulae. The data are admittedly far from complete, but the later investigation by Hardcastle (*Monthly Notices*, **74**, 699, 1914), using the Franklin-Adams plates, confirms in general the earlier results.

[7] *Harvard Annals*, **60**, No. VIII, 1908.

[8] *Op. cit.*, p. 701.

[9] *Memoirs of the Royal Astronomical Society*, **49**, Part I, 1888; **51**, 185, 1895; **59**, 105, 1910.

[10] *Op. cit.*, Plate I.

There is, however, in all the data upon which these estimates rest a certain lack of homogeneity, and accordingly the very recent counts by Melotte[4] on the Franklin-Adams chart plates are of greater value than the earlier work. All clusters with diameters greater than 1′ and effective photographic magnitudes brighter than 16 or 17 are included, with the result that 83 globular and 162 open clusters are listed. This enumeration can be improved only through the study of fainter objects or through the individual examination on a larger scale of some of the doubtful clusters.[5] Then, however, considerable modification may be expected. For instance, the cluster Messier 11, classed by Bailey as globular,[6] is distinctly open and is limited in the number of its stars; while Messier 14, chosen by Melotte as the type for the open cluster of regular outline, appears definitely globular on Mount Wilson plates.

Since the time of the Herschels very few globular clusters have been discovered, notwithstanding the considerable increase in telescopic power and the great increase in the known number of stars and nebulae. This leads Bailey to suggest that the limit of the region of globular clusters has been reached.[7] Closer examination of small faint nebulae and of nebulous stars, however, may add to the number. Pease finds, for example, that the supposed variable nebula N.G.C. 6760 is a faint cluster containing more than a thousand stars.[1]

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[1] *System of the Stars*, p. 227, London, 1905.

[2] *Problems in Astrophysics*, p. 428, London, 1903.

[3] Bailey puts the total number of clusters at nearly seven hundred (*Publications of the Astronomical and Astrophysical Society of America*, **1**, 269, 1910). In his most recent paper the number of clusters definitely globular is placed at 76 (*Harvard Annals*, **76**, 43, 1915).

[4] *Loc. cit.*

[5] A start in this direction has been made by Knox Shaw, *Helwan Observatory Bulletins*, No. 9, 1912; No. 15, 1915. See also *Monthly Notices*, **76**, 105, 1915.

[6] *Harvard Annals*, **60**, 214, 1908. "Perhaps a globular cluster. Somewhat irregular" (*Harvard Annals*, **76**, 50, 1915).

[7] *Publications of the American Astronomical Society*, "Report of the Seventeenth Meeting," p. 8, 1914.

II. PREVIOUS OBSERVATIONAL AND THEORETICAL STUDIES

A. *Introduction.*—Practically all the detailed observational work on individual clusters, with the exception of the investigations of cluster variables, relates to counts of the stars and to determinations of positions. No serious attempts have been made to determine accurate magnitudes,[2] chiefly because of the lack of dependable magnitude scales for the fainter stars.

To a certain extent the theoretical work based on star-counts has been premature and meaningless; and, while the cataloguing of positions has been of high value, the attempt to obtain relative proper motions has proved largely in vain. There are, moreover, rather evident reasons why these two negative results should be the outcome:

1. The counts and subsequent theoretical discussions of star-density have always dealt with the globular clusters. The distribution in open clusters, where the chances of dealing with all the members of the system are better, has received only cursory attention, and then mostly from the standpoint of reading spiral forms or radial structure into arrangements which more safely and naturally can be interpreted as chance distributions.

The over-interpretation of superficial arrangement has been a still greater burden to the globular clusters. In fact, the law of probabilities is generally ignored in such considerations, for as soon as the slightest departure from circular symmetry is noted among the thousand or two brighter stars of a globular cluster, definite and highly significant "lines of cleavage" and "lanes of nebulosity," or "spiral paths" and "channels of force" are observed. The significance of chance groupings or chance vacancies decreases remarkably with increasing exposure time. The spiral structure or asymmetrical arrangements may be present, but to conclude definitely that such is the case is impossible when we deal with a very small percentage of the total number of stars.

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[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, 13, 258, 1914.

[2] Exceptions to this statement include the studies by Schwarzschild and others on certain bright open groups (Praesepe, Pleiades, etc.), which were made mainly to investigate magnitude scales.

In this use of the brighter objects as representative of the whole of the cluster lies the prematurity and weakness of much of the theoretical work on the laws of distribution; for it has rarely been noted that such work applies only to the relatively few stars whose absolute brightness may be more than one hundred times that of our sun. We could hope to obtain only very incomplete and unsatisfactory results if our studies of the galactic system were so limited. Prior to the work by Pease[1] on the distribution in clusters, using plates with exposures varying from one minute to several hours, few of the counts involved more than two or three thousand stars. If stellar density had been investigated from the standpoint of magnitudes and colors (spectra) as well as numbers, we could place more confidence in the results of the customary comparison of globular clusters with gases in equilibrium. A much more serious objection to the current methods of discussing star-densities will be revealed in the following study of colors in Messier 13.

2. Admitting that there are many thousands of stars in a globular cluster, and assigning them any known type of spectrum and dimensions reasonably comparable with those of other known stellar bodies, we must conclude from the magnitudes and angular separations that the cluster systems are very distant. Proper motions determined within an interval of a few years would accordingly correspond to enormously great real velocities, and it is not surprising, therefore, that practically no relative motion has yet been found in globular clusters. The matter will be discussed numerically in a subsequent paper.[2]

B. *Variables, distribution, cluster catalogues.*—The more important investigations of clusters can be very briefly summarized.

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[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, **12**, 213, 1913; **13**, 258, 1914.

[2] The second paper of this series, Section XIV. Moulton has also considered this matter numerically (*Publications of the Astronomical and Astrophysical Society of America*, **1**, 329, 1909).

The most valuable contributions have been made by Bailey[1] and von Zeipel.[2] The former has accumulated vast amounts of important information relative to cluster variables (practically all we know of these objects, in fact), as well as valuable data on classification, distribution of stars in clusters, and of clusters in the sky. Von Zeipel, in the study of Messier 3, has published the largest of all cluster catalogues, and has made investigations of considerable importance on the distribution of the brighter stars in several clusters, and on the possible physical relationships of the individual stars. E. C. Pickering, H. C. Plummer, See, Eddington, and Strömgren and Drachmann have contributed to the theoretical interpretation of the arrangement of stars in clusters.[3] Further reference to their results is reserved for a later paper, where observed distributions and their interpretation will be considered in connection with magnitudes and colors.

Photographic catalogues of the positions of individual stars now exceed both in number and accuracy the lists based on visual observations. The most exhaustive determination of positions, however, is probably represented by the unpublished visual measures made by Barnard with the 40-inch refractor of the Yerkes Observatory. In the seventeen years during which his micrometrical work has continued, the only definite and appreciable motion found is for two stars in Messier 92, and of course it is not certain that the moving objects are actual members of the cluster.[4]

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[1] "Variable stars in the Cluster ω Centauri," *Harvard Annals*, **38**, 1–252, 1902; "A Catalogue of Bright Clusters and Nebulae," *ibid.*, **60**, No. 8, 1908; "Variable Stars in the Cluster Messier 3," *ibid.*, **78**, Part 1, 1–98, 1913; "Globular Clusters," *ibid.*, **76**, No. 4, 1915. For other articles by Bailey, refer to bibliography given in the last-mentioned paper.

[2] "La théorie des gas et les amas globulaires," *Comptes Rendus*, **144**, 361, 1907; "Catalogue de 1571 étoiles contenues dans l'amas globulaire Messier 3 (N.G.C. 5272)," *Annales de l'Observatoire de Paris*, **25**, 1–101, 1908; "Recherches sur la constitution des amas globulaires," *Kungl. Svenska Vetenskapsakademiens Handlingar*, **51**, No. 5, 1913.

[3] A fairly complete bibliography of the papers dealing with this phase of the cluster problem is given by Strömgren and Drachmann, *Publikationer og mindre Meddelelser fra Københavns Observatorium*, No. 16, 1914. See also *Harvard Annals*, **76**, No. 4, p. 77, 1915, and *Monthly Notices*, **76**, 107, 1915.

[4] *Astronomische Nachrichten*, **176**, 17, 1907; *ibid.*, p. 21; *ibid.*, **182**, 305, 1909; *Astrophysical Journal*, **40**, 173, 1914.

The more important catalogues (excluding the various lists of the Pleiades, of stars in Orion, and of similar configurations) are listed at the end of this section. The bright double cluster *h* and χ Persei has been measured many times both visually and photographically; bibliographies are given by Schur[1] and Van Maanen;[2] more recent catalogues have been published by Messow[3] and Helffrich.[4]

C. *Colors and spectra.*—The investigation of the spectra and colors in clusters has not advanced very far. The spectra of the brighter stars in a number of the brighter, very open clusters were classified by Mrs. Fleming.[5] Several different types were found in each, some showing a preference for blue stars, others for yellow and red. Of the spectra of individual stars in the fainter open clusters[6] and in globular clusters, nothing is known aside from the work of Pease and Adams,[7] who have classified provisionally 40 spectra in Messier 13. All spectral classes from A to K were recorded, with an average of type F, agreeing with the work of Fath[8] and of Slipher[9] on integrated spectra of various globular clusters.

It has been frequently suggested that certain clusters are relatively yellow, others white. But the first definite observation of color was made qualitatively by Barnard,[10] who compared photographic magnitudes with visual and photo-visual estimates for a number of stars in the Hercules cluster. Hertzsprung,[1] comparing the relative intensity between the limits λ 3761 and λ 4555 in photographic spectral images of Messier 3 as a whole, and of neighboring stars, concludes that the integrated color of the cluster corresponds closely to spectral type F. Recently Hertzsprung[2] has published effective wave-lengths for 184 stars in the very open cluster N.G.C. 1647, and for 57 of these stars Seares[3] has derived magnitudes and color-indices.

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[1] *Astronomische Mitteilungen von der Königlichen Sternwarte zu Göttingen*, No. 6, 1900.

[2] *Dissertation*, Utrecht, 1911; *Recherches Astronomiques de l'Observatoire d'Utrecht*, No. 5, 1911.

[3] *Astronomische Abhandlungen der Hamburger Sternwarte in Bergedorf*, **2**, No. 2, 1913.

[4] *Veröffentlichungen der Grossherzoglichen Sternwarte zu Heidelberg*, **7**, No. 2, 1913.

[5] *Harvard Annals*, **26**, 260, 1891.

[6] Adams and Van Maanen have published spectra of some of the brighter stars in the double cluster in Perseus (*Astronomical Journal*, **27**, 187, 1913).

[7] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, **12**, 219, 1913; **13**, 268, 1914.

[8] *Mt. Wilson Contr.*, No. 67; *Astrophysical Journal*, **37**, 198, 1913. Earlier papers by Fath are cited in this article.

[9] *Bulletins of the Lowell Observatory*, **2**, 59, 1914.

[10] *Astrophysical Journal*, **12**, 176, 1900; **29**, 72, 1909; **40**, 173, 1914.

VISUAL CATALOGUES OF STARS IN CLUSTERS

1. Comas Solá. Messier 8, *Astronomische Nachrichten*, **148**, 97, 1898. 15 stars.
2. Hahn. Σ 762, *Abhandlungen der mathematisch-physischen Klasse der Königlich-Sächsischen Gesellschaft der Wissenschaften*, **17**, 151, 1891. 60 stars.
3. Helmert. Messier 11, *Publicationen der Hamburger Sternwarte*, No. 1, 1874. 200 stars.
4. Kruse. N.G.C. 6633, *Veröffentlichungen der Grossherzoglichen Sternwarte zu Heidelberg*, **7**, No. 3, 1913. 71 stars.
5. Lamont. Messier 11, *Annalen der Königlichen Sternwarte bei München*, **17**. 184 stars.
6. Nangle. κ Crucis, *Journal of the British Astronomical Association*, **18**, 384, 1908; **19**, 27, 141, 1908. 48 stars.
7. Peter. N.G.C. 2264, *Abhandlungen der mathematisch-physischen Klasse der Königlich-Sächsischen Gesellschaft der Wissenschaften*, **15**, No. 1, 1889. 44 stars.
8. Peter. N.G.C. 6738, *ibid.* 28 stars.
9. Russell. κ Crucis, *Monthly Notices*, **33**, 66, 1872. 130 stars.
10. Schultz. N.G.C. 6885, *Kungl. Svenska Vetenskapsakademiens Handlingar*, **21**, No. 3, 1873. 104 stars.
11. Schultz. Messier 92, *Bihang till Kungl. Svenska Vetenskapsakademiens Handlingar*, **12**, *Afd.*, No. 2, 1886.
12. Valentiner. N.G.C. 6633, *Astronomische Beobachtungen auf der Grossherzoglichen Sternwarte zu Mannheim, III. Abteilung.* 71 stars.

PHOTOGRAPHIC CATALOGUES OF STARS IN CLUSTERS

1. Bailey, ω Centauri, *Harvard Annals*, **38**, 1–251, 1902. Variable stars. This volume also contains the positions of variables and of reference stars for sixteen other globular clusters.
2. Bailey. Messier 3, *Harvard Annals*, **78**, Part I, 1913. Variable stars.
3. Bellamy. Index No. 4996, *Monthly Notices*, **64**, 662, 1904. 103 stars.

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[1] *Astrophysical Journal*, **41**, 10, 1915.

[2] *Mt. Wilson Contr.*, No. 100; *Astrophysical Journal*, **42**, 92, 1915.

[3] *Ibid.*, No. 102; **42**, 120, 1915.

4. Bohlin. Messier 92, *Astronomiska Iakttagelser och Undersökningar å Stockholms Observatorium*, **8**, No. 3, 1907; *Astronomische Nachrichten*, **174**, 203, 1907. 348 stars.
5. Donner and Backlund. N.G.C. 6885 = 20 Vulpeculae, *Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg*, Série 2, **5**, No. 2. 140 stars.
6. Fagerholm. Messier 67, Dissertation, Upsala, 1906. 295 stars.
7. Fagerholm. Messier 103, *Arkiv för Matematik, Astronomi och Fysik*, **5**, No. 14, 1909. 283 stars.
8. Fenet. Messier 11, *Bulletin de la Société Astronomique de France*, **9**, 85, 1895. Chart of 395 stars without catalogue.
9. Giebeler. Messier 37, *Veröffentlichungen der Königlichen Sternwarte zu Bonn*, No. 12, 1914. 1231 stars.
10. Gould. *Photographic Observations of Star Clusters*, 1897. 9144 stars in 37 open clusters, chiefly in the Southern Hemisphere.
11. Ludendorff. Messier 13, *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **15**, No. 6, 1905. 1136 stars.
12. Meyer. N.G.C. 6802, Dissertation, Breslau, 1902; *Astronomische Nachrichten*, **167**, 321, 1905. 59 stars.
13. Nijland. N.G.C. 6633, Dissertation, Utrecht, 1897. 28 stars (also visually measured).
14. Nordlund. Messier 37, *Arkiv för Matematik, Astronomi och Fysik*, **5**, No. 17, 1909. 840 stars.
15. Olsson. Messier 67. *Astronomiska Iakttagelser och Undersökningar anstälda på Stockholms Observatorium*, **6**, No. 4, 1898. 148 stars.
16. Plummer, H. C. Messier 13, *Monthly Notices*, **65**, 79, 1904. 70 stars.
17. Plummer, W. E. Messier 13, *Monthly Notices*, **65**, 801, 1905. 54 stars.
18. Pingsdorf. Messier 52, Dissertation, Bonn, 1909. 132 stars.
19. Pummerer. N.G.C. 663, *Publikationen der von Kuffner'schen Sternwarte*, **6**, No. 7, 1913. 328 stars.
20. Scheiner. Messier 13, *Abhandlungen der Königlich-Preussischen Akademie der Wissenschaften zu Berlin*, Anhang, 1892. 833 stars.
21. Stratonoff. Messier 11. *Publications de l'Observatoire Astronomique et Physique de Tachkent*, No. 1, 1899. 861 stars.
22. Wolf. N.G.C. 6633, *Astronomische Nachrichten*, **126**, 297, 1890. 73 stars.
23. Von Zeipel. Messier 3, *Annales de l'Observatoire de Paris*, **25**, 1, 1908. 1571 stars.
24. Zurhellen. Messier 46, *Veröffentlichungen der Königlichen Sternwarte zu Bonn*, No. 11, 1909. 527 stars.

III. CORRELATION BETWEEN DIAMETERS AND BRIGHTNESS IN CLUSTERS

From Bailey's "Catalogue of Bright Clusters and Nebulae," which may be considered homogeneous over the field it covers, we obtain the data in Table I relative to mean diameters of globular clusters (outside the Magellanic Clouds) and the magnitudes of the brightest stars in each.

TABLE I

Brightest Magnitude	Number of Clusters	Mean Diameter
9	1	20′
10	0	....
11	1	18
12	7	9.4
13	25	10.0
14	5	4.4
15	3	3.3
16	1	3

The tabulation is illustrated in Fig. 1, where the full line refers to the globular clusters. The broken line refers to the 37 open clusters (outside the Magellanic Clouds) designated by Bailey as "fairly condensed."

A. The striking result that magnitude increases with decreasing apparent diameter is to be expected if distance is correlated with apparent size. It suggests at once the possibility of deriving relative mean parallaxes for clusters from measures either of diameters or of magnitudes.

Assuming real dimensions and absolute magnitudes approximately equal for all globular clusters, a decrease of 1.5 apparent magnitudes would correspond to doubling the distance or halving the apparent diameter. But the observed change in diameter (Fig. 1) is less than would be expected on this basis. Assuming that our data will bear close interpretation, we might conclude that this discrepancy indicates: (1) that the more distant clusters are larger than the nearer ones; or, (2) that the stars of the more distant clusters are absolutely fainter than those of the nearer ones; or, (3) that the light of the stars of the more distant clusters is diminished by space absorption; or, since Bailey's magnitude estimates are photographic, (4) that the brighter stars of the more distant clusters are intrinsically redder than those of the nearer clusters; or, (5) some combination of the foregoing. The third or fifth interpretation of this excess of diameter relative to brightness over what would be expected for equal absolute dimensions and magnitudes would probably be the most acceptable.

B. If, on the other hand, the observed change in diameter is not connected with distance, we are left the rather improbable alternative interpretation that the smaller and more condensed globular clusters (which contain no fewer stars so far as present indications go than those apparently large) are formed of stars of relatively small intrinsic brightness.[1]

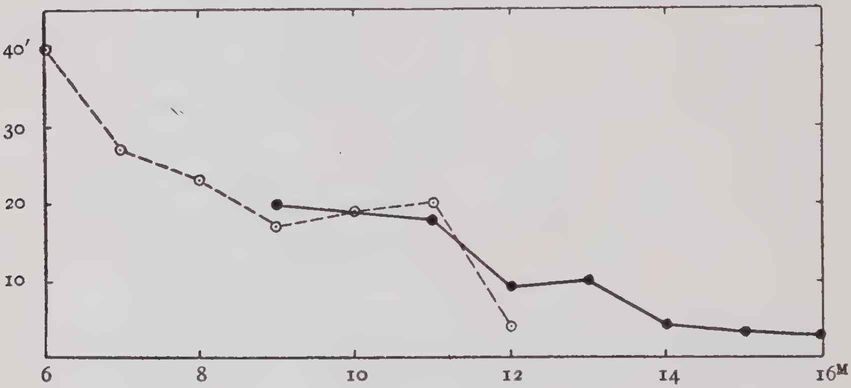

FIG. 1.—Diameters and magnitudes of clusters
Abscissae: Magnitudes of brightest stars
Ordinates: Apparent diameters of clusters

C. For the reasons cited below we are not fully justified in this close interpretation, and for the present the preceding remarks must be held only as tentative explanations of a phenomenon not definitely proved to exist:

1. Only a few more than half of the known globular clusters could be included in Table I, and most of those not included are outside the Magellanic Clouds.

[1] But suppose that the dimensions of a cluster diminish with age. Then we need only make the assumption, at present allowable, that the individual stars also diminish in size and light with time, and we have, if the ages of clusters differ, the phenomenon as observed, and need not appeal to distance. In a later paper a method will be discussed and applied by means of which this hypothesis can be tested. In this connection it is of obvious importance to consider theoretically and, as far as possible, observationally the historical development of a cluster with respect to its dimensions.

2. The number of clusters involved in the extreme ends of the curve is small. One-half the total number have diameters differing by less than 4′ from the mean of all.

3. The magnitudes make no claim to accuracy, being merely estimates which may not maintain, uniformly over the whole sky, their value as relative magnitudes.

If the fainter stars are estimated relatively too faint,[1] the discrepancy noted above under A may be much diminished, or even vanish, in which case magnitude or diameter or both together can be effectively used to estimate mean relative distance.

The relation here discussed is not apparent for the globular or open clusters in the Magellanic Clouds, but the available material is of small amount. There seems also to be in the clusters at large no marked correlation between estimated number of stars and apparent diameter, nor between number of stars and brightest or mean magnitude. It is hoped that further information concerning these relations can be derived in the course of time from the Mount Wilson photographs.

IV. THE PURPOSE OF THE PRESENT STUDY OF CLUSTERS

The object in taking up the investigation of the magnitudes and colors in stellar clusters is twofold. First, it is hoped that considerable advance can be made in our understanding of the internal arrangement and physical characteristics of these objects. Secondly, and probably of more importance, it is hoped that through the acquisition of a clearer appreciation of stellar clusters, particularly of globular systems, some contribution can be made to the knowledge of our own galactic system. For it is quite obvious that a globular cluster, whether considered as entirely extraneous to our galactic domain or not, is in itself a stellar system on a great scale—a stellar unit which without doubt must be comparable to our own galactic system in many ways, though possibly differing from it fundamentally in others.

A. In the study of clusters themselves the results which may be derived only from a knowledge of magnitudes and colors are in part as follows: (1) laws of distribution or density for different magnitudes and colors; (2) relation connecting absolute magnitude and color, and its variation from cluster to cluster; (3) total number of stars in the faint globular clusters predicted from observed luminosity-curves; (4) dependence of stellar variability on color and magnitude; (5) change of color with light-variation, and the consequent opportunity of investigating on a large scale the causes underlying Cepheid variation.

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[1] In a private letter Professor Bailey suggests that this may be the case, at least for Messier 3.

B. In the study of the galactic system the advantages of an investigation of cluster magnitudes and colors include the following: (1) elimination of the factor of distance-selection. (In the individual globular cluster differences in distance are negligible. Hence, for the study of the frequency of spectral types and of absolute magnitudes in a stellar system, and for the study of relationships between spectrum and luminosity, the most troublesome unknown, the relative distance of the stars under consideration, is eliminated. Relative apparent magnitudes are, with but little error, also relative absolute magnitudes); (2) the possibility of dealing with complete stellar systems—unfortunately not literally true for globular clusters, because of the great number and extreme faintness of the stars; (3) the availability of large numbers for statistical discussions; (4) the possible evaluation of absorption of light in space and of color-factors depending on absolute magnitude.

C. In addition to those suggested above the following general astronomical problems are closely connected with the present study of clusters: (1) frequency of absolute magnitudes and spectra in stellar systems—luminosity-curves; (2) the order of stellar evolution; that is, the probable character of the progression of spectral type (color) with age; (3) the distances of clusters; (4) relation of the globular cluster to the open cluster and to the galaxy.

V. REMARKS ON THE METHODS AND PROGRESS OF MAGNITUDE DETERMINATIONS

The derivation of photographic and photo-visual magnitudes, and hence also of the colors of stars in clusters, has been made possible through two important factors. The first is the 60-inch reflector of the Mount Wilson Observatory; the second and more necessary one is the fundamental work by Seares in establishing photographic and photo-visual magnitude scales for the faint stars.

The 60-inch reflector allows the speed necessary for successful intercomparison of the fields of faint stars, and at the same time the scale, even at the principal focus, permits the study of the stars at the very center of many of the globular clusters. The magnitudes determined with the reflector from focal images make no claim to high accuracy. And, indeed, for the present at least, high accuracy is not at all requisite. It is sufficient if we obtain color-indices with a probable error of a tenth of a magnitude. The crucial point is not the size of the accidental errors nor the precision of the brightness for any given cluster star, but rather the unimportance of the systematic errors and the availability of dependable magnitude scales over the entire interval of brightness investigated.

In the succeeding papers much weight must be put on the reliability of the Mount Wilson magnitude scales. Frequently relations between magnitude and other properties will be derived, which would be meaningless if serious systematic error existed in the comparison magnitudes or in the method of work. The principles underlying the determination of the magnitude standards, and the derivation and discussion of the results have been fully described by Seares in recent papers.[1] The methods of photographic photometry with the 60-inch reflector have also been discussed in detail, so that little need be added. A matter concerning the comparison of the Mount Wilson and Harvard magnitude standards may be taken up briefly, however, because of its importance in interpreting some of the results of later papers.

Beyond the twelfth magnitude no scale of standard visual or photo-visual magnitudes exists except the Mount Wilson sequence, which extends to magnitude 17.5. At the twelfth magnitude the Mount Wilson photo-visual and Harvard visual scales are in agreement. As the extension of the photo-visual scale to fainter magnitudes closely parallels the photographic both in method and in accuracy, we are led to place much the same confidence in the two scales, even though we cannot check the photo-visual results with other determinations.[1]

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[1] *Mt. Wilson Contr.*, Nos. 70, 80, 97, 98; *Astrophysical Journal*, **38**, 241, 1913; **39**, 307, 1914; **41**, 206, 259, 1915.

For the photographic scale, however, comparison can be made directly with the Harvard Polar Sequence. The considerable outstanding differences in the final results have been discussed critically by Seares in a recent paper.[2] Fig. 2 illustrates the deviation of the two scales between the tenth and twentieth magnitudes. The abscissae are Mount Wilson photographic magnitudes. The full black line represents the difference Mount Wilson *minus* Harvard Polar Sequence (*Harvard Annals*, 71, Part III, 1914). The broken line indicates the difference between Mount Wilson and the Har-

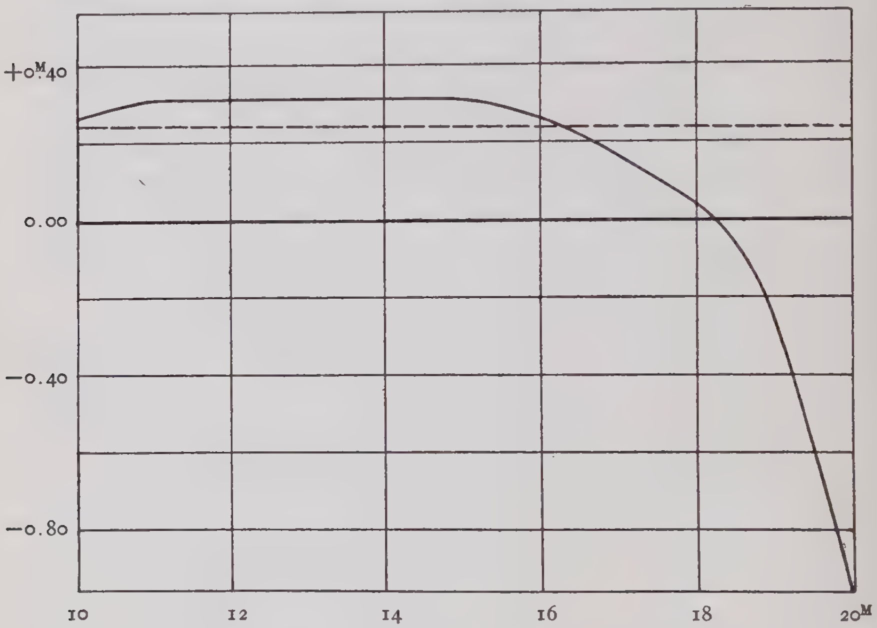

FIG. 2.—Comparison of Mount Wilson and Harvard photographic scales
Abscissae: Mount Wilson photographic magnitudes
Ordinates: Scale differences, Mount Wilson *minus* Harvard

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[1] Provisional photo-visual magnitudes for a few stars are given by Miss Leavitt in *Harvard Annals*, 71, 142, 1914.

[2] *Mt. Wilson Contr.*, No. 98; *Astrophysical Journal*, 41, 259, 1915.

vard magnitudes as corrected by Seares for (1) residual color-errors depending upon the various instruments used; (2) reduction from the Harvard color-system to that of the reflector; (3) distance of stars from the center of the plate; (4) misapplication of a correction depending on the order of exposure.

It is evident that the adoption of Harvard photographic magnitudes, at least as far as magnitude 18, would give smaller values and hence smaller color-indices. It will be shown in the next paper, however, that diminishing all the observed color-indices by even a quarter of a magnitude would lead to an incredible result.[1]

The few details in which the determination of cluster magnitudes differs from the ordinary methods will be mentioned in the discussion of the individual clusters. Because of the great number of images on some of the plates, special precautions must be taken against systematic errors of measurement. Practically all of the plates, except those made for the study of variables, have one exposure on the field of the cluster preceded and followed by one of the same duration on the field of the Pole. The plates are made as far as possible only under the best observing conditions, and at the time when the altitudes of the cluster and Pole are comparable. Exposures of different duration are made in order to vary the

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[1] King's discussion of the Polar Standards (*Harvard Annals*, **76**, No. 10) was received some months after the above discussion was written. The bearing of his results on the present problem is indicated in the following note contributed by Mr. Seares: "The difference of 0.24 mag. between Mount Wilson and Harvard, the latter revised as indicated above by Mr. Shapley, has its origin in a divergence of scale for the brighter stars of the form

$$\text{Mt. W.}-\text{Harvard}=+0.061\ (M-6.0)$$

which holds to $M=10.0$. The re-reduction of some of the Harvard results connects the divergence with the stars brighter than the eighth magnitude, i. e., the constant difference of 0.24 mag. originates between magnitudes 6 and 8 and applies to all portions of the scale between the eighth and twentieth magnitudes.

"King has now redetermined the photographic scale for the brighter stars, using the 24-inch reflector and methods similar to those employed at Mount Wilson. The direct comparison of his magnitudes with the Mount Wilson scale (*Harvard Annals*, **76**, No. 10, p. 186) gives for the interval 2.7–9.3

$$\text{Mt. W.}-\text{King}=-0.004\ (M-6.0)$$

"The adoption of King's results would therefore remove the outstanding unexplained difference of a quarter of a magnitude between the Mount Wilson and Harvard scales."

measuring conditions near the center of the cluster. For extremely faint stars it is not advisable to make direct comparisons with the polar standards, but rather to extend the scales downward by means of diaphragm plates or by long and short exposures.

Thus far more than a hundred photographs, nearly all made at the 25-foot focus on 4×5 plates, have been secured. Many of these, however, are single exposures on clusters known to contain many variable stars. For photographic magnitudes Seed 27 plates are used, and for photo-visual, Cramer Instantaneous Iso with a yellow filter placed in contact with the plate. The measurement and reduction proceed slowly, partly because of the large number of stars, but chiefly owing to the difficulty of identification and measurement in the very condensed clusters.

The possibility of the following systematic errors is kept in mind, and efforts are made to avoid or minimize them by varying the methods of observing, measuring, and making the reductions: (1) uncertainty of atmospheric extinction—dependence on azimuth as well as altitude; (2) errors connected with tilt, depending on orientation of plate-carrier or tube extension; (3) variation of the correction for distance from the center of plate; (4) effects of non-symmetrical field arising from astigmatism of the mirror; (5) influence of neighboring stars in measures of dense fields; (6) influence of hazy background of stars on measures near the center of globular clusters; (7) increase with magnitude of errors depending on fatigue; (8) miscellaneous prejudices.

VI. HYPOTHETICAL SPECTRA

Since the color of a star is intimately related to its spectrum, it is convenient for the discussions of color-indices to establish color-classes analogous to the spectral classes of the Draper catalogue.[1] Corresponding to the spectra B, A, F, G, K, M, we may use the small letters *b*, *a*, *f*, *g*, *k*, *m*, to designate classes of color, defining the latter so that, under certain average conditions, stars of spectral type B have the color *b*, A corresponds to *a*, etc. We may also appropriately refer to color-classes as hypothetical spectra.

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[1] Seares, *Proceedings of the National Academy of Sciences*, 1, 481, 1915.

The relation between color-index and spectrum depends upon the instrumental equipment, and definitive determination for the 60-inch reflector has not been made, but provisionally we may adopt

$$\text{Pg. Mag.} - \text{Pv. Mag.} = \text{Color-Index} = 0.4 \times \text{Spectrum}$$

where the color-index is expressed in magnitudes and the spectrum has the values -1, 0, $+1$, $+2$, etc., corresponding to B0, A0, F0, G0, etc.

The foregoing relation holds only for the mean of a large number of stars; it represents, in fact, the average change of color with spectrum for those stars (mostly near and relatively bright) upon which it is based. For any individual star we must substitute *hypothetical spectrum* for *spectrum*, observing that these may differ through (1) scattering of light in space and (2) changes in spectral intensity dependent on absolute magnitude.

The quantity derived from magnitudes is the hypothetical spectrum. This may be compared with the directly observed spectrum and the difference, defined as the color-excess, is a measure of the effect on the color of the luminosity and absorption factors.

Table II defines the adopted limits of color-index for the corresponding color-class or hypothetical spectrum.

TABLE II

Color-Index	Color-Class or Hypothetical Spectrum	Color-Index	Color-Class or Hypothetical Spectrum
$-0^{\mathrm{M}}.40$ to $-0^{\mathrm{M}}.37$.....	*b*0	$0^{\mathrm{M}}.00$ to $+0^{\mathrm{M}}.39$.....	*a*0–9
-0.36 " -0.33.....	*b*1	$+0.40$ " $+0.79$.....	*f*0–9
. .	.	. .	.
. .	.	. .	.
. .	.	. .	.
. .	.	. .	.
-0.04 to -0.01.....	*b*9	$+1.60$ to $+1.99$.....	*m*0–9

VII. DIFFICULTIES AND LIMITATIONS IN THE STUDY OF CLUSTERS

There are a number of more or less serious obstacles in the way of making the cluster investigation as ideally exhaustive and expeditious as could be wished. The most obvious limitations are as follows:

1. The extreme faintness of many of the globular clusters. With our present equipment of telescopic power and photographic plates it will not be practicable to carry the determination of star colors below photo-visual magnitude 17.5, and this limit can be reached only with serious difficulty. For the present, then, it will be impossible in the truly globular clusters to cover a range of more than five or six magnitudes. In the galactic system there are stars known to differ in absolute brightness by more than 15 magnitudes.

2. The probable presence, in photographs of the centers of condensed clusters, of the Eberhard effect,[1] or similar photographic obstacles imperfectly understood and controlled with difficulty.

3. The southern declination of most of the globular clusters, including the brightest and largest, and also the similarly unfavorable location of the Magellanic Clouds.

4. The indefinite nature of the correction for background and foreground stars, particularly near the Milky Way.

5. The uncertainty of atmospheric extinction, and of absorption of light in space. The magnitudes in the cluster must be compared with those of the Pole or some other standard region. This involves possible troubles with atmospheric absorption, which, on Mount Wilson at least, may depend on azimuth as well as altitude.

6. The absence of data relative to motions in clusters.

7. The apparently distinct differences between the physical characteristics of the globular clusters and the galactic system.

Summarizing briefly, we may remark that the preceding pages are preliminary to a general investigation of the magnitudes and colors of stars in clusters. Previous investigations have dealt chiefly with position catalogues, proper motions, variable stars, and considerations of star-density in individual clusters. By determining the magnitudes and colors it should be possible to add largely to our knowledge of the physical nature of such systems and to contribute toward the solution of various general stellar problems. The subjects for study will include both globular clusters, of which there are perhaps eighty quite asymmetrically situated with respect to the galaxy, and open clusters, which are located almost entirely in the Milky Way. An interesting fact not previously emphasized is the relation of brightness to diameter of clusters, illustrated in Fig. 1. The globular clusters contain enormous numbers of stars—perhaps hundreds of thousands or millions—while the open clusters range from fairly condensed symmetrical groups to very open irregular aggregations which are apparently only slight galactic condensations.

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[1] *Physikalische Zeitschrift*, **13**, 288, 1912.

Mount Wilson Solar Observatory
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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

Second Paper: Thirteen Hundred Stars in the Hercules Cluster (Messier 13)

By HARLOW SHAPLEY

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

SECOND PAPER: THIRTEEN HUNDRED STARS IN THE HERCULES CLUSTER (MESSIER 13)

By HARLOW SHAPLEY

I. PRELIMINARY REMARKS ON THE HERCULES CLUSTER[1]

The preceding paper[2] contains a brief general summary of previous investigations of stellar clusters and an outline of the object, methods, advantages, and limitations of the proposed determination of magnitudes and colors at the Mount Wilson Observatory. The present communication treats of the results for the Hercules cluster. It is of necessity long and detailed, but in succeeding papers the descriptions of data and methods can be much abbreviated and perhaps the observational material itself decreased. Sections II–VI deal chiefly with the observations, the last ten sections with discussions of the data. This initial section contains a synopsis of the main results of previous work on the Hercules cluster.

The best known of all the globular clusters was considered first for several reasons: (1) its position in the sky when observations were begun was favorable for comparison with the Pole; (2) two good catalogues of the positions of the brighter stars were available; (3) only two variable stars had been recorded among the thousand brightest objects,[3] an obvious advantage in deriving colors from non-simultaneous determinations of photographic and photo-visual magnitudes; (4) spectra of about forty of the individual stars were known approximately from direct classification.

No other globular cluster has been studied so frequently and extensively as Messier 13. It was one of the earliest placed on record,[1] being discovered accidentally by Edmund Halley 201 years ago.[2] He observed that "it is but a little patch . . . . and similar to the lucid spots around Theta Orionis, in Andromeda, and in the Centaur . . . . most of them but of few minutes in diameter; yet since they are among the fixed stars . . . . they cannot fail to occupy spaces immensely great, and perhaps not less than our whole solar system." Messier saw the Hercules cluster as a brilliant circular nebula, but the elder Herschel resolved it partially in 1783,[3] and later completely with the 20-foot telescope. An account of the earlier studies by Messier, D'Arrest, and others is given by Scheiner in his monograph "Der Grosse Sternhaufen im Herkules Messier 13."[4]

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[1] Messier 13 = G.C. 4230 = N.G.C. 6205; $\alpha = 16^{h}\,38^{m}.1$, $\delta = +36^{\circ}\,39'$, 1900; galactic latitude = +40°, galactic longitude = 26°.

[2] *Mt. Wilson Contr.*, No. 115.

[3] Discovered by Bailey, *Harvard Annals*, **38**, 2, 1902; Barnard independently discovered one of these variables, *Astrophysical Journal*, **12**, 182, 1900.

Originally in the drawings and verbal descriptions, and later in the early photographs, much attention was paid to the dissymmetrical arrangement of the brighter stars in the cluster and to the possible existence of visible nebulosity. Contributions to these subjects have been made by Secchi,[5] the Earl of Rosse,[6] Trouvelot[7] working at Harvard with Winlock, and Harrington[8] assisted at Ann Arbor by a professional artist. Holden[9] reported 13 different intersections of distinct lanes, Schaeberle[10] interpreted the arrangement as forming a spiral with one arm clockwise, the other counter-clockwise, while Scheiner[1] and Barnard[2] have voiced conservative views relative to the structure and nebulosity, respectively. Palmer,[3] Miss Clerke,[4] Perrine,[5] and Ludendorff[6] have also discussed different phases of these matters. The conclusion from all the former studies, which is strengthened by the results from plates made at Mount Wilson, is that no nebulous material can be detected in the Hercules cluster (at least unless it is fainter than the twentieth magnitude), and that the so-called "channels" among the brighter stars, while apparent in some form to the eye and on photographic plates of the proper exposure and contrast, do not appear among the fainter magnitudes and are probably to be considered of no particular importance in the structure of the cluster.

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[1] Messier 22 was discovered by Ihle in 1665; ω Centauri was noted as a lucid spot in the sky by Halley in 1677, though it was previously known to Bayer as a hazy star, and even to Ptolemy as the star in the Cloud on the Horse's Back; in 1681 Kirch discovered Messier 11 (not globular, but rich and considerably condensed) and Messier 5 in 1702. All of these earlier discoveries consisted in noting that the objects were nebulous spots rather than single stars. The resolution into clusters awaited in most cases the telescopes of the Herschels.

[2] *Philosophical Transactions* (abridged), **4**, 226, 1700–1720.

[3] *Ibid.* for 1818, p. 439 (no volume number).

[4] *Abhandlungen zu der Königl. Preuss. Akademie der Wissenschaften zu Berlin*, Anhang, 1892.

[5] *Memorie dell' Osservatorio del Collegio Romano*, 1852–1855.

[6] *Philosophical Transactions* for 1861, p. 732 (no volume number). The drawing, *ibid.*, Plate 28, was the beginning of the controversy as to the existence, nature, and significance of dark lanes.

[7] *Harvard Annals*, **8**, Part II, Plate 25, 1876.

[8] *Astronomical Journal*, **7**, 156, 1887.

[9] *Publications of the Astronomical Society of the Pacific*, **3**, 375, 1891.

[10] *Astronomical Journal*, **23**, 226, 1903.

A few of the earlier observers made rough counts of the stars in Messier 13, and Trouvelot[7] plotted without great accuracy the position of 171 stars. But the first attempt to record the positions of the individual stars with sufficient accuracy to give future observers a trustworthy basis for the study of proper motions was made by Scheiner in 1891–1892. His catalogue, cited above, contains 833 stars—all that were measurable on two plates with exposures of 2 hours and 1 hour, made with the 32.5 cm refractor of the Potsdam Observatory. A second catalogue of somewhat higher accuracy was made by Ludendorff,[8] and contains 1136 stars. It is based on two plates taken with the same instrument at Potsdam in 1900 and 1902, but with exposures of 2.5 and 2 hours, respectively. Comparison of the positions derived by Scheiner and Ludendorff show no definite proper motion during the interval of ten years. The star numbers and positions given in Ludendorff's catalogue are used in the present work.

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[1] *Op. cit.*, p. 31.

[2] *Monthly Notices*, **57**, 14, 1897; see also papers cited in Section II of the First Paper.

[3] *Astrophysical Journal*, **10**, 246, 1899.

[4] *Problems in Astrophysics*, chaps. xxvii–xxx, inclusive, London, 1903.

[5] *Astrophysical Journal*, **20**, 354, 1904. [6] *Loc. cit. postea.* [7] *Loc. cit.*

[8] "Der Grosse Sternhaufen im Herkules Messier 13," *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **15**, Part 6, pp. 1–56, 1905. See also, *Astronomische Nachrichten* **178**, 369, 1908.

The positions of 70 selected stars in the cluster have been measured on a plate made at Oxford and published by H. C. Plummer.[1] On a plate from the Yerkes Observatory, W. E. Plummer[2] has measured more than two thousand stars, but of this list only the 54 have been published which appear also in the lists by Scheiner and H. C. Plummer. No proper motions were found from this work (see Section II of the First Paper[3] and Section XIV below).

In none of the foregoing investigations has serious consideration been paid to the determination of magnitudes.

E. C. Pickering[4] determined the distribution of stars in Messier 13, and some valuable theoretical considerations of his results are contributed by H. C. Plummer.[5] A much more exhaustive consideration of the star-density is made by von Zeipel,[6] who used copies of plates made at the Yerkes Observatory and also, for the outer portions of the cluster, photographs made with the Upsala refractor. About 2500 stars were discussed, but only roughly from the standpoint of brightness. Of the many important results and conclusions derived by von Zeipel for this and other globular clusters only the following need be mentioned at this point:

1. From the ratio of the number of bright stars to faint stars in different regions it is necessary to conclude that the physical state near the center differs from that in the exterior regions.

2. So far as these brighter stars are concerned, the density in Messier 13 shows that the cluster is comparable to a spherical nebula (gaseous sphere) in convective equilibrium conforming to certain adiabatic laws.

3. It is entirely possible that the globular clusters have attained their adiabatic equilibrium solely by virtue of the mutual attraction of the constituent stars.

4. Each cluster is a gigantic system containing approximately a million stars.

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[1] *Monthly Notices*, **65**, 79, 1904.

[2] *Ibid.*, **65**, 801, 1905.

[3] *Mt. Wilson Contr.*, No. 115.

[4] *Harvard Annals*, **26**, 215, 1897.

[5] *Monthly Notices*, **71**, 460, 1911.

[6] "Recherches sur la constitution des amas globulaires," *Kungl. Svenska Vetenskapsakademiens Handlingar*, **51**, No. 5, pp. 1–51, 1913.

This last conclusion is admittedly quite speculative. It is based upon purely theoretical assumptions that now appear to be inadmissible; but it will be shown from other considerations, which have a good observational basis, that the conclusion itself may not be far wrong. Later it will be shown why No. 1 above is a sound result, other evidence being presented to show that conditions may exist in the center of the cluster which are not evident in the outer regions. This same evidence will suggest that near the center, at least, von Zeipel and others are dealing with a distribution which is not real, and that the theoretical results, including Nos. 2 and 3 above, must be modified accordingly.

The only papers dealing with magnitudes or colors, aside from the incidental treatments in the position catalogues, are those by Palmer,[1] Perrine,[2] and Barnard. Barnard's work on Messier 13 is discussed in some detail in Sections VII and XII. Palmer divides the stars visible on one of Keeler's Lick Observatory plates into two groups with respect to magnitude, counting 1016 bright and 4466 faint stars, with very few of intermediate brightness. It is suggested that the law of distribution is different for the two groups. Perrine goes farther and finds this separation into two classes of magnitudes a marked characteristic in 30 of the 35 globular clusters examined by him (including Messier 13), and states further: "Perhaps a third of the whole number [of stars in each cluster] lies between the eleventh and thirteenth photographic magnitudes, while almost all of the remainder are very faint, being about the sixteenth magnitude." There can be little doubt that this last statement is quantitatively very much in error. The idea of the division of cluster magnitudes into two distinct groups has always been prominent, especially in visual work, but there is room for a strong suspicion that in most cases it is only a subjective phenomenon, and that in the others it is not so conspicuous as commonly supposed. Some of the Mount Wilson observations are being made on clusters in which this condition is said to be the most marked.

[1] *Loc. cit.*

[2] *Loc. cit.* and *Lick Observatory Bulletins*, **5**, 102, 1909.

The integrated spectrum of Messier 13 has been observed by Fath at the Lick Observatory[1] and at Mount Wilson.[2] The work by Pease on the spectra of individual stars in the cluster will be discussed in a later section of this paper.

II. THE OBSERVATIONAL DATA

The magnitudes contained in the catalogue (Section V) are derived from the complete measurement of two pairs of plates, made on different nights and each consisting of one isochromatic and one ordinary photographic plate. In order to avoid, in the derivation of color-indices, errors depending on the presence of undetected light-variations in some of the stars, the plates of a pair are always taken in succession so that only uncommonly rapid light-changes can affect the results.[3] It is believed that errors arising from this source are negligibly infrequent and small.

In addition to the four standard magnitude plates, the results in the catalogue are indirectly based on the partial measurement of two other pairs of plates, as described below in Section IV. Also two additional plates containing each a long and a short exposure have been used to determine the magnitudes of some of the fainter stars (Section XI), and two preliminary plates have been of service in deriving magnitudes of newly discovered variables (Section XII). The data relative to all of these plates are given in Table I. The observations were made at the principal focus of the 60-inch reflector, with the assistance of Mr. Hoge in the manipulation of the telescope and in the accurate timing of the exposures. The plates were made with full aperture, and, with the exception of those on Plates 2072 and 2073, all exposures on one plate are identical in duration.

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[1] *Lick Observatory Bulletins*, **5**, 74, 1908.

[2] *Mt. Wilson Contr.*, No. 49; *Astrophysical Journal*, **33**, 61, 1911.

[3] In order to facilitate polar comparisons where two kinds of plates are being used a second plate-holder especially adapted for the photo-visual work was made in the Pasadena shop. It was so constructed that the focus need not be changed, as was formerly necessary, to allow for the color-filter. A pair of polar comparisons is now made with but one change from Pole to cluster, the two cluster exposures being made in immediate succession.

TABLE I

LIST OF PLATES

Plate	Kind	Date	G.M.T.	Length of Exposure	No. Exposures on Pole	Seeing	Quality	Use
		1914						
1820...	S 27	Aug. 23	19^h12^m	2^m	0	6	Good	For variables
1821...	S 27	23	19 16	2	1	6	Drift	For variables
1900...	Iso.	Sept. 18	17 24	5	2	4	Good	Standard mags.
1901...	S 27	18	17 46	2	2	5	Good	Standard mags.
1920...	Iso.	19	16 53	20	2	5	Good	Standard mags.
1921...	S 27	19	17 46	4	2	5	Good	Standard mags.
2072...	S 27	Oct. 18	14 34	2, 12	0	4	Good	Faint stars
2073...	Iso.	18	15 36	8, 93	0	4	Medium	Faint stars
		1915						
2319...	Iso.	Mar. 15	20 46	5	2	2	Medium	Check on mags.
2320...	S 27	15	20 50	1	2	1	Poor	Check on mags.
2321...	Iso.	15	21 38	.5	2	3	Good	Check on mags.
2322...	S 27	15	21 42	2	2	3	Medium	Check on mags.

III. MEASUREMENT OF PLATES AND DERIVATION OF MAGNITUDES

A. The plates were measured chiefly by the method described by Seares in *Mt. Wilson Contr.*, No. 80, Section 6.[1] The movable scale, placed in the common focal plane of the objective and eyepiece of the measuring microscope, contains a graded series of 23 stellar images corresponding ordinarily to a range of about seven magnitudes. As estimates of the size and density of stellar images are made to the tenth part of the interval separating two scale-images with an uncertainty of but two- or three-tenths, the probable uncertainty of a single measurement does not exceed a tenth of a magnitude and may be much less. This, however, includes neither the errors arising from uncertainties in the distance correction, nor, of more importance still, those affecting one plate compared with another, which may depend on any or many of the more or less obscure photographic effects that frequently disturb determinations of magnitude. In fact, the possible plate errors are such that it is inadvisable to attempt an extensive reduction of the accidental errors of measurement through the multiplication of settings.

In addition to the direct measurement of the images in a stellar cluster with the graded scale, the writer has found it of great advantage to carry on simultaneously a series of estimates—an independent visual determination of the relative brightness of adjoining images by the Argelander method, using as steps tenths of intervals between scale-images. To illustrate: In the field of the measuring microscope at any time are the images of a great many cluster stars, a number of which are of comparable size and density. Several of these are measured in succession with the scale, and, at the same time, without referring to the scale, the comparable images are mentally arranged in order of "brightness." Perhaps the order, or the number of steps between two stars, may not be the same by estimate and by direct measurement; the mean of the two results is then adopted. This double method can be carried over the whole cluster without much trouble; so that often nearly all of the star-images whose scale readings are, for instance, between 16 and 17 have been directly compared with each other. It seems, moreover, that the estimates are often more dependable than the direct measurement, especially when the strong central condensation prohibits the comfortable manipulation of the scale-images relative to the stars to be measured.

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[1] *Astrophysical Journal*, **39**, 325, 1914.

The secret of the success of such estimates is practice, and more particularly a convenient number of reference stars differing but little in magnitude. After some experience the comparison of neighboring images is carried on subconsciously, and but little greater time is consumed than in making simple direct measurements with the scale alone, while the precision is probably doubled. The relatively high accuracy for the work on Messier 13 is to be attributed in part at least to the method of estimates.

B. In addition to the double measurement of the plates described above, series of completely independent duplicate measures were made to test for the various suspected systematic errors. Effects of fatigue and other personal-equation errors were detected early and avoided. During measurement the plates are illuminated artificially, so that variations in weather and sky illumination disturb the work little, if at all.

The possibility of systematic error of measurement depending on the presence and arrangement of near-by images—an error of paramount importance, since it would be related directly to the distance from center—was tested in various ways: (1) by rotation of the plate through different angles;[1] (2) by varying the exposure times[2] so that the aspect of the field surrounding any given star is greatly different from plate to plate; (3) by plotting artificial spots around the stellar image to be measured; (4) by covering parts of the negatives with a black paper and measuring the images up to the very edge of the occulting diaphragm.

In the last two tests the accidental errors were somewhat increased, but there was in none of the experiments except (2) (cf. Fig. 2) definite evidence of systematic error. It is important to know that this is the case because of the possible dependence of cluster phenomena on distance from the center. It could not be expected, however, that the tests would show for photographic images the well-known disastrous errors met with in measuring visually the brightness of the components of double or multiple stars. In the latter case color-phenomena enter; in the former, the measures are independent of spectral types and of scintillation.

A result similar to the foregoing has been obtained by Bailey.[3] His special investigation of the effect of the relative positions of neighboring stars on estimates of magnitude showed an extreme error of about 0.01 mag.

C. The recorded mean scale readings both for polar and cluster stars were first corrected for distance from the center of the plate[4] and for local irregularities in the measuring scale (the latter determined by a detailed comparison with the scale used by Seares in the work on the North Polar Standards). To determine the magnitudes for the cluster stars, the corrected scale readings for the polar standards on each plate were plotted against their known magnitudes, as illustrated in Fig. 1 for the Seed "27" plate No. 1921 and for the isochromatic plate No. 1920. The co-ordinates of the resulting smooth curves were tabulated for each plate and, with their corrected scale readings for the argument, the magnitudes of the cluster stars were taken directly from the tables.

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[1] If the images were the least elongated there could be no doubt of the presence of systematic errors depending on orientation of the plate. On the plates used for Messier 13, however, the images are sensibly circular.

[2] See the discussion of errors in the next section.

[3] *Harvard Annals*, **38**, 203, 1902.

[4] Using the revised mean distance correction determined by Seares, *Mt. Wilson Contr.*, No. 97, pp. 4 ff.; *Astrophysical Journal*, **41**, 209, 1915.

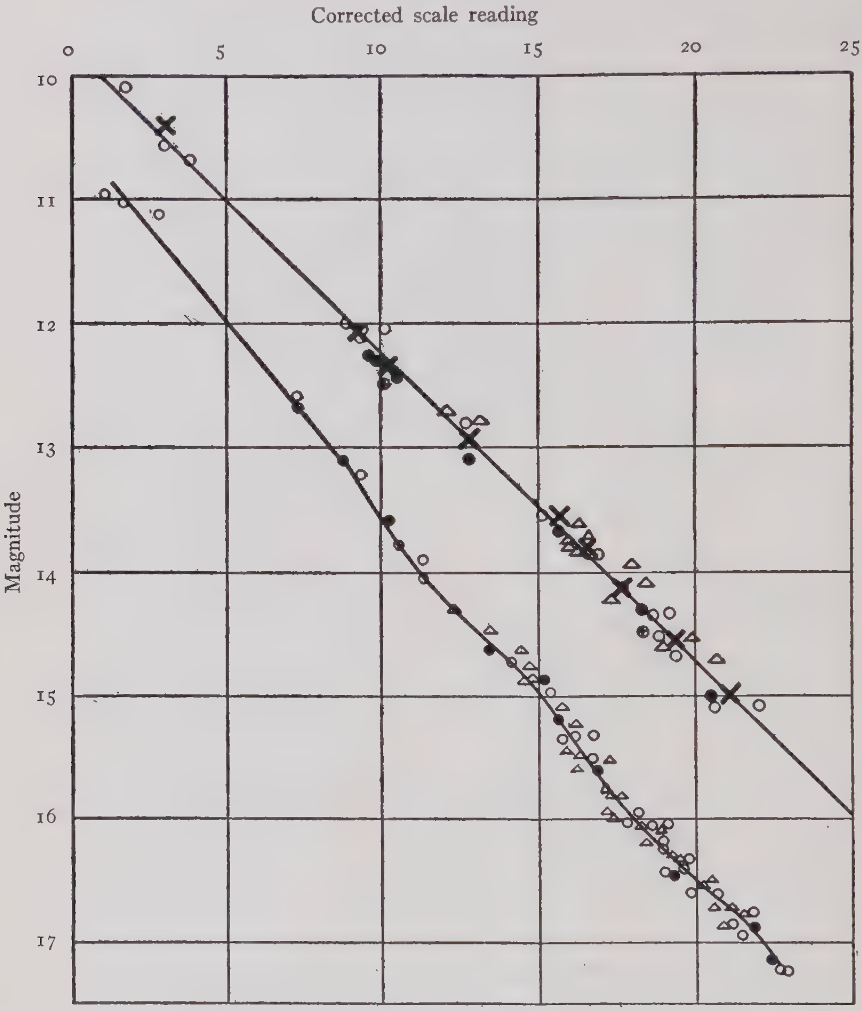

FIG. 1.—Scale reading and magnitude for Polar Standards—above, Plate 1920; below, Plate 1921. Triangles refer to stars more distant from the center than 9′.

Fig. 1 shows also the decrease of accuracy with increasing distance from the center of the plate. For the full aperture no stars are measured at a distance greater than 5 units = 25 mm = 11′.3.

The magnitudes derived from the special tables had finally to be corrected, before entering in the catalogue, for differential atmospheric absorption due to the small differences in altitude of the cluster field and the Pole at the time of the exposures. For the photo-visual magnitudes the corrections were taken directly from the Potsdam tables,[1] interpolating for the altitude of Mount Wilson. The correction for the photographic magnitudes is assumed to be double that for the visual. The extinction corrections for the four standard plates are as follows:

Plate	Pole−Cluster=
Plate 1900,	0.00 mag.
1901,	+0.06
1920,	−0.04
1921,	+0.09

IV. RESIDUALS AND PROBABLE ERRORS[2]

A. *Discussion for standard plates.*—The catalogue, Section V, contains the mean photographic and photo-visual magnitudes, followed by the differences between these means and the separate determinations on the two plates of each kind. The large number of residuals permits a thorough inquiry into the nature of the errors; in fact, it has been deemed unnecessary to make use of the whole mass of data in all of the following discussions. For both photographic and photo-visual results the errors have been discussed in their relation to astigmatism, magnitude, zero-point, and distance from the center.

1. The Photographic Magnitudes—Plates 1901 and 1921: The exposure time on the Seed "27" plates Nos. 1901 and 1921 were two and four minutes, respectively. The latter shows, within the area measured, possibly 2500 stars; the former, not quite half so many. Practically all the stars in Ludendorff's catalogue, however, and most of those recorded on the best standard isochromatic plate, are measurable on Plate 1901. The shorter exposure, in fact, yields values for nearly 90 per cent of all the stars measured on any plate, and after discarding a considerable number of the

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[1] Müller, *Die Photometrie der Gestirne* (Leipzig, 1897), p. 515.

[2] Cf. also Section III, B.

least dependable values,[1] there remain more than a thousand magnitudes based on determinations from both of the standard Seed "27" plates. In the following discussions only the residuals Plate 1921 *minus* Mean are employed. The differences Plate 1901 *minus* Mean are practically the same with reversed sign, since equal weight was given in the majority of cases.[2]

In the first four columns of Table II the residuals are collected into nearly equal groups in order of right ascension. Most of the stars involved in the fifth, sixth, seventh, and eighth groups are in the densest part of the cluster, and, as might be expected, the average deviation is there considerably enhanced. The definite run of the systematic deviations for separate groups, shown in the fourth column of the table, is represented also in the upper part of Fig. 2.

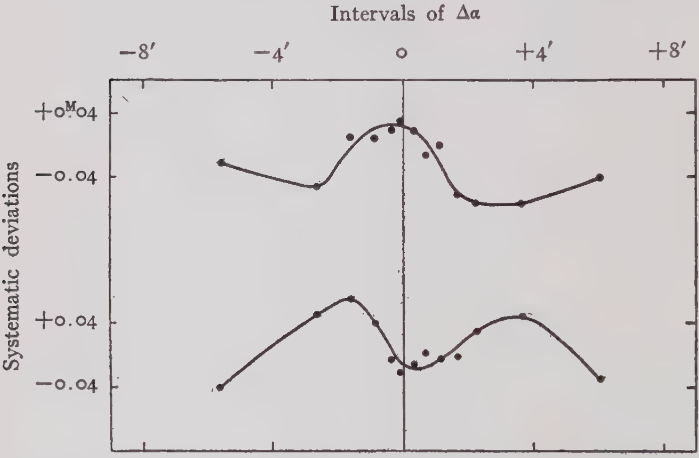

FIG. 2.—The relation between systematic deviations and right ascension—photographic above, photo-visual below.

Table II does not give a direct measure of the systematic error depending on distance from the center, for the right-ascension

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[1] See Section VI for the criteria which were used in discarding certain magnitudes before making the statistical studies.

[2] For the system of weighting the observations see p. 25.

TABLE II

RESIDUALS AND RIGHT-ASCENSION INTERVALS

Interval of Right Ascension	Photographic, 1921—Mean			Photo-visual, 1920—Mean		
	Number Residuals	Average Deviation	Systematic Deviation	Number Residuals	Average Deviation	Systematic Deviation
		mag.	mag.		mag.	mag.
−8′.0 to −3′.3...	85	±0.055	−0.02	8	±0.05	−0.04
−3.3 " −2.1...	87	.072	− .05	14	.05	+ .05
−2.1 " −1.2...	87	.069	+ .01	21	.08	+ .07
−1.2 " −0.6...	90	.074	+ .01	23	.06	+ .04
−0.6 " −0.3...	83	.099	+ .02	15	.04	− .00
−0.3 " +0.1...	86	.080	+ .03	27	.05	− .02
+0.1 " +0.5...	78	.098	+ .02	26	.08	− .01
+0.5 " +0.9...	85	.102	− .01	26	.06	+ .00
+0.9 " +1.3...	84	.081	.00	29	.05	− .00
+1.3 " +1.9...	78	.086	− .06	28	.02	.00
+1.9 " +2.6...	78	.091	− .07	15	.05	+ .03
+2.6 " +4.6...	76	.078	− .07	9	.05	+ .05
+4.6 " +7.5...	54	±0.066	−0.04	3	±0.03	−0.03
Whole cluster....	1051	±0.081	−0.016	244	±0.055	+0.011

groups containing the central stars also include a number of those from outlying regions. Table III gives a partial but sufficient treatment of the residuals from the standpoint of distance alone.

TABLE III

RESIDUALS AND DISTANCE FROM CENTER

Interval of Distance	Photographic, 1921—Mean			Photo-visual, 1920—Mean		
	Number of Residuals	Average Deviation	Systematic Deviation	Number of Residuals	Average Deviation	Systematic Deviation
		mag.	mag.		mag.	mag.
<0′.5..........	36	±0.107	+0.08	33	±0.062	+0.02
0.5 to 0.9.......	42	.073	+ .03			
1.0 " 1.9.......	41	.076	− .01			
2.0 " 2.9.......	42	.065	− .04	33	±0.054	+0.05
3.0 " 3.9.......	41	.081	− .07			
4.0 " 4.9.......	40	.063	− .05	16	±0.048	+0.03
≧5.0..........	45	±0.063	−0.03			
Whole cluster....	1051	±0.081	−0.016	244	±0.055	+0.011

The result is the same as before, though more marked (fourth column of Table III). It is to be noted that the error is in the sense of stars near the center of the cluster being measured relatively too faint on Plate 1921.

The dependence of the accidental and systematic errors on magnitude is shown in Table IV. For this table residuals were taken at random from all parts of the cluster so that the systematic deviations attributable to astigmatism or distance are eliminated almost entirely. Those connected with distance correction doubtless enter to some extent, since distance correction depends on brightness. There appears, however, to be no systematic error connected with magnitude, and the only notable relation between magnitude and accidental error is the unexpectedly greater accuracy for the faintest images.[1] This fortunate result is probably to be attributed to the adaptability to the numerous faint images of the method of estimates discussed in Section III.

TABLE IV

Residuals and Magnitude

Magnitude	Photographic, 1921−Mean			Photo-visual, 1920−Mean		
	Number of Residuals	Average Deviation	Systematic Deviation	Number of Residuals	Average Deviation	Systematic Deviation
		mag.	mag.		mag.	mag.
11.5 to 12.0				4	±0.06	+0.04
12.0 " 12.5				19	.08	+ .05
12.5 " 13.0				31	.05	.00
13.0 " 13.5	10	±0.10	−0.02	35	.05	+ .01
13.5 " 14.0	30	.07	+ .02	35	.06	+ .03
14.0 " 14.5	29	.10	+ .03	12	.04	+ .01
14.5 " 15.0	29	.07	− .025			
15.0 " 15.5	30	.09	+ .01			
15.5 " 16.0	29	.08	+ .01			
16.0 " 16.5	30	±0.03	−0.00			
Whole cluster	1051	±0.081	−0.016	244	±0.055	+0.011

2. The Photo-visual Magnitudes—Plates 1900 and 1920: The exposure times on the isochromatic plates, Nos. 1900 and 1920, were 5 and 20 minutes, respectively. The latter shows, with a few exceptions, all of the stars catalogued by Ludendorff (and many not listed by him), but on the former only about one-fourth of that number are recorded. We are not in a position, therefore, to discuss the photo-visual errors for stars fainter than 14.5;

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[1] Perhaps this would justify giving full weight to the magnitudes from Plate 1901 based on scale readings less than 21.0. See Section V, D.

but we are led to believe that the magnitudes depending on Plate No. 1920 alone are reliable, partly by the following discussion of those residuals available, and partly by other considerations discussed in later sections of this paper. It is proposed, however, to verify them directly through further measurement of the faint stars. The plates necessary for this and other related purposes have been secured and will be discussed in a future paper.

Following the methods used above in discussing photographic magnitudes, the results for the photo-visual residuals can be briefly outlined. The differences Plate 1920 *minus* Mean, taken from Column 7 of the catalogue of magnitudes, are collected into Tables II, III, and IV, along with the photographic results.

The run of the systematic deviations when the residuals are arranged in order of right ascension (last column of Table II) is illustrated in the lower part of Fig. 2. Table III shows for representative groups of residuals the direct dependence of the errors on distance from the center. It indicates the increased difficulty of measurement toward the center, the small zero-point error, and the smaller systematic errors relative to photographic results. As noted by Seares in his work on the North Polar Standards, the clean-cut images on the isochromatic plates allow more accurate determinations of magnitude than is possible from the ordinary photographic plate. This justifies the use of fewer measures and less material in determining photo-visual magnitudes. Table IV shows that the photo-visual errors are practically independent of the brightness of the stars.

3. Summary of Probable Errors: From the 1051 photographic and 244 photo-visual residuals grouped in Table II the probable errors were derived which are summarized in Table V. Since only 10 per cent of the photographic magnitudes depend on but one plate, and since for half of these there are no corresponding color-indices, the average probable error is less than ±0.05 mag. for 95 per cent of the magnitudes used in the statistical discussions. This error includes not only uncertainties of observation in the cluster magnitudes, but also the errors in the polar standards, uncertainties in the distance correction and zero-point, and other plate errors. If the individual residuals for each of the groups of

Table II be corrected for the systematic deviation, thus excluding the plate errors, the probable errors become as in the last two lines of Table V.

TABLE V

AVERAGE PROBABLE ERROR OF A MAGNITUDE DETERMINATION

	PHOTOGRAPHIC		PHOTO-VISUAL	
	Whole Cluster	Distance > 2′	Whole Cluster	Distance > 2′
Probable error:				
One plate	±0.068	±0.057	±0.046	±0.044
Two plates	±0.048	±0.041	±0.033	±0.031
Corrected for systematic deviations:				
One plate	±0.061	±0.044	±0.040	±0.035
Two plates	±0.043	±0.031	±0.029	±0.025

The average probable error[1] of a tabulated color-index (column 8 of the catalogue) varies from ±0.082 to ±0.058, and for about 75 per cent of the stars[2] is ±0.067. Excluding stars within 2′ of the center the limits become ±0.072 and ±0.051, and the median value is ±0.060.

B. *Discussion for control plates.*—One of the most important factors in the measurement of color-indices is the determination of the zero-points, since any error in them enters with full weight into the colors. As each of the cluster plates was exposed to the Pole, we should have, under ideal conditions, the zero-point of the Polar Standards registered on every plate; and the derived cluster magnitudes, when corrected for differential extinction, should be on the international scale. But uncertainties in the adopted extinction correction may enter, as well as possible differences in the atmospheric conditions in the direction of the two fields.

To check the zero-points derived from the standard plates, two additional pairs were made, with different values of the zenith distances and with the cluster field above the eastern instead of the western horizon. The data describing the control plates have already been given in Table I. All of the polar images and a selected

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[1] Without correcting for systematic deviations.

[2] Those measured on Plates 1901, 1920, and 1921.

TABLE VI

Control of Zero-Point

Star	Distance	Color-Index	Photographic			Photo-visual		
			Adopted Mag.	Observed − Adopted		Adopted Mag.	Observed − Adopted	
				2320	2322		2319	2321
				mag.	mag.		mag.	mag.
172....	3′.1	+1.04	14.41	+0.12	....	13.37	−0.08	+0.05
189....	3.6	+0.60	14.56	....	+0.26	13.96	....	....
199....	3.4	+1.42	13.66	− .05	.00	12.24	− .11	− .04
205....	2.7	+0.70	15.24	....	+ .08	14.54	....	....
236....	3.3	+0.97	16.63	....	− .07	15.66	....	....
237....	2.9	(+0.20)	16.04	....	+ .01	(15.84)	....	....
239....	3.0	+0.93	15.58	+ .04	− .08	14.65	....	....
240....	2.8	+1.34	13.71	+ .08	+ .21	12.37	+ .08	− .05
243....	2.8	+1.01	14.15	− .03	+ .05	13.14	+ .08	− .11
244....	2.9	+1.16	13.80	+ .01	+ .21	12.64	− .05	− .08
247....	2.5	+0.24	15.37	− .17	− .20	15.13	....	....
250....	1.8	+1.27	13.71	....	....	12.44	− .03	− .07
252....	2.8	+1.25	13.84	+ .20	+ .14	12.59	+ .03	− .01
258....	2.7	+1.16	14.53	− .11	− .27	13.37	− .01	+ .09
260....	2.5	+0.99	15.64	.00	− .28	14.65	....	....
263....	3.3	+0.06	15.24	− .18	− .24	15.18	....	....
268....	2.6	+1.20	14.46	− .16	+ .13	13.26	+ .19	+ .04
277....	2.4	−0.05	15.08	− .14	− .15	15.13	....	....
296....	1.5	+1.05	13.59	....	....	12.54	− .09	+ .02
316....	2.7	+1.16	13.68	− .03	+ .12	12.52	+ .02	− .05
324....	1.5	+1.11	13.06	....	....	11.95	− .17	− .14
326....	3.3	−0.12	13.76	− .02	....	13.88	....	....
337....	2.0	+1.02	14.73	+ .11	....	13.71	− .14	+ .08
345....	1.4	+1.07	13.85	+ .06	....	12.78	− .03	− .02
348....	3.0	+0.92	14.90	.00	....	13.98	....	....
353....	2.1	+0.97	13.84	− .12	....	12.87	− .10	− .16
370....	1.5	+1.00	14.22	....	....	13.22	+ .17	− .09
398....	1.0	+1.18	13.67	....	....	12.49	+ .08	+ .10
403....	5.1	+1.07	14.08	+ .11	+ .12	13.01	+ .12	− .03
414....	2.1	+1.38	13.46	....	....	12.08	+ .01	+ .04
423....	3.0	+0.92	15.06	− .01	....	14.14	....	....
436....	1.2	+0.86	14.06	+ .36	+ .27	13.20	− .01	− .11
481....	0.8	+1.43	13.76	....	....	12.33	− .08	+ .04
510....	1.7	+0.17	14.84	− .04	− .08	14.67	....	....
526....	1.4	+0.57	14.09	− .04	+ .10	13.52	.00	− .03
541....	1.7	+0.07	14.74	+ .19	+ .06	14.67	....	....
553....	1.9	+0.71	14.64	....	+ .12	13.93	....	....
687....	5.2	+0.80	13.76	.00	+ .11	12.96	+ .05	− .17
726....	4.3	+0.81	14.38	+0.08	+0.18	13.57	....	.00
973....	2.3	+1.27	13.35	....	....	12.08	− .10	− .10
1087...	5.9	+0.40	13.14	....	....	12.74	−0.05	0.00
Average difference................				±0.09	±0.14	....	±0.08	±0.07
Number of stars..................				27	25	....	25	26
Extinction correction.............				+0.10	+0.17	....	+0.06	+0.08

number from the cluster were measured by the methods used for the standard plates. In selecting the stars pains were taken that different degrees of magnitude, color-index, and distance from the center should be included.

The results for the four plates are given in Table VI. The star-number is taken from Ludendorff's catalogue; the distance from the center is expressed in minutes of arc; the color-index and adopted magnitudes are taken from the catalogue in Section V, and the residuals for the different plates, under the heading "Observed—Adopted," are the differences between the catalogue magnitudes and those derived directly from the control plates.

TABLE VII

DEVIATION OF ZERO-POINTS FROM ADOPTED MEANS

Photographic			Photo-visual		
Plate	O.—C.	No. Stars	Plate	O.—C.	No. Stars
	mag.			mag.	
1901	+0.016	1051	1900	—0.011	244
1921	—0.016	1051	1920	+0.011	244
2320	+0.02	27	2319	—0.01	25
2322	+0.04	25	2321	—0.03	26

Examining the results for the individual stars on each plate, no evidence is found of appreciably deviating scales or of errors depending on magnitude or distance. The rather large average differences are due to the inferior quality of some of the plates, to errors in the standard magnitudes, and to the presence of small systematic deviations.

The probable errors in Table V only partially include the uncertainties of the zero-points; but the errors from this source are small, as is shown by Table VII, which gives results for all the plates, interpreting mean systematic deviations as corrections to the zero-points. The data of Table VII indicate that small corrections might be made to the catalogued magnitudes, and in particular to those which are based on one plate only. It can be deduced from this and preceding tables that for 10 per cent of the stars (those measured neither on Plate 1900 nor on Plate 1901) an average correction of +0.03 mag. should be made to the color-indices, and that for about 75 per cent of the remainder a correction of +0.01 mag. would be appropriate. None of these corrections has been applied, however, for the following reasons, in addition to the obvious one of little gain for much labor: (1) the corrections are derived from a small amount of data, and the largest depends on results from a poor plate; (2) the true corrections in different parts of the cluster vary by considerably larger amounts than these small mean corrections; (3) the statistical investigations based on the observed color-indices involve uncertainties considerably in excess of a hundredth of a magnitude.

C. *Possible systematic error in magnitudes of central stars.*—The foregoing discussion of errors shows for the denser central region of the cluster not only the expected larger accidental deviations, but also a systematic error apparently inherent in the plates themselves. At first it appeared not serious enough to introduce unallowable errors into the magnitudes and colors. Further investigation, however, and especially the statistical work in later sections, suggests that a photographic phenomenon analogous to the one discovered and studied by Eberhard[1] may influence the results to a considerable extent. Eberhard found that the intensity of a photographic image depends on the density of neighboring parts of the plate.

Further discussion of this phenomenon in its probable bearing on the magnitudes in the center of the Hercules cluster will be postponed to a later paper. For the present the reasons for suspecting its appearance will only be enumerated: (1) the nature of the field photographed and the method of development; (2) certain systematic errors referred to in connection with Tables II and III and illustrated in Fig. 2; (3) the discrepancy between directly observed spectra and color-indices in the center of the cluster; (4) the conspicuous increase of average color-index in the central regions (discussed at length in Section VIII); (5) some laboratory experiments in copying and enlarging photographs of clusters. In (2), (4), and (5) the effect appears only as a differential factor from plate to plate.

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[1] *Physikalische Zeitschrift*, **13**, 288, 1912.

The five reasons just cited for believing that a systematic photographic error may affect the magnitudes within 1′.5 or 2′ of the center, with obvious and slight changes in wording, are also valid reasons for believing that there is no such effect outside the crowded central regions. In addition we have still stronger assurance that the magnitudes of stars more distant than 2′ are reliable in the fact that the Polar Standards, which are very generally distributed on all plates throughout the outer regions of the cluster, show no trace of systematic error depending on distance.[1] In fact, Fig. 1 shows that no important systematic error related to distance from the center has entered into the determination of the magnitudes beyond distance 2′.

A quantitative correction for an Eberhard effect is not possible, for as yet very little is known about its amount, variation with plate emulsions, or its control. In view of this circumstance it has been considered best to withhold publication of the magnitudes derived for the central stars of the cluster until later study can make them more definite. The data derived from them will be retained to some extent in the statistical tables and discussion, for it is through the analysis of this material that the peculiarity of the central region is best revealed.

V. THE CATALOGUE OF MAGNITUDES AND COLORS

A. *The system of numbering the stars.*—The main catalogue (Table IX) contains results for all the stars with distance $\geqq 2'$ for which positions are given by Ludendorff and for a number of others for which the brightness on the best isochromatic plate (No. 1920) seemed comparable with the brightness of those listed in the Potsdam catalogue. In fact, all stars visible on Plate 1920 were measured for magnitudes. In addition a considerable number of those given as single by Ludendorff proved upon the closer examination permitted by the Mount Wilson plates to be double or multiple stars. In fact, as many as five or six components were found for some of the objects near the center of the cluster recorded as single by Scheiner and Ludendorff. The total number of stars in the catalogue is 616. In a supplementary table, p. 73, are given magnitudes of about 30 very faint stars. The number with distance less than 2′ is 659; hence the total number of magnitudes investigated is slightly in excess of 1300.

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[1] The only one falling within the crowded zones is, by chance, superposed on cluster stars and cannot be accurately measured.

Ludendorff's numbers are retained throughout and appear in the first column of the present catalogue. Additional stars are designated by postscript letters. When an image has been separated into two or more parts the individual components are assigned the original Ludendorff number, each with an appended Greek letter. Postscript Roman letters are assigned to neighboring stars unnumbered by Ludendorff, and to close companions evidently not considered by him in measuring the primary's position. Thus a number may appear several times in succession with both Greek and Roman letters, one referring to component and the other to companion stars. In Section XI another system of postscript letters is used for fainter stars, but none of these objects appears in the main catalogue.

B. *The adopted center of the cluster.*—The origin of co-ordinates chosen by Scheiner and adopted by Ludendorff was admittedly not in the exact center of the cluster. As *Normalstern*, Scheiner chose a star to which later was assigned the number 382. It is No. 516 in Ludendorff's list, and the latter finds its position appreciably different from that recorded by Scheiner. On the Mount Wilson plates the star is a very close double. The possible variation of one of the components may account for the difference between Scheiner's and Ludendorff's measures. On a smaller scale the images of the two components coalesce to form a fairly bright center of reference. The extreme visual faintness of the *Normalstern*, however, led Barnard to adopt as reference point a much brighter star, Lud. 509, which he considered more nearly the center of the cluster.[1]

The center used in the present work for the application of distance corrections—the point upon which the plates were centered in making exposures—follows the origin of co-ordinates used by Ludendorff by 15″. But the point finally adopted as center of the cluster (determined by counts of stars of different magnitudes and by inspection of long- and short-exposure plates) is south of Ludendorff's origin of positions 10″ and follows it by 15″. The adopted center of the cluster for the epoch 1900.0 is, therefore,

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[1] Lud. 509 is about 20″ from the center adopted in the present work; for the brighter stars Barnard's choice of center is probably the better.

$$\text{R.A.} = 16^{h}\,38^{m}\,7\overset{s}{.}14, \qquad \text{Dec.} = +36^{\circ}\,39'\,9\overset{''}{.}06$$

which is about half-way between the stars 555 and 576. All the distances in the second column of the catalogue are referred to this point. The uncertainty of a distance estimate should not exceed one-tenth of a minute of arc.

C. *Evaluation of Ludendorff's system of magnitude estimates.*—The determination of magnitudes in Messier 13 by Ludendorff is based upon a series of estimates.[1] The stars were arranged in 16 classes in order of decreasing brightness, no attempt being made to obtain absolute values. It is now possible to transform the relative estimates into photographic magnitudes, thus giving a record of brightness for an early epoch, July 20, 1900, which will assist in the derivation of periods of the variable stars in the cluster.

When based on relative estimates, brightness will presumably depend upon the surrounding field. Hence values of the various Ludendorff classes were determined for four different intervals of distance from the center of the cluster. The results are summarized in Table VIII. Classes brighter than 3 occur but seldom, and when recorded generally refer to a double or multiple star. Nor can much reliance be put upon the few magnitudes designated by the numbers 3 and 4. For instance, stars differing by as much as a whole magnitude are included in the 3's. Columns 2–9 of the table contain for the various distances from the center the number of stars considered and the mean photographic magnitude derived from them.[2] The average deviation of a single determination is approximately ±0.15 mag., so that the probable error of an average mean magnitude is about ±0.05 mag.

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[1] For details of the method and probable value of the estimates, see Ludendorff, *op. cit.*, pp. 12, 15, 49.

[2] Columns 3, 5, and 10 are subject to revision when the Mount Wilson photographic magnitudes of the central stars are freed from a possible Eberhard effect.

The expected dependence of the value of an estimate upon the condensation of stars is distinctly shown. The values in the last two columns of the table were finally adopted—one for the center of the cluster, the other for the exterior regions. The uncertainty of the reduced Ludendorff estimates is of the order of a tenth of a magnitude.

TABLE VIII

EVALUATION OF LUDENDORFF'S MAGNITUDE ESTIMATES

Lud. Class	Distance from Center of Cluster								Adopted Photographic Magnitude	
	0'.0 to 1'.0		1'.0 to 3'.0		3'.0 to 5'.0		>5'.0		Distance 0 to 0'.'0 3	Distance >3'.0
	No.	Mean Pg. Mag.	No.	Mean Pg. Mag.	No.	Mean Pg. Mag.	No.	Mean Pg. Mag.		
3....	1	13.82	4	13.62	2	13.56	2	13.10	13.5	
4....	7	13.89	7	13.83	5	13.71	4	14.02	13.9	
5....	7	14.04	6	14.12	7	14.47	4	14.42	14.1	14.5
6....	8	14.50	7	14.61	10	14.76	3	14.74	14.5	14.7
7....	7	14.68	12	14.71	9	14.92	4	14.90	14.7	14.9
8....	8	15.01	9	14.94	12	14.94	10	15.00	14.9	15.0
9....	8	15.08	9	14.96	10	15.19	10	15.20	15.1	15.2
10....	7	15.30	13	15.28	8	15.36	10	15.41	15.3	15.4
11....	7	15.28	13	15.57	10	15.56	8	15.64	15.5	15.6
12....	8	15.53	10	15.61	13	15.77	6	15.78	15.6	15.8
13....	6	15.76	4	15.92	8	16.00	7	15.95	15.8	16.0
14....	3	15.67	9	15.90	8	16.05	6	16.09	15.9	16.1
15....	1	15.97	10	16.19	9	16.16	9	16.37	16.1	16.3
16....	......	....	9	16.16	8	16.38	5	16.59	16.2	16.5

It is to be noted that Ludendorff's scale diverges very little from a true scale of magnitudes. Scheiner's estimates, on the other hand, besides being systematically about two magnitudes too bright, are referred to a scale which deviates conspicuously from the Mount Wilson standards.

D. *System of weighting observations.*—The fourth and sixth columns of the catalogue contain the photographic and photovisual mean magnitudes, respectively, and the fifth and seventh columns the deviations from the means of the individual determinations on the standard plates. The first number in the fifth column refers to Plate 1901, the second to Plate 1921; in the seventh column the numbers refer to Plates 1900 and 1920, respectively. Equal weights are given in the majority of cases, but when for faint stars the scale reading for Plate 1900 or Plate 1901 was equal to or greater than 21.0, half-weight was given the corresponding magnitude in forming the mean. When a colon was placed after the mean scale reading because of a diffuse or irregular image or for other reason, half-weight was assigned to the magnitude; for double colons one-fourth weight was used. The cases of unequal weights may be determined by inspection of the residuals, for instance, Stars 14, 19, and 24.

Systematic remeasurements were made whenever the full-weight residuals for either photographic or photo-visual results exceeded a quarter of a magnitude. This led to the discovery of a few variable stars, and also to the elimination of errors that had escaped the checks.

Magnitudes in parentheses in the fourth and sixth columns do not depend on any of the standard plates, because either of faintness or of superposition of polar stars. A discussion of these magnitudes which are of relatively low weight is given in Section XI and in the "Notes to the Catalogue." A blank in the sixth column indicates that the star was too faint for measurement on the isochromatic plates.

E. *Color-index and notes.*—The color-index in the eighth column, which is the difference between the magnitudes in the fourth and sixth columns, is marked with a colon when either the photographic or photo-visual magnitude is uncertain. Values in parentheses were derived from provisional results and have been excluded from the statistical discussion.

The numbers in the last column refer to the notes succeeding the catalogue.

In order to make sure that color-indices of exceptionally large or small value are not the result of accidental errors, all greater than +1.5 or less than −0.3 were redetermined. The mean of the original and the new measures was adopted, except where an obvious error was discovered. A few mistakes in identifications were detected, but the number of large and small color-indices was little changed. The remeasurements for all causes involve nearly a hundred stars and are designated by Notes 9, 10, and 11.

TABLE IX

Catalogue of Stars

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
1	7.2	15.0	14.85	− 8, + 8	14.14	....	+0.71	
2	7.1	14.9	14.96	−16, +17	15.08	....	−0.12	
3	6.7	16.3	16.41	....	(15.54)	....	(+0.87)	17
4	6.6	15.4	15.28	+ 2, − 1	14.54	....	+0.74	
5	6.5	14.5?	16.74	....	(16.50)	....	(+0.24)	17, 18, 20
6	6.9	14.9	14.94	− 8, + 9	14.59	....	+0.35	
7	8.8	15.2	15.04	− 1, + 2	15.16	....	−0.12	
8	8.4	15.8	15.69	+ 2, − 1	14.92	....	+0.77	
9	7.4	16.3	16.10	....	(15.42)	....	(+0.68)	17
10	6.0	15.2	15.14	+ 4, − 4	13.69	....	+1.45	
11	7.2	16.5	(17.25)	....	....	....	....	19, 20
12	6.1	15.6	15.52	+16, −15	14.56	....	+0.96	
13	6.2	15.6	15.50	+ 5, − 6	14.46	....	+1.04	21
14	5.4	16.1	15.94	+ 5, − 2	15.05	....	+0.89	
15	7.6	13.5	13.06	− 3, + 2	12.80	+ 5, − 4	+0.26	
16	6.5	15.2	15.26	+10, −10	14.40	....	+0.86	22
17	6.8	14.9	14.86	− 6, + 7	14.97	....	−0.11	
18	5.5	13.9	14.52	0, + 1	13.58	0, − 1	+0.94	
19	6.2	16.0	15.91	+11, − 5	15.03	....	+0.88	
20	5.2	15.0	14.97	− 5, + 6	14.56	....	+0.41	
21	6.8	15.8	16.01	− 4, + 2	15.37	....	+0.64	
22	7.4	15.6	15.56	+23, −22	14.54	....	+1.02	
23	5.8	16.0	16.08	− 6, + 2	(16.63)	....	(−0.55)	17
24	5.4	16.1	16.17	+ 9, − 4	(16.54)	....	(−0.37)	17
25	5.3	15.0	15.06	+ 3, − 3	15.11	....	−0.05	
26	5.3	14.5	14.43	+ 3, − 2	13.49	....	+0.94	
27	5.6	15.2	15.12	+ 3, − 2	15.26	....	−0.14	
28	5.0	16.0	15.58	+15, −14	14.72	....	+0.86	
29	5.1	15.2	15.18	+ 3, − 2	14.27	....	+0.91	
30	4.8	15.0	15.12	− 3, + 4	15.21	....	−0.09	
31	4.8	15.4	15.33	0, + 1	14.56	....	+0.77	
32	4.7	15.8	15.77	+ 2, − 1	15.03	....	+0.74	
33	6.4	15.0	15.21	+ 6, − 5	15.23:	....	−0.02:	
34	4.6	15.6	15.60	+17, −16	14.70	....	+0.90	
35	5.0	15.6	15.70	− 5, + 6	15.56	....	+0.14	
36	5.0	16.1	16.13	....	15.38	....	+0.75	
37	7.2	15.4	15.31	+ 5, − 4	14.38	....	+0.93	
38	4.5	15.4	15.21	+ 3, − 2	14.51	....	+0.70	
39	6.0	15.8	15.44:	....	15.26:	....	+0.18::	23
40	5.6	15.4	15.48	− 7, + 8	15.34	....	+0.14	
41α	4.6	......	15.25	−16, +16	14.21	....	+1.04	9, 24
41β	4.6	......	14.91	− 2, + 2	14.82	....	+0.09	9, 24
41γ	4.6	......	15.80	− 2, + 3	15.42	....	+0.38	24
42	5.2	16.1	15.60	+17, −16	14.54	....	+1.06	
43	5.3	14.5	14.52	+ 8, − 7	13.59	− 1, 0	+0.93	
44	5.6	15.0	14.92	− 3, + 4	14.75	....	+0.17	
45	4.8	15.4	15.44	+ 8, − 7	15.11	....	+0.33	
46	4.6	15.6	15.57	+27, −13	15.56	....	+0.01	
47	5.5	16.0	15.89	+13, − 6	14.93	....	+0.96	
48	5.7	16.5	16.64	....	(16.92)	....	(−0.28)	17

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
49....	4.2	14.5	14.83	+ 3, − 2	15.00	....	−0.17	
50....	4.2	14.5	14.80	+ 3, − 2	14,65	....	+0.15	
51....	4.2	16.5	16.63	....	15.58	....	+1.05	
52....	4.4	14.7	15.00	+ 5, − 4	15.18	....	−0.18	
53....	4.7	15.2	15.18	0, + 1	15.31	....	−0.13	
54....	5.0	16.3	16.20	....	15.56	....	+0.64	
55....	4.1	14.9	15.18	+ 9, − 8	15.31	....	−0.13	
56....	4.0	15.8	15.44	− 3, + 4	14.65	....	+0.79	
57....	6.3	16.0	16.07	+ 1, − 1	(16.42)	....	(−0.35)	
58....	4.0	15.8	15.41	+ 1, 0	14.59	....	+0.82	
59....	5.1	15.0	15.08	+ 6, − 5	15.13	....	−0.05	
60....	4.4	16.5	16.31	....	(15.85)	....	(+0.46)	
61....	4.3	16.0	16.13	....	(16.44)	....	(−0.31)	
62....	3.8	15.8	15.76	− 2, + 3	15.05	....	+0.71	
63....	4.4	16.5	16.24	....	(15.54)	....	(+0.70)	
64....	6.5	16.5	16.85	....	(16.65)	....	(+0.20)	
65....	3.7	16.3	16.41	....	(15.97)	....	(+0.44)	
66....	3.7	14.5	14.54	+ 4, − 4	13.98	....	+0.56	
67....	3.7	15.8	15.83	+ 1, 0	14.87	....	+0.96	
68....	4.0	15.8	15.94	+17, − 8	15.39	....	+0.55	
69....	4.6	14.9	15.15	+ 6, − 5	15.13	....	+0.02	
70....	4.4	13.9	13.45	+11, −10	12.04	+ 3, − 2	+1.41	
71....	3.7	16.3	16.21	....	15.61	....	+0.60	
72....	4.0	13.9	13.74	− 8, + 9	12.42	−16, +16	+1.32	
73....	4.1	15.0	15.10	+ 8, − 7	15.34	....	−0.24	
74....	3.5	15.2	14.96	+ 7, − 6	15.16	....	−0.20	
75....	4.5	16.0	15.87	+ 8, − 4	15.13	....	+0.74	
76....	3.7	16.3	16.04	+ 9, − 4	15.31	....	+0.73	
77....	3.8	13.9	13.82	− 4, + 4	12.80	− 7, + 6	+1.02	
78....	4.0	15.0	14.89	0, + 1	14.21	....	+0.68	
79....	3.8	14.7	14.72	+17, −16	13.93	−10, + 5	+0.79	
80....	4.6	15.6	15.58	+15, −14	14.62	....	+0.96	
81....	4.1	14.7	14.69	+11, −11	13.89	− 8, + 4	+0.80	
82....	4.1	16.0	16.10	+ 4, − 2	15.13	....	+0.97	
83....	3.3	15.0	15.00	+ 5, − 4	15.00	....	0.00	
84....	3.2	15.8	15.78	+12, − 6	(16.00)	....	(−0.22)	
85....	3.5	16.1	16.24	+ 9, − 4	(15.73)	....	(+0.51)	
86α...	5.7	......	15.02	− 8, + 8	15.16	....	−0.14	
86β...	5.7	......	16.04	− 7, + 6	15.34	....	+0.70	
87....	3.3	15.8	15.94	+17, − 8	15.08	....	+0.86	
88....	3.7	15.2	15.40	− 4, + 4	15.37	....	+0.03	
89....	3.3	15.6	15.41	+ 1, 0	14.59	....	+0.82	
90....	3.4	16.3	16.22	+ 1, 0	15.39	....	+0.83	
91....	5.1	15.4	15.18	+ 9, − 8	14.56	....	+0.62	
92....	3.2	15.8	15.91	+11, − 5	15.11	....	+0.80	
93....	3.6	15.4	15.43	− 4, + 5	14.59	....	+0.84	
94....	3.2	14.7	14.78	+11, −10	14.59	....	+0.19	
95....	3.0	15.8	15.20	+10, −10	14.46	....	+0.74	
96....	2.9	13.9	13.70	− 2, + 3	12.54	− 3, + 4	+1.16	
97....	3.1	15.6	15.21	+ 3, − 2	14.59	....	+0.62	
98....	2.9	14.9	15.31	−10, +10	15.51	....	−0.20	11
99....	3.1	16.3	16.03	+ 7, − 3	15.26	....	+0.77	

TABLE IX—*Continued*

STAR	DIS-TANCE	LUD. MAG.	PHOTOGRAPHIC		PHOTO-VISUAL		COLOR-INDEX	NOTES
			Mag.	Residuals	Mag.	Residuals		
100...	3.1	15.8	15.61	+26, −13	15.23	....	+0.38	
101...	3.4	15.8	15.83	+ 9, − 4	15.05	....	+0.78	
102...	5.4	15.4	15.61	+13, −13	14.56	....	+1.05	
103...	6.7	15.2	15.32	− 5, + 5	15.29:	....	+0.03:	25
104...	2.8	15.1	14.93	+ 1, 0	15.03	....	−0.10	
105...	2.8	15.9	16.17	+ 4, − 2	15.08	....	+1.09	
106...	3.2	16.1	16.01	+13, − 6	15.37	....	+0.64	
107...	3.5	15.4	15.33	+ 6, − 6	15.05	....	+0.28	
108α..	2.9	......	15.36	− 6, + 5	14.27	....	+1.09	
108β..	2.9	......	15.68	+19, −20	15.00	....	+0.68	
109...	3.4	14.5	13.99	+11, −10	13.14	+ 1, − 1	+0.85	
110...	5.3	15.0	15.06	+ 3, − 3	14.32	....	+0.74	
111...	3.0	16.1	16.18	+ 3, − 1	(16.25)	....	(−0.07)	
112...	2.8	15.1	15.00	+ 3, − 4	14.65	....	+0.35	
113...	5.5	16.5	16.22	+ 4, − 2	15.48	....	+0.74	
114...	2.7	14.7	14.23	− 6, + 7	13.36	− 5, + 5	+0.87	
115...	3.5	15.2	15.36	+ 3, − 2	14.29	....	+1.07	
116...	6.1	16.3	16.48	....	(16.74)	....	(−0.26)	
117...	2.6	14.9	14.66	+ 5, − 6	13.92	− 9, + 4	+0.74	
118...	2.7	15.1	15.03	0, 0	15.21	....	−0.18	
119...	4.1	15.2	15.18	+ 9, − 8	15.34	....	−0.16	
120...	2.6	14.9	15.02	+10, − 9	15.05	....	−0.03	
121...	2.8	16.2	15.95	+13, − 6	15.23	....	+0.72	
122...	2.6	15.1	15.26	+ 4, − 3	15.37	....	−0.11	
123...	2.5	15.3	15.54	+10, −10	14.70	....	+0.84	
124...	4.4	15.4	15.18	+ 9, − 8	14.59	....	+0.59	
125...	2.5	16.1	16.22	....	(15.52)	....	(+0.70)	17, 25
126...	2.8	14.9	15.12	+ 9, − 9	14.03	....	+1.09	
127...	2.6	15.3	15.38	− 5, + 6	15.53:	....	−0.15:	
128...	4.7	15.8	15.79	+29, −15	15.11	....	+0.68	
129...	2.5	15.3	15.45	− 6, + 7	14.65	....	+0.80	
130...	3.1	15.0	15.21	+ 6, − 5	14.32	....	+0.89	
131...	2.8	15.5	15.96	+15, − 7	15.51:	....	+0.45:	
132...	3.2	15.0	14.95	+11, −11	14.80	....	+0.15	
133...	2.4	15.9	15.99	+ 9, − 4	15.39	....	+0.60	
134...	4.6	15.8	15.65	+25, −13	15.00	....	+0.65	
135...	2.4	15.6	15.10	....	15.18	....	−0.08	7, 26
136...	2.3	16.1	16.24	....	15.66	....	+0.58	
137...	5.0	15.4	15.34	− 7, + 7	14.32	....	+1.02	
138...	4.0	15.2	15.23	+ 4, − 4	15.29	....	−0.06	
139...	4.8	14.5	14.35	+ 5, − 5	13.93	....	+0.42	
140...	2.5	14.9	14.80	+ 3, − 2	13.98	....	+0.82	
141...	2.7	14.9	14.88	+ 4, − 4	14.59	....	+0.29	
142...	2.3	16.2	16.29	....	(15.85)	....	(+0.44)	
143...	2.3	15.5	15.65	+ 9, − 9	15.66	....	−0.01	
144...	3.8	14.7	14.54	+ 6, − 6	14.14	....	+0.40	
145...	2.6	16.1	16.17	+ 9, − 4	(15.91)	....	(+0.26)	27
146...	4.7	15.6	15.71	+31, −15	15.03	....	+0.68	
147...	3.5	15.2	15.58	+29, −14	15.08	....	+0.50	
148...	3.4	16.1	16.12	+17, − 9	15.31	....	+0.81	9
149...	3.0	15.8	15.96	+15, − 7	15.34	....	+0.62	
150...	2.2	16.1	16.24	....	(15.82)	....	(+0.42)	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
151	4.0	14.9	15.08	+16, −15	15.24	....	−0.16	
152	2.3	16.2	16.08	0, 0	15.45	....	+0.63	
153	3.0	16.0	15.82	+ 5, − 3	15.37	....	+0.45	
154	2.8	15.9	16.02	+14, − 7	15.26	....	+0.76	9
155	5.0	16.5	16.56	....	....	....	....	
156	2.4	15.5	15.46	−10, +10	15.66	....	−0.20	
157	2.4	15.9	16.01	+18, − 9	15.39	....	+0.62	
158	3.0	13.9	13.86	−15, +16	12.70	+ 3, − 2	+1.16	
159	5.8	16.3	16.09	+ 2, − 1	....	....	....	
160	2.2	14.7	14.49	+ 6, − 6	13.52	−10, + 9	+0.97	9
161	2.7	16.2	16.30	+17, − 4	....	....	....	
162	3.0	14.9	14.68	+15, −15	13.96	....	+0.72	28
163	4.1	15.2	15.36	0, + 1	15.61	....	−0.25	11
164	2.1	16.1	16.99	....	....	....	....	
165	3.2	14.9	14.65	+ 9, − 9	13.90	....	+0.75	
166	2.5	15.6	15.56	+13, −12	14.92	....	+0.64	
167	3.2	16.1	15.88	+ 4, − 2	15.37	....	+0.51	9
168	3.3	14.5	14.39	+ 1, 0	13.53	−11, +11	+0.86	
169	2.7	13.9	13.71	+ 5, − 5	12.74	+ 2, − 1	+0.97	
170	2.1	14.7	14.98	+ 2, − 2	15.16	....	−0.18	
171	2.3	16.3	15.98	....	15.42	....	+0.56	
172	3.1	14.5	14.41	+ 8, − 7	13.37	− 5, + 4	+1.04	
173	2.4	15.9	15.70	+20, −10	14.97	....	+0.73	
174	2.1	14.9	14.65	+12, −12	13.92	− 7, + 5	+0.73	
175	2.3	15.3	15.30	− 6, + 7	15.16	....	+0.14	
176	2.1	13.9	14.08	+ 9, − 9	13.14	− 6, + 7	+0.94	
177	2.2	15.3	15.44	−11, +12	15.39	....	+0.05	
178	2.4	14.9	14.67	+19, −19	13.77	−13, +13	+0.90	
179	3.8	16.1	16.17	....	15.39	....	+0.78	
180	2.4	15.5	15.34	+12, −11	14.56	....	+0.78	
181	2.6	14.7	14.71	+ 6, − 6	13.95	− 5, + 3	+0.76	
182	2.0	15.9	15.98	+ 7, − 3	....	....	....	
183	2.2	15.3	15.12	+14, −13	15.34	....	−0.22	
184	2.2	15.9	15.89	+13, − 6	15.66:	....	+0.23:	
185	2.7	16.1	16.20	+ 6, − 3	15.39	....	+0.81	29
185*a*	2.9	......	16.15	....	15.26	....	+0.89	
186	3.2	16.1	(15.93)	(+22)	(16.44)	....	(−0.51)	11, 30
187	2.3	15.3	15.10	+ 8, − 7	15.01	....	+0.09	31
188	3.1	16.3	16.06	+13, − 6	....	....	....	
189	3.6	14.7	14.56	+15, −15	13.96	....	+0.60	9
190	2.3	15.9	16.25	+ 1, − 1	....	....	....	
191	4.2	16.3	16.25	+ 6, − 3	(15.49)	....	(+0.76)	
192	2.3	16.1	16.24	....	15.31	....	+0.93	
193	2.3	14.5	14.48	+10, − 9	13.41	− 1, 0	+1.07	
194	2.1	13.9	13.58	− 2, + 1	12.16	− 8, + 7	+1.42	
195	2.0	16.1	15.90	− 8, + 8	15.26	....	+0.64	
196	2.0	15.8	15.91	+ 4, − 2	15.16	....	+0.75	
197	4.9	16.0	15.88	+17, − 9	(16.13)	....	(−0.25)	
199	3.4	13.9	13.66	0, 0	12.24	−14, +14	+1.42	
201	2.3	14.1	14.10	+20, −21	13.30	−20, +19	+0.80	
202	2.5	15.5	15.24	+ 9, − 8	15.42	....	−0.18	
204	2.0	15.8	15.99	+25, −13	15.29	....	+0.70	

TABLE IX—*Continued*

STAR	DISTANCE	LUD. MAG.	PHOTOGRAPHIC		PHOTO-VISUAL		COLOR-INDEX	NOTES
			Mag.	Residuals	Mag.	Residuals		
205...	2.7	15.5	15.24	+15, −14	14.54	....	+0.70	
207...	2.3	15.6	15.34	+ 2, − 3	14.59	....	+0.75	
208...	3.7	16.0	16.12	+17, −91	(16.25)	....	(−0.13)	
211...	2.0	15.9	16.26	0, 0	(16.19)	....	(+0.07)	
213...	2.1	15.5	15.02	+ 4, − 3	14.06	....	+ 0.96	
219...	5.0	15.4	15.31	−10, +10	14.32	....	+0.99	
220...	2.3	15.8	15.93	+ 7, − 4	15.39	....	+0.54	
221...	2.1	15.6	15.67	+15, −15	14.65	....	+1.02	
222...	2.2	......	12.54	− 8, + 7	13.06	− 2, + 2	−0.52	
224...	4.3	15.0	15.03	+11, −10	....	....	....	32
224*a*..	4.3	......	16.13	....	....	....	....	32
228...	5.1	16.5	16.53	....	(15.90)	....	(+0.63)	
229...	2.0	15.3	15.30	−12, +11	15.21	....	+0.09	9
230...	2.1	15.3	15.04	+ 2, − 1	14.32	....	+0.72	
234...	4.0	16.1	16.16	0, − 1	15.34	....	+0.82	33
236...	3.3	16.3	16.63	....	15.66	....	+0.97	
237...	2.9	15.9	16.04	+17, − 9	(15.84)	....	(+0.20)	34
238α..	2.0	......	15.34	− 4, + 3	14.83	....	+0.51	35
238β..	2.0	......	15.72	+ 5, − 4	15.42	....	+0.30	35
239...	3.0	15.6	15.58	− 6, + 6	14.65	....	+0.93	
240...	2.8	13.9	13.71	− 7, + 8	12.37	−11, +11	+1.34	
243...	2.8	14.1	14.15	+ 8, − 7	13.14	− 8, + 7	+1.01	
244...	2.9	13.9	13.80	+ 1, − 1	12.64	− 4, + 4	+1.16	
246...	2.0	15.8	15.49	+ 9, − 8	15.53	....	−0.04	
247...	2.5	15.1	15.37	−10, +11	15.13	....	+0.24	
252...	2.8	13.9	13.84	− 8, + 9	12.59	− 2, + 2	+1.25	
255...	5.2	16.1	15.95	....	15.26	....	+0.69	
258...	2.7	14.1	14.53	+11, −10	13.37	− 4, + 4	+1.16	36
260...	2.5	15.3	15.64	0, 0	14.65	....	+0.99	
263...	3.3	14.9	15.24	−16, +17	15.18	....	+0.06	
265...	3.1	16.3	16.24	....	15.63	....	+0.61	
267...	2.0	13.9	13.74	− 5, + 5	13.93	....	−0.19	
268...	2.6	14.1	14.46	+ 9, − 7	13.26	+ 3, − 3	+1.20	
271...	3.8	15.6	15.36	0, + 1	14.56	....	+0.80	
274...	2.0	14.7	15.00	− 3, + 3	13.98	....	+1.02	
275...	4.2	16.1	16.24	....	15.42	....	+0.82	
276...	4.1	14.9	14.82	+ 7, − 7	14.90	....	−0.08	
277...	2.4	14.7	15.08	− 2, + 2	15.13	....	−0.05	11
282...	4.3	15.8	15.92	+13, − 6	(16.44)	....	(−0.52)	37
283...	4.0	16.5	16.26	....	15.51	....	+0.75	
285...	4.7	16.3	16.74	....	(16.30)	....	(+0.44)	
291...	2.0	15.1	15.02	+13, −12	15.45	....	−0.43	11
300...	2.3	15.5	15.48	− 7, + 8	14.59	....	+0.89	
307...	3.8	15.2	15.08	+ 6, − 5	14.03	....	+1.05	
310...	2.0	15.3	15.30	0, + 1	15.34	....	−0.04	
313...	2.0	15.5	15.42	− 6, + 6	15.48	....	−0.06	
316...	2.7	13.9	13.68	0, + 1	12.52	− 1, + 1	+1.16	
326...	3.3	13.5	13.76	0, 0	13.88	+ 1, 0	−0.12	
328...	4.8	15.4	15.56	+20, −19	(15.74)	....	(−0.18)	
330...	3.3	15.4	15.26	+ 7, − 7	14.56	....	+0.70	
331...	4.0	16.3	16.13	− 2, 0	15.34	....	+0.79	
331*a*..	4.0	......	16.44	....	....	....	....	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
331*b*...	4.0	......	16.43:	....	....	....	....	7
335...	2.6	14.5	14.60	+11, −10	14.40	....	+0.20	
337...	2.0	14.5	14.73	+16, −15	13.71	−19, +19	+1.02	
337*b*..	2.0	......	16.21	− 2, + 1	15.53	....	+0.68	
338...	3.3	15.4	15.61	+18, −17	....	....	....	
341...	2.8	15.6	15.95	+ 5, − 3	15.18	....	+0.77	
348...	3.0	15.2	14.90	+10, − 9	13.98	− 6, + 3	+0.92	
351...	2.5	14.7	14.96	+13, −12	14.95	....	+0.01	
353...	2.1	13.9	13.84	+ 8, − 8	12.87	− 9, + 9	+0.97	
363...	3.8	15.0	15.18	+15, −15	15.34	....	−0.16	
364...	5.1	16.3	16.58	....	(16.75)	....	(−0.17)	38
365...	4.8	16.0	16.04	+ 7, − 4	15.45	....	+0.59	
379...	3.1	15.4	14.93	+10, − 9	14.19	....	+0.74	
382...	3.5	16.5	16.27	....	15.51	....	+0.76	
389...	4.9	16.5	16.56	....	(16.02)	....	(+0.54)	
391...	2.3	15.8	15.69	+25, −13	15.23	....	+0.46	
392...	2.3	15.9	15.81	+ 3, − 2	14.92	....	+0.89	
394...	3.2	16.5	16.20	....	15.63	....	+0.57	
395...	2.2	15.5	15.60	+24, −12	14.72	....	+0.88	
401α..	4.1	......	(17.27)	....	....	....	....	19, 39
401β..	4.1	......	(17.42)	....	....	....	....	19, 39
401*a*..	3.8	......	15.69	+18, − 9	15.04	....	+0.65	40
403...	5.1	13.9	14.08	+15, −15	13.01	− 2, + 2	+1.07	
413...	4.7	15.6	15.77	+25, −13	(15.85)	....	(−0.08)	41
414...	2.1	13.9	13.46	+ 4, − 4	12.08	− 3, + 3	+1.38	
415...	3.3	16.5	(17.09)	....	....	....	....	19, 42
418...	2.7	15.5	15.46	+ 9, − 9	15.53	....	−0.07	
423...	3.0	15.2	15.06	+ 8, − 7	14.14	....	+0.92	
434...	3.0	15.4	15.84	+ 3, − 1	(15.67)	....	(+0.17)	
443...	2.6	16.1	16.41	....	15.42	....	+0.99	
444...	3.7	15.6	15.36	0, + 1	14.56	....	+0.80	
450...	2.3	14.9	15.14	− 2, + 2	15.18	....	−0.04	31, 44
469...	4.4	15.4	15.26	+ 7, − 7	14.38	....	+0.88	
473...	3.4	16.0	16.02	− 2, + 1	15.05	....	+0.97	
497...	2.4	15.8	15.89	+ 5, − 2	15.08	....	+0.81	
499...	5.2	15.0	15.05	+13, −12	14.21	....	+0.84	
500...	2.3	15.3	15.44	−11, +12	15.42	....	+0.02	
505...	4.6	15.8	16.16	+ 5, − 3	15.13	....	+1.03	
507...	2.7	15.6	16.12	− 1, + 1	15.45	....	+0.67	
512...	3.0	15.6	15.15	+ 6, − 5	14.32	....	+0.83	
514...	2.4	15.8	16.21	+ 3, − 1	15.39	....	+0.82	
517...	2.8	15.6	15.48	+ 7, − 7	14.65	....	+0.83	
525...	2.0	15.3	15.40	− 4, + 4	14.54	....	+0.86	
527...	3.7	16.3	16.44	....	(16.72)	....	(−0.28)	45
528...	2.5	15.3	14.96	+ 4, − 3	14.21	....	+0.75	
529...	2.2	15.9	16.20	+ 6, − 3	(16.51)	....	(−0.31)	
530...	3.9	15.4	15.14	+19, −18	14.40	....	+0.74	
533...	2.4	16.2	16.27	....	(15.70)	....	(+0.57)	46
534...	3.7	15.0	15.11	+19, −18	15.11	....	0.00	
538...	3.1	15.6	15.63	+19, −19	15.61	....	+0.02	
542...	5.0	16.5	16.68	....	(16.59)	....	(+0.09)	
557...	6.5	15.0	15.11	−11, +23	13.98	....	+1.13	

TABLE IX—*Continued*

STAR	DISTANCE	LUD. MAG.	PHOTOGRAPHIC		PHOTO-VISUAL		COLOR-INDEX	NOTES
			Mag.	Residuals	Mag.	Residuals		
560...	2.0	15.9	15.84	+13, − 6	15.63:	....	+0.21:	
561...	2.4	15.8	16.17	+14, − 7	(16.25)	....	(−0.08)	
572...	3.2	16.5	16.88	....	(16.76)	....	(+0.12)	
582...	2.5	15.3	15.40	− 4, + 4	14.59	....	+0.81	
583...	5.5	16.3	16.31	....	(15.72)	....	(+0.59)	
585...	2.0	16.2	16.04	+17, − 9	15.18	....	+0.86	7
588...	3.1	15.0	15.05	+ 7, − 6	14.24	....	+0.81	
590...	5.0	16.3	16.20	....	15.42	....	+0.78	
596...	2.4	15.5	15.90	+ 2, − 1	14.85	....	+1.05	
599...	4.6	15.8	16.17	....	15.31	....	+0.86	
604...	6.1	15.6	15.82	+13, − 6	15.00	....	+0.82	
615...	2.8	16.1	16.20	....	15.66	....	+0.54	
618...	2.0	15.3	14.76	+17, −16	14.11	....	+0.65	
619...	2.2	15.9	15.96	+ 1, − 1	14.80	....	+1.16	
621...	2.4	15.3	15.20	+10, −10	14.27	....	+0.93	
627...	5.2	16.3	16.15	....	15.45	....	+0.70	
631...	3.5	16.5	(17.59)	....	....	....	....	19, 43
637...	2.3	15.3	15.46	− 5, + 6	14.56	....	+0.90	
638...	2.0	15.6	15.32	+10, − 9	14.56	....	+0.76	
641...	3.8	15.0	15.14	+25, −24	15.37	....	−0.23	
648...	3.3	15.0	15.16	+ 7, − 6	14.56	....	+0.60	
653...	3.4	15.0	15.18	+15, −15	15.37	....	−0.19	
664...	2.5	14.7	14.90	+16, −15	14.67	....	+0.23	
671...	2.0	15.6	15.83	+14, − 7	15.00	....	+0.83	
676...	4.2	16.0	16.21	− 2, + 1	(15.45)	....	(+0.76)	
677...	2.0	14.1	14.50	+11, −11	13.66	+11, − 5	+0.84	
681...	2.5	15.5	15.26	+10, −10	14.46	....	+0.80	
682...	2.6	15.5	15.77	+10, − 5	14.87	....	+0.90	47
686...	2.2	15.5	15.42	+19, −19	15.56	....	−0.14	
687...	5.2	13.9	13.76	− 6, + 7	12.96	0, 0	+0.80	
691...	3.0	15.6	15.36	+13, −13	14.67	....	+0.69	
701...	5.7	15.2	15.80	+ 2, − 1	15.42:	....	+0.38:	
711...	2.5	16.2	16.37	....	....	....	....	
713...	2.5	15.9	16.00	+16, − 8	15.34	....	+0.66	
716...	2.2	16.2	16.45	−16, + 8	(16.64)	....	(−0.19)	
726...	4.3	14.5	14.38	+ 7, − 6	13.57	+ 2, − 1	+0.81	
727...	2.2	15.9	16.17	+ 4, − 2	15.34	....	+0.83	
731...	2.3	15.3	15.08	+10, − 9	14.29	....	+0.79	
739...	3.6	16.5	16.13	+ 6, − 3	15.34	....	+0.79	
749...	3.7	16.0	16.03	+16, − 8	15.21	....	+0.82	
750...	2.5	16.1	16.43	....	15.66	....	+0.77	
752...	2.2	15.9	15.54	+10, −10	14.75	....	+0.79	
761...	2.0	14.7	14.78	− 2, + 3	14.56	....	+0.22	
762...	4.2	16.1	15.95	+ 7, − 3	14.75	....	+1.20	
763...	5.0	15.0	14.70	+13, −12	13.96	....	+0.74	
765...	2.3	16.2	16.27	....	15.61	....	+0.66	48
766...	3.5	14.5	14.35	+23, −22	13.39	+ 1, − 1	+0.96	
771...	3.2	16.3	16.03	....	15.73	....	+0.30	
773...	2.6	13.9	13.91	+ 9, − 8	13.08	− 4, + 5	+0.83	
775...	2.5	14.7	14.91	+ 1, − 1	15.05	....	−0.14	
775*a*..	2.5	......	16.10	+ 1, 0	....	....	....	
777...	2.4	13.9	13.87	+ 5, − 4	12.86	−4, + 5	+1.01	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
783...	3.0	14.9	14.72	+17, −16	13.88	− 9, + 5	+0.84	
787...	4.2	15.2	15.12	+ 3, − 2	14.27	....	+0.85	
789...	5.1	14.9	14.82	+ 4, − 4	13.95	−12, + 6	+0.87	9
790...	5.5	16.0	16.17	....	15.37	....	+0.80	
791...	4.8	16.3	16.10	....	15.58	....	+0.52	
796...	2.1	14.5	14.93	+ 1, 0	14.97	....	−0.04	
800...	4.7	14.7	14.49	+ 9, − 8	13.56	− 9, + 8	+0.93	
808...	4.5	15.2	15.30	− 6, + 7	15.13	....	+0.17	
809...	2.2	16.2	16.63	....	....	....	....	
810...	2.0	14.9	14.93	+ 1, 0	14.01	....	+0.92	
813...	4.1	16.3	16.22	....	15.48	....	+0.74	
814...	2.8	16.1	16.20	+ 6, − 3	15.37	....	+0.83	
818...	2.8	14.9	15.20	− 2, + 3	15.16	....	+0.04	
821...	2.4	16.1	15.91	+ 4, − 2	14.95	....	+0.96	
822...	5.7	14.9	15.09	0, 0	15.11	....	−0.02	
824...	2.0	15.1	15.09	+ 6, − 6	14.21	....	+0.88	
828...	2.6	15.5	15.71	+11, −11	14.75	....	+0.96	
831...	2.9	15.8	16.06	+13, − 6	15.26	....	+0.80	
833...	3.8	14.5	14.52	+ 9, − 9	13.99	− 5, + 2	+0.53	
834...	2.1	15.5	15.29	+ 7, − 6	14.54	....	+0.75	
835...	2.0	13.9	13.23	+16, −15	12.17	− 8, + 8	+1.06	
836...	2.8	15.8	15.75	+15, − 7	15.11	....	+0.64	
848...	2.8	13.9	14.05	+12, −12	13.10	− 5, + 6	+0.95	
849...	2.0	15.6	15.71	+31, −15	15.13	....	+0.58	
852...	2.3	14.9	15.17	+ 7, − 7	15.42	....	−0.25	
854...	2.4	16.2	16.88	....	....	....	....	
855...	2.4	15.6	15.47	+24, −24	14.75	....	+0.72	
857...	3.6	16.3	16.37	....	(15.82)	....	(+0.55)	
860...	5.5	16.5	16.53	....	(16.55)	....	(−0.02)	
861...	2.6	14.9	14.86	+ 8, − 8	14.95	....	−0.09	
862...	3.2	16.3	16.08	....	15.56	....	+0.52	7
866...	2.6	15.5	15.89	+ 1, 0	15.53	....	+0.36	
867...	2.9	15.3	15.11	+16, −15	14.62	....	+0.49	
869...	4.3	15.2	15.17	+ 7, − 7	14.29	....	+0.88	
870...	2.2	15.9	15.72	....	15.37	....	+0.35	
871...	3.1	14.7	14.62	+ 9, − 9	13.71	+ 5, − 2	+0.91	
872...	2.0	13.9	14.05	....	13.80	....	+0.25	49
873...	2.5	14.9	15.09	+ 6, − 6	15.13	....	−0.04	
876...	2.1	15.6	15.80	+17, − 8	(15.80)	....	(0.00)	
877...	2.5	13.9	13.98	+16, −15	13.06	0, 0	+0.92	
878...	4.0	16.1	16.10	....	(16.59)	....	(−0.49)	
879...	4.1	16.0	16.20	....	15.73:	....	+0.47:	
880...	2.2	14.5	15.14	+ 4, − 4	15.21	....	−0.07	
881...	4.5	16.5	....	....	....	....	....	50
882...	4.9	15.4	15.67	+12, −11	(15.74)	....	(−0.07)	
883...	4.1	15.4	15.39	+ 3, − 2	14.46	....	+0.93	
886...	4.3	15.4	15.38	+11, −11	14.70	....	+0.68	
888...	2.1	15.8	15.59	+31, −15	15.26	....	+0.33	51
890α..	2.4	......	14.93	+ 1, 0	14.59	....	+0.34	52
890β..	2.4	......	15.04	+ 5, − 5	15.11	....	−0.07	52
892...	3.9	15.0	15.55	+22, −21	15.39	....	+0.16	
893...	2.2	15.3	15.06	+ 3, − 3	15.18	....	−0.12	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
895...	2.7	16.2	16.37	....	....	....	....	
896...	3.4	15.4	15.65	+17, −17	15.05	....	+0.60	
897...	2.5	16.1	16.13	....	15.68	....	+0.45	
898...	3.7	16.3	16.71	....	(16.21)	....	(+0.50)	
899...	2.5	14.7	15.14	+ 4, − 4	15.16	....	−0.02	
903...	2.0	15.9	16.05	+ 3, − 2	15.39	....	+0.66	
905...	2.6	16.1	16.08	....	15.61	....	+0.47	
906...	2.0	15.5	15.39	+ 3, − 2	15.56	....	−0.17	53
909...	4.3	16.5	16.29	....	15.37	....	+0.92	
910...	3.5	16.3	16.35	....	....	....	....	
915...	5.3	16.5	16.58	....	....	....	....	
916...	2.0	15.3	15.33	+ 3, − 2	14.46	....	+0.87	53
917...	4.1	15.6	15.91	+ 9, − 5	15.48:	....	+0.43:	
919...	3.3	13.9	13.88	+ 6, − 5	12.92	−11, +11	+0.96	
920...	3.0	14.5	14.31	+21, −20	13.58	−12, +11	+0.73	
921...	2.7	14.1	14.16	+14, −14	13.16	− 6, + 5	+1.00	
922...	4.7	16.3	16.03	+11, − 5	15.31	....	+0.72	
924...	2.3	16.2	16.43	....	15.66:	....	+0.77:	
925...	2.4	14.7	15.07	+ 8, − 8	15.21	....	−0.14	
929...	2.1	15.6	15.63	+21, −11	14.80	....	+0.83	
930...	2.2	15.3	15.21	+ 6, − 5	14.49	....	+0.72	
932...	3.5	14.5	14.52	+ 9, − 9	13.93	....	+0.59	
933...	2.3	14.7	15.01	+11, −11	15.00	....	+0.01	
935...	2.2	14.5	14.50	+14, −14	14.51	....	−0.01	
942...	3.2	15.6	15.90	+ 7, − 4	(16.20)	....	(−0.30)	
943...	2.9	14.9	14.80	+23, −22	14.09	....	+0.71	
945...	2.4	15.5	15.21	+ 6, − 5	14.40	....	+0.81	
946...	2.7	14.7	14.71	+21, −21	14.01	....	+0.70	
947...	2.0	15.1	14.72	+22, −22	14.06	....	+0.66	
948...	3.0	13.9	13.94	+ 6, − 5	13.04	−13, +12	+0.90	
949...	4.4	16.0	16.24	....	....	....	....	
951...	2.3	15.9	16.29	....	15.45	....	+0.84	
953...	2.2	14.9	15.20	+ 4, − 4	15.23	....	−0.03	
954...	3.8	13.5	13.45	+14, −14	12.14	− 9, + 9	+1.31	
955...	4.3	14.5	14.44	+ 8, − 8	13.80	− 9, + 8	+0.64	
958...	2.0	15.5	15.56	+31, −15	14.90	....	+0.66	
959...	2.3	15.1	15.04	+11, −11	15.11	....	−0.07	
960...	2.0	15.9	16.17	....	15.63	....	+0.54	
961...	2.7	13.9	14.19	+11, −11	13.24	− 7, + 7	+0.95	
962...	2.7	14.9	15.73	+11, − 5	15.42	....	+0.31	
963...	3.2	15.2	15.23	+10, −10	15.42	....	−0.19	
964...	2.3	15.6	15.80	+15, − 8	15.18	....	+0.62	
965...	2.5	15.5	15.15	+12, −12	14.46	....	+0.69	
966...	2.6	16.1	16.29	....	15.66:	....	+0.63:	
967...	2.0	13.9	13.92	+13, −13	13.13	+ 2, − 2	+0.79	
968...	2.2	15.5	15.67	+ 7, − 7	15.73	....	−0.06	
969...	2.0	15.1	15.01	+ 8, − 8	15.23	....	−0.22	
971...	4.0	14.9	15.04	+11, −11	15.11	....	−0.07	
972...	5.9	14.9	14.88	+ 4, − 4	14.56	....	+0.32	
973...	2.3	13.9	13.35	+15, −14	12.08	− 4, + 5	+1.27	
974...	3.3	16.3	15.99	+ 9, − 4	15.48	....	+0.51	
976...	2.0	16.1	16.34	+ 2, − 1	....	....	....	

TABLE IX—*Continued*

STAR	DIS-TANCE	LUD. MAG.	PHOTOGRAPHIC		PHOTO-VISUAL		COLOR-INDEX	NOTES
			Mag.	Residuals	Mag.	Residuals		
977	2.0	16.2	16.24	+ 2, 0	....	....	....	
978	2.1	15.1	15.04	+ 2, − 1	15.39	....	−0.35	11
979α	2.3	......	14.65	+15, −15	14.01	....	+0.64	
979β	2.3	......	15.44	−11, +12	14.56	....	+0.88	
980	2.6	15.3	15.24	....	14.35	....	+0.89	58
981	2.2	15.1	15.02	+10, − 9	15.21	....	−0.19	
982	2.5	16.2	16.71	....	....	....	....	
983	2.1	15.6	15.99	− 7, + 4	15.71	....	+0.28	
984	2.8	13.9	14.48	+10, − 9	13.74	....	+0.74	
985	2.1	15.3	14.74	+18, −18	14.03	....	+0.71	8
986	2.2	15.5	15.33	+ 3, − 2	15.48	....	−0.15	
987	3.8	14.7	14.88	+ 6, − 7	14.87	....	+0.01	9
988	3.0	16.1	16.41	....	....	....	....	
989	2.5	15.3	15.60	+24, −12	15.66	....	−0.06	
990	3.3	16.5	16.27	....	15.63	....	+0.64	
991	5.4	14.5	14.34	+15, −15	13.37	− 9, + 9	+0.97	
992	2.4	16.2	16.26	....	....	....	....	
992a	2.4	......	16.60	....	....	....	....	
993	2.5	13.9	13.90	+ 7, − 7	13.92	− 5, + 6	−0.02	
994	3.1	14.7	14.90	+ 7, − 6	15.21	....	−0.31	11
995	5.4	15.4	15.75	+ 4, − 3	14.92	....	+0.83	7
996	3.1	15.2	15.30	+ 3, − 3	15.23	....	+0.07	
997	2.9	16.1	15.95	....	15.45	....	+0.50	
998	3.3	15.8	15.74	+ 3, − 2	14.97	....	+0.77	
999	2.3	16.1	16.09	+ 5, − 3	15.66	....	+0.43	
1000	2.4	16.2	16.13	....	15.66	....	+0.47	
1001	2.4	14.9	14.65	+21, −20	13.93	−16, + 8	+0.72	
1002	2.9	15.1	15.51	+ 4, − 3	15.66	....	−0.15	
1003	2.3	15.9	16.16	+ 5, − 3	15.58	....	+0.58	
1004α	2.4	......	15.02	+10, − 9	14.35	....	+0.67	
1004β	2.4	......	15.98	+13, − 6	....	....	....	
1005	2.4	15.5	15.43	− 1, + 1	14.62	....	+0.81	
1006	2.4	15.9	16.24	....	15.66:	....	+0.58:	
1007	3.9	14.9	15.05	+13, −12	15.21	....	−0.16	
1008	2.4	15.6	15.87	+ 8, − 4	15.76	....	+0.11	54
1009	3.3	15.2	15.04	+11, −11	14.35	....	+0.69	
1010	3.8	16.5	16.26	....	15.73:	....	+0.53:	
1011	3.9	15.6	15.23	+13, −13	14.56	....	+0.67	
1012	2.6	15.5	15.41	+23, −22	15.21	....	+0.20	
1013	2.7	15.9	16.17	− 1, 0	15.63	....	+0.54	
1014	4.6	15.4	15.53	+ 5, − 5	15.34	....	+0.19	9
1015	4.7	15.8	16.14	− 3, + 1	15.39	....	+0.75	
1016	2.9	14.9	14.66	+ 8, − 8	13.96	....	+0.70	
1017	2.5	15.8	16.05	+26, −13	15.53	....	+0.52	7
1018	4.6	15.6	15.64	+13, −12	15.00	....	+0.64	
1019	2.8	13.5	12.98	− 6, + 6	12.63	− 3, + 3	+0.35	
1020	2.8	15.9	16.01	+ 1, − 1	15.58	....	+0.43	
1021	3.0	16.1	15.30	− 6, + 7	14.54	....	+0.76	
1022	2.5	14.7	14.81	+11, −11	14.70	....	+0.11	8
1023	3.0	13.9	13.90	+ 7, − 7	13.02	− 3, + 4	+0.88	
1024	2.7	15.3	15.20	+10, −10	14.59	....	+0.61	
1025	3.6	15.8	15.91	+ 4, − 2	15.16	....	+0.75	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
1026	3.0	16.0	16.07	+14, − 7	15.71	....	+0.36	
1027	2.9	14.5	14.51	+10, −10	13.98	....	+0.53	
1028	2.7	15.9	16.22	....	....	....	....	
1029	3.5	16.1	16.11	− 3, + 2	15.63	....	+0.48	
1030	2.8	13.9	14.12	+11, −10	13.15	−3, + 3	+0.97	
1031	3.7	16.3	16.07	−20, +10	15.37	....	+0.70	
1032	2.8	14.7	14.45	+15, −15	13.77	....	+0.68	
1033	2.8	16.1	16.03	....	....	....	....	
1034	5.5	16.1	16.23	+ 3, − 1	....	....	....	
1035	4.2	15.6	15.85	+12, − 6	15.31	....	+0.54	
1036	2.8	14.9	15.05	+16, −15	15.21	....	−0.16	
1037	4.5	16.5	16.74	....	15.45	....	+1.29	
1038α	3.1	......	14.93	+10, − 9	14.97	....	−0.04	55
1038β	3.1	......	15.07	+14, −14	15.37	....	−0.30	55
1039	2.8	16.1	16.27	....	....	....	....	
1040	3.0	15.6	16.00	....	15.58	....	+0.42	56
1041	2.8	15.3	15.36	+ 3, − 2	15.48	....	−0.12	
1042	2.8	15.1	15.09	+ 6, − 6	15.37	....	−0.28	
1042*a*	2.8	......	16.31	....	15.63	....	+0.68	
1043	6.3	14.5	14.09	+14, −13	13.37	−7, + 7	+0.72	
1044	2.9	16.2	16.17	....	15.51	....	+0.66	
1045	3.0	15.0	15.00	+ 6, − 7	15.16	....	−0.16	9
1046	3.8	16.1	16.22	....	15.39	....	+0.83	
1047	3.0	16.0	15.84	+ 3, − 1	....	....	....	11
1048	3.9	14.9	14.86	+ 6, − 5	14.24	....	+0.62	
1049	3.5	16.3	16.27	....	....	....	....	
1050	5.5	15.0	14.85	+ 1, − 1	14.19	....	+0.66	
1051	4.0	14.7	14.65	+21, −20	13.93	−6, + 3	+0.72	
1052	3.7	15.8	(15.82)	....	(16.09)	....	(−0.27)	59
1053	7.0	14.7	14.85	+ 1, − 1	14.70	....	+0.15	
1054	3.7	16.3	(16.47)	....	(15.88)	....	(+0.59)	59
1055	3.3	14.7	14.76	+18, −18	14.70	....	+0.06	
1056	4.8	14.9	15.07	+17, −17	15.34	....	−0.27	
1057	3.9	15.8	15.86	+ 6, − 3	15.08	....	+0.78	
1058	5.0	15.6	15.92	+13, − 6	....	....	....	
1059	4.4	15.6	15.59	+18, −18	15.48	....	+0.11	9
1060	3.6	14.7	14.69	+17, −16	13.93	....	+0.76	
1061	3.5	16.1	15.84	+11, − 5	15.34	....	+0.50	
1062	3.5	15.2	15.05	+13, −12	14.32	....	+0.73	
1063	3.9	14.9	15.08	+13, −12	15.45	....	−0.37	11
1064α	5.0	......	16.80	....	....	....	....	
1064β	5.0	......	16.83	....	....	....	....	
1065	3.8	15.2	15.05	+13, −12	14.49	....	+0.56	
1066	6.0	15.4	15.60	+17, −16	(15.88)	....	(−0.28)	
1067	3.6	14.9	14.49	+ 9, − 8	14.35	....	+0.14	
1068	6.0	16.1	16.15	....	(15.38)	....	(+0.77)	
1069	4.0	16.3	16.26	....	....	....	....	
1070	3.7	15.4	15.68	+32, −16	15.66	....	+0.02	
1071	4.7	15.8	16.07	+ 1, − 1	(16.34)	....	(−0.27)	
1072	4.5	15.0	14.81	+19, −18	14.21	....	+0.60	
1073	5.7	13.9	13.75	− 4, + 4	12.82	+5, − 4	+0.93	
1074	4.0	15.8	15.61	+13, −13	15.23	....	+0.38	

TABLE IX—*Continued*

Star	Distance	Lud. Mag.	Photographic		Photo-visual		Color-Index	Notes
			Mag.	Residuals	Mag.	Residuals		
1075..	5.4	15.8	15.79	...	15.00	...	+0.79	
1076..	6.9	16.1	16.27	...	15.51	...	+0.76	
1077α.	4.0	...	15.73	+19, − 9	15.27	...	+0.46	
1077β.	4.0	...	15.85	+12, − 6	(16.13)	...	(−0.28)	
1078..	4.2	15.0	15.15	+12, −12	15.37	...	−0.22	
1079..	7.2	15.0	15.14	+ 4, − 4	15.23	...	−0.09	
1080..	6.0	15.2	15.12	+ 9, − 9	14.51	...	+0.61	
1081..	4.1	15.6	15.52	+ 3, − 4	14.65	...	+0.87	9
1082..	4.3	15.0	15.01	+ 5, − 5	15.05	...	−0.04	
1083..	4.7	16.3	16.10	...	15.39	...	+0.71	
1084..	4.4	16.3	16.48	...	15.56	...	+0.92	
1085..	5.8	16.0	15.80	+ 2, − 1	14.95	...	+0.85	
1086..	4.4	16.3	16.50	...	(16.72)	...	(−0.22)	
1087..	5.9	13.5	13.14	+ 3, − 3	12.74	0, − 1	+0.40	
1088..	4.3	15.4	15.69	+26, —13	15.58	...	+0.11	
1089..	6.4	16.3	16.44	...	...	...	...	
1090..	4.5	16.1	16.28	−14, + 7	15.51	...	+0.77	
1091..	6.4	15.6	15.34	− 4, + 3	14.56	...	+0.78	
1092..	4.5	15.8	15.88	+ 4, − 2	15.37	...	+0.51	
1093..	5.8	16.3	16.35	...	...	...	...	
1094..	4.5	15.8	15.98	+ 7, − 3	(16.50)	...	(−0.52)	
1095..	4.4	15.6	15.26	+13, −13	14.56	...	+0.70	
1096..	4.8	14.9	14.96	+ 1, 0	14.21	...	+0.75	
1097..	5.1	15.2	14.93	+10, − 9	14.21	...	+0.72	
1098..	4.7	16.0	16.16	+13, − 6	(16.56)	...	(−0.40)	
1099..	4.7	16.3	16.56	...	(15.93)	...	(+0.63)	
1100..	4.9	16.3	16.80	...	(16.64)	...	(+0.16)	
1101..	5.6	15.4	15.51	+15, −14	(15.70)	...	(−0.19)	
1102..	6.2	15.8	16.06	...	15.31	...	+0.75	
1103..	7.0	15.6	15.78	+12, − 6	15.23	...	+0.55	
1104..	5.1	15.2	15.30	− 6, + 7	15.53	...	−0.23	
1105..	5.3	15.4	15.67	+ 7, − 7	15.23	...	+0.44	
1106..	6.6	15.2	15.18	0, + 1	15.31	...	−0.13	
1107..	6.9	16.3	16.53	...	(16.02)	...	(+0.51)	
1108..	5.8	16.3	16.77	...	(16.73)	...	(+0.04)	
1109..	5.7	14.7	14.88	− 5, + 5	14.59	...	+0.29	
1110..	5.8	15.4	15.59	+15, −15	15.16	...	+0.43	
1111..	6.4	16.1	16.10	...	15.34	...	+0.76	
1112..	6.3	15.4	15.52	+19, −18	14.95	...	+0.57	
1113..	6.7	15.4	15.26	+ 7, − 7	15.31	...	−0.05	
1114..	7.6	14.5	14.63	− 5, + 5	13.90	...	+0.73	
1115..	8.1	15.8	15.71	+13, − 7	15.00	...	+0.71	
1116..	6.1	15.6	15.30	− 6, + 7	14.59	...	+0.71	
1117..	7.9	16.3	16.46	...	(15.93)	...	(+0.53)	
1118..	6.7	15.6	15.22	− 4, + 5	14.54	...	+0.68	
1119..	2.7	16.2	16.22	...	...	...	...	
1120..	2.7	16.2	16.17	+ 4, − 2	15.39	...	+0.78	
1121..	2.2	16.2	16.10	+ 4, − 2	...	...	...	7
1124..	2.0	16.2	16.08	0, 0	15.51	...	+0.57	
1126..	2.3	16.1	16.24	+15, − 4	...	...	...	
1127..	5.4	16.5	15.82	0, + 1	15.33	...	+0.49	57

NOTES TO THE CATALOGUE

1. Image irregular; probably several stars.

2. Image elongated; probably double star.

3. Image diffuse; may not be a single star.

4. Plate 1900, image diffuse.

5. Plate 1901, image diffuse.

6. Plate 1920, image diffuse.

7. Plate 1921, image diffuse.

8. Spectrum star; objects designated by this note are on or near the median lines along which the slit of the spectroscope was set in the spectroscopic work by Pease.

9. Remeasured because of large residuals for photographic magnitudes.

10. Remeasured because of large residuals for photo-visual magnitudes.

11. Remeasured because of exceptional color-index.

12. Plate 1901, image elongated; probably double.

13. Plate 1920, image elongated; probably double.

14. Plate 1921, image elongated; probably double.

15. Images of components overlapping.

16. Image of companion star overlapping.

17. The photo-visual magnitudes in parentheses were determined from the long-exposure plate, No. 2073, the stars being too faint for measurement on the standard photo-visual plates Nos. 1900 and 1920. The magnitudes and color-indices thus obtained are not used in the general statistical study of the cluster, but are discussed in Section XI.

18. The declination of this star is 14″ less than that catalogued by Ludendorff. This indicates either a mistake in the catalogue, or that the star is a variable of large range. The possibility of large proper motion is practically excluded, for the star appears in its present position on the eleven-hour exposure by Ritchey in June 1910. On that plate, however, a nineteenth-magnitude star appears in the catalogued position. The star now called 5 must be at the very limit of visibility on the Potsdam plates.

19. The photographic magnitudes in parentheses were determined from the long-exposure plate, No. 2072, the stars being too faint for measurement on the standard photographic plates, Nos. 1901 and 1921. In such cases it is probable that the star is a variable of considerable range or, more likely, that a defect in the plate was mistaken for a stellar image. But see stars 1052 and 1054, and note.

20. Measured on only one plate by Ludendorff and not at all by Scheiner.

21. Very bad image on Plate 1901.

22. Plate 1901, measures difficult; image of polar star very close; half-weight.

23. Plate 1921, defective image.

24. Very close triple; measures definite.

25. Probably double; image elongated on Plate 2073.

26. This is a new variable star. The magnitudes and color-indices refer to the consecutive plates, Nos. 1920 and 1921. The Pg. mag. on Plate 1901 is 16.10. See Table XXVI.

27. Plate 1920, Pv. mag. = 15.53:. Rejected as a possibly wrong identification since the observed image was at the limit of visibility.

28. "Perhaps close double" (Ludendorff). Clearly a single star on Mount Wilson plates.

29. From the long-exposure plates, Nos. 2072 and 2073: Pg. mag. = 16.41, Pv. mag. = 15.45, C. I. = +0.96.

30. The adopted Pg. mag. is derived from Plates 1921 (16.15) and 2072 (15.71). The large difference, which was checked by remeasuring, suggests variability. On Plate 2072 a close faint companion is visible.

31. This star is included in Barnard's list of yellow stars (*Astrophysical Journal*, **40,** 175, 1914) and, therefore, is possibly a variable star. See Section XII and footnote on p. 52.

32. Plate 1920, image of polar star superposed.

33. "Elongated" (Ludendorff). Definitely a single star on Mount Wilson plates.

34. Plate 2073, image slightly diffuse.

35. 238α = Scheiner 167; 238β = Scheiner 160.

36. Suspected of being a variable star by Barnard; see Section XII.

37. Plate 1920, Pv. mag. = 15.61:. Rejected as a wrong identification.

38. Plate 1920, Pv. mag. = 14.87. Rejected as a possibly wrong identification.

39. Judging from the observed faintness, one or both components of this close double must be variable through a large range, or else there is a mistake in the catalogued position. Ludendorff observed it on only one plate. Ritchey's plate shows the magnitudes about as given here. See the next note.

40. It is hardly likely that this star is meant for 401 (preceding note) as it is distant nearly 15″ from the catalogued position and is much brighter than the value assigned to 401. It is difficult to explain the absence of 401*a* from both Ludendorff's and Scheiner's catalogues except by assuming the star to be variable. Several neighboring stars of the same brightness are recorded in both catalogues. Ritchey's plate gives it the same relative magnitude as recorded here.

41. Plate 1920, Pv. mag. > 15.61, so that C. I. < +0.16.

42. Measured on only one plate by Ludendorff; variable or defect?

43. Variable?

44. No evidence of variability from Mount Wilson plates. In Barnard's list of yellow stars.

45. This may be a variable star. On Plate 2072 the Pg. mag. is 16.13.

46. Plate 2073, diffuse, may be double; two independent measures gave Pv. mag. 15.62 and 15.78.

47. Suspected by Barnard of being variable. My plates suggest variability also, but show no change in excess of 0.3 or 0.4 mag.

48. From Plates 2072 and 2073: Pg. mag. = 16.20, Pv. mag. = 15.51, C. I. = +0.69.

49. This is a new variable star. Magnitudes and color-index are derived from Plates 1920 and 1921. On Plate 1901, Pg. mag. = 14.74. Image invisible on 1900, therefore fainter than 14.2 Pv. mag.

50. Invisible on all my plates, therefore fainter than 17.5 Pg. mag. Probably error, or variable star.

51. "Probably two stars" (Ludendorff).

52. 890α = Scheiner 678; 890β = Scheiner 679. Overlapping slightly on Plate 1921, but all measures definite.

53. Plate 1901, 906 and 916 overlapping and difficult.

54. "Probably several stars" (Ludendorff). On Plate 1921 a close faint companion is south and following.

55. 1038α=Scheiner 775; 1038β=Scheiner 774.

56. "Somewhat elongated" (Ludendorff).

57. Plate 2073, Pv. mag.=15.45.

58. Plate 1921, image of polar star superposed. If this happens to be a variable of short period, the color-index, depending on non-successive plates, is erroneous.

59. Plate 1921, image of polar star superposed; too faint on Plate 1920.

F. *Identification positions of postscript stars.*—Three different systems of appended letters have been used in designating stars not included in Ludendorff's catalogue.[1] The "α-postscript stars" are the components of doubles, triplets, etc. The "*a*-postscript stars" are the unlisted companions and neighbors of catalogue stars. The "*x*-postscript stars" are uncharted faint objects in the vicinity of stars of catalogued position. For the last no positions have been determined (see Section XI for discussion). For the first two groups Table X contains positions sufficiently accurate for identification. They were determined from an enlarged glass positive, made from Plate 1921, upon which was imprinted a reseau with squares 30″ on a side. The position of any image is readily deduced from the positions already known for surrounding stars. The origin of co-ordinates is the same as for Ludendorff's catalogue.

TABLE X

CO-ORDINATES OF POSTSCRIPT STARS

Star	Δα	Δδ	Star	Δα	Δδ
41α........	−5′ 15″	−1′ 15″	401*a*......	−0′ 23″	−4′ 2″
41β........	−5 17	−1 13	775*a*......	+1 21	−2 34
41γ........	−5 14	−1 14	890α......	+1 59	−2 6
86α........	−3 34	−5 0	890β......	+2 2	−2 5
86β........	−3 36	−5 0	979α......	+2 45	+0 50
108α........	−3 6	+0 49	979β......	+2 43	+0 51
108β........	−3 6	+0 51	992*a*......	+3 0	−1 15
185*a*........	−2 11	+1 56	1004α......	+3 1	−0 53
224*a*........	−1 36	−4 17	1004β......	+3 3	−0 51
238α........	−1 35	+1 4	1038α......	+3 46	+1 8
238β........	−1 37	+1 6	1038β......	+3 43	+1 5
331α........	−0 50	−4 16	1042*a*......	+3 42	−0 7
331*b*........	−1 5	−3 59	1064α......	+4 38	−3 48
337*b*........	−0 58	+1 33	1064β......	+4 39	−3 43
401α........	−0 30	−4 13	1077α......	+5 11	+0 18
401β........	−0 31	−4 12	1077β......	+5 8	+0 20

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[1] See Subsection A above.

VI. STATISTICAL TABLES

A. *Basis of rejection.*—Of the 1275 stars originally listed in the main catalogue, the 1049 for which both color-index and magnitude are definitely reliable within the admitted limits of error are employed in the statistical discussion.[1] The rejections were made, with the help of the notes to the catalogue, on a definite basis, and in such a manner that no material selective effect on the remaining data results.

Of the 226 stars discarded:

64 are "tf" stars; that is, because of faintness no photo-visual magnitudes could be obtained on the standard plates, and none was derived from the long-exposure isochromatic plate No. 2073.

77 are "parentheses" stars; that is, because of faintness either the photographic or photo-visual magnitudes were not determined from the standard plates, but were derived from the long-exposure plates, No. 2073 or No. 2072; the resulting magnitudes and colors are of relatively low weight.

8 are stars missing or too diffuse to measure, probably representing errors in Ludendorff's catalogue, or possibly variables of great range.

These discarded stars are discussed to the extent of their value in Section XI. The reasons for omitting them from the general discussion are obvious.

Of the remaining rejected stars:

34 have magnitudes followed by colons or double colons, indicating considerable uncertainty of measurement.

43 are miscellaneous objects whose magnitudes or colors, according to the notes, may have very low weight.

The last group contains (1) about 15 stars whose images are diffuse, elongated, or irregular on two or more plates (Notes 1–7); (2) about 10 stars with overlapping images, which were not excluded for other reasons; (3) a few stars (Note 6) too faint to be checked on the second isochromatic plate, and (4) certain others for which there is a possibility of questionable results because of suspected variability, poor images, indefinite measurements, uncertainty of identification, or similar causes.

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[1] Cf. Section IV.

All of the rejections were made (without regard to the values of the color-indices) before any of the statistical arrangements were begun, and are presumably without prejudice. There will be a slight but unimportant preference of the rejections for the center of the cluster because of the difficulty of measures on the closer groups, but so far as color and magnitude are directly concerned all grades are affected equally.

B. *Arrangement of tabular data.*—In the main statistical tables (XI and XII), and in all discussions and subsidiary tables based upon them, the data are arranged according to the photo-visual magnitude. Because of certain instrumental limitations this practice will be followed in all the cluster studies. To obtain a given limiting magnitude the exposure time on the isochromatic plate must be eight or ten times that on the Seed "27" plate. It is always possible, therefore, to extend the photographic magnitudes far beyond the practicable limit of the isochromatic plates. In the present case all the stars visible on the best standard photo-visual plate (No. 1920) are also present on the photographic negative No. 1921, regardless of the color, but not vice versa.

A satisfactory discussion of all the data from the standpoint of photographic magnitude would be impossible because of a strong selection of those faint stars red enough to appear on the isochromatic plates. Blue stars of equal or even much brighter photographic magnitude might not appear on the latter photographs. But in dealing with the photo-visual magnitudes, down to the fainter limit of the plate, there is no color selection, and to that limit the material is complete. Photo-visual magnitudes are also an advantage for the comparison of cluster luminosity-curves with those of the brighter stars, which are always expressed in terms of visual magnitude.

Table XI contains for successive annular regions the relations between color-class, average color-index, average magnitude, and distance. Various tabular summaries of the data will appear in later sections.[1] Table XII contains for larger annular regions detailed information relative to the frequencies of magnitudes and colors. The tabulated quantities are numbers of stars. No

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[1] In particular, Table XVII.

TABLE XI

COLOR-CLASS AND DISTANCE

DISTANCE	TABULATED QUANTITY	COLOR-CLASS						ALL COLORS
		b	*a*	*f*	*g*	*k*	*m*	
0′.1.....	No. stars.....	0	0	0	1	1	2	4
	Av. C. I......	....	....	....	+1.12	+1.24	+1.82	+1.50
	Av. Pv. mag...	....	....	....	13.54	13.50	13.54	13.53
0′.2.....	No. stars.....	0	0	0	6	5	2	13
	Av. C. I......	....	....	....	+0.98	+1.39	+1.73	+1.25
	Av. Pv. mag...	....	....	....	13.98	13.41	12.57	13.55
0′.3.....	No. stars.....	0	0	3	4	7	5	19
	Av. C. I......	....	....	+0.66	+0.94	+1.40	+1.76	+1.28
	Av. Pv. mag...	....	....	14.59	14.62	13.22	13.62	13.84
0′.4.....	No. stars.....	0	0	6	5	11	3	25
	Av. C. I......	....	....	+0.58	+1.03	+1.39	+1.71	+1.16
	Av. Pv. mag...	....	....	14.46	14.06	13.44	13.13	13.77
0′.5.....	No. stars.....	0	5	6	15	10	3	39
	Av. C. I......	....	+0.21	+0.70	+0.96	+1.38	+1.76	+0.99
	Av. Pv. mag...	....	15.00	14.06	14.10	13.74	13.02	14.03
0′.6.....	No. stars.....	0	2	5	6	6	0	19
	Av. C. I......	....	+0.32	+0.62	+1.02	+1.34	....	+0.94
	Av. Pv. mag...	....	15.15	14.42	14.21	13.85	....	14.25
0′.7.....	No. stars.....	0	5	8	16	6	1	36
	Av. C. I......	....	+0.31	+0.64	+0.98	+1.34	+1.60	+0.89
	Av. Pv. mag...	....	14.26	14.71	13.98	13.68	12.45	14.09
0′.8.....	No. stars.....	1	10	9	15	8	2	45
	Av. C. I......	−0.07	+0.24	+0.60	+1.00	+1.34	+1.82	+0.82
	Av. Pv. mag...	15.11	15.04	14.51	14.11	13.44	11.95	14.25
0′.9.....	No. stars.....	1	7	6	11	1	0	26
	Av. C. I......	−0.11	+0.29	+0.63	+0.98	+1.48	....	+0.69
	Av. Pv. mag...	13.85	14.71	15.00	13.95	12.76	....	14.35

detailed description of either table is necessary. Their discussion will be taken up in the following sections.

VII. FREQUENCY OF COLOR-INDICES; ON THE ABSORPTION OF LIGHT IN SPACE

A. *Introductory remarks on the statistical studies.*—As the treatment of the results for Messier 13 is but one of several similar discussions planned for the future, it will be advisable to treat some parts of the present material in a rather summary fashion in

TABLE XI—*Continued*

Distance	Tabulated Quantity	Color-Class						All Colors
		b	*a*	*f*	*g*	*k*	*m*	
1′.0	No. stars	0	5	12	20	2	0	39
	Av. C. I.	....	+0.18	+0.64	+1.01	+1.36	....	+0.81
	Av. Pv. mag.	....	15.11	14.76	13.96	12.70	....	14.29
1′.1	No. stars	2	7	7	10	1	0	27
	Av. C. I.	−0.09	+0.20	+0.65	+0.99	+1.32	....	+0.63
	Av. Pv. mag.	15.26	14.81	14.53	14.66	14.43	....	14.70
1′.2	No. stars	4	6	7	23	1	1	42
	Av. C. I.	−0.14	+0.22	+0.67	+0.93	+1.34	+1.78	+0.71
	Av. Pv. mag.	15.06	15.48	14.72	13.94	12.96	11.92	14.33
1′.3	No. stars	2	8	12	15	3	0	40
	Av. C. I.	−0.24	+0.13	+0.59	+0.96	+1.48	....	+0.66
	Av. Pv. mag.	14.90	14.96	14.94	14.01	12.33	....	14.40
1′.4	No. stars	2	7	10	8	0	0	27
	Av. C. I.	−0.05	+0.12	+0.64	+0.90	....	....	+0.53
	Av. Pv. mag.	15.30	15.29	14.83	13.99	....	....	14.74
1′.5	No. stars	3	10	13	21	1	0	48
	Av. C. I.	−0.03	+0.19	+0.60	+0.92	+1.35	....	+0.63
	Av. Pv. mag.	14.78	15.20	15.00	14.01	12.29	....	14.54
1′.6	No. stars	4	4	8	8	1	0	25
	Av. C. I.	−0.15	+0.26	+0.62	+0.91	+1.42	....	+0.56
	Av. Pv. mag.	15.39	14.59	15.01	14.22	14.85	....	14.74
1′.7	No. stars	2	9	12	9	0	1	33
	Av. C. I.	−0.22	+0.16	+0.65	+0.94	....	+1.64	+0.57
	Av. Pv. mag.	15.56	14.84	14.71	14.11	....	12.08	14.55
1′.8	No. stars	2	3	9	10	1	0	25
	Av. C. I.	−0.27	+0.30	+0.65	+0.97	+1.27	....	+0.69
	Av. Pv. mag.	15.18	15.07	14.62	14.12	12.44	....	14.43
1′.9	No. stars	4	3	9	6	0	0	22
	Av. C. I.	−0.10	+0.21	+0.70	+0.95	....	....	+0.56
	Av. Pv. mag.	15.39	15.07	14.30	14.41	....	....	14.63

the first analysis of cluster magnitudes.[1] With the addition of analogous data from other clusters we can return to the results here tabulated and make a more thorough study than is now possible of the various subjects involved. In view of the proposed additions, therefore, the results are to a certain extent provisional, but in most

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[1] The first announcement of the results for Messier 13 was made at the California meeting of the American Astronomical Society, August 2–7, 1915.

TABLE XI—*Continued*

DISTANCE	TABULATED QUANTITY	COLOR-CLASS						ALL COLORS
		b	*a*	*f*	*g*	*k*	*m*	
2′.0	No. stars	7*	4	14	10	0	0	35
	Av. C. I.	−0.16	+0.22	+0.65	+0.92	......	......	+0.52
	Av. Pv. mag.	15.22	14.75	14.89	14.09	......	......	14.71
2′.1	No. stars	3	2	4	5	2	0	16
	Av. C. I.	−0.19	+0.30	+0.73	+0.94	+1.40	......	+0.65
	Av. Pv. mag.	15.17	15.48	14.20	13.90	12.12	......	14.19
2′.2	No. stars	10†	2	2	4	0	0	18
	Av. C. I.	−0.15	+0.20	+0.76	+0.96	......	......	+0.24
	Av. Pv. mag.	15.05	15.38	14.62	15.10	......	......	15.05
2′.3	No. stars	4	4	11	8	1	0	28
	Av. C. I.	−0.09	+0.06	+0.60	+0.90	+1.27	......	+0.53
	Av. Pv. mag.	15.34	15.15	15.11	14.51	12.08	......	14.87
2′.4	No. stars	5	2	7	9	0	0	23
	Av. C. I.	−0.11	+0.22	+0.66	+0.90	......	......	+0.55
	Av. Pv. mag.	15.26	15.18	14.95	14.47	......	......	14.85
2′.5	No. stars	6	4	7	7	0	0	24
	Av. C. I.	−0.08	+0.15	+0.64	+0.85	......	......	+0.44
	Av. Pv. mag.	15.06	14.86	15.11	14.30	......	......	14.82
2′.6	No. stars	3	3	3	5	1	0	15
	Av. C. I.	−0.08	+0.25	+0.66	+0.91	+1.20	......	+0.55
	Av. Pv. mag.	15.12	15.05	14.49	14.49	13.26	......	14.65
2′.7	No. stars	2	2	6	7	0	0	17
	Av. C. I.	−0.12	+0.30	+0.67	+0.99	......	......	+0.67
	Av. Pv. mag.	15.37	15.00	14.94	13.40	......	......	14.36
2′.8	No. stars	4	3	9	7	2	0	25
	Av. C. I.	−0.16	+0.25	+0.66	+0.97	+1.30	......	+0.62
	Av. Pv. mag.	15.27	14.15	15.01	14.07	12.48	......	14.48
2′.9	No. stars	2	0	7	5	0	0	14
	Av. C. I.	−0.18	......	+0.61	+1.02	......	......	+0.64
	Av. Pv. mag.	15.58	......	14.66	13.99	......	......	14.55

* Contains one star with C. I. = −0.43.

† Contains one star with C. I. = −0.52.

cases they are to be so considered only because the later data should render more secure the present interpretations of the observations.

It has been found convenient in discussing the results to divide the cluster into ten concentric annular regions and seven intervals of brightness. Data relative to these divisions are contained in Tables XIII and XIV, in addition to the more detailed information in Tables XI and XVI.

TABLE XI—*Continued*

DISTANCE	TABULATED QUANTITY	COLOR-CLASS						ALL COLORS
		b	*a*	*f*	*g*	*k*	*m*	
3′.0	No. stars	4	4	12	12	0	0	32
and	Av. C. I.	−0.20	+0.21	+0.66	+0.92	....	....	+0.59
3′.1	Av. Pv. mag.	15.18	15.44	14.76	13.78	....	....	14.53
3′.2	No. stars	2	6	11	7	0	0	26
and	Av. C. I.	−0.16	+0.13	+0.63	+0.91	....	....	+0.53
3′.3	Av. Pv. mag.	14.65	15.00	15.03	14.21	....	....	14.77
3′.4	No. stars	2	1	8	6	1	0	18
and	Av. C. I.	−0.20	+0.28	+0.62	+0.92	+1.42	....	+0.65
3′.5	Av. Pv. mag.	15.26	15.05	14.99	14.53	12.24	....	14.72
3′.6	No. stars	0	4	9	4	0	0	17
and	Av. C. I.	....	+0.05	+0.67	+0.86	....	....	+0.57
3′.7	Av. Pv. mag.	....	15.12	14.74	14.81	....	....	14.85
3′.8	No. stars	4	2	11	4	1	0	22
and	Av. C. I.	−0.23	+0.08	+0.66	+0.92	+1.31	....	+0.52
3′.9	Av. Pv. mag.	15.34	15.08	14.57	14.20	12.14	....	14.58
4′.0	No. stars	9	1	8	6	1	0	25
and	Av. C. I.	−0.15	+0.38	+0.68	+0.87	+1.32	....	+0.44
4′.1	Av. Pv. mag.	15.28	15.23	14.97	14.68	12.42	....	14.92
4′.2	No. stars	3	2	3	6	1	0	15
and	Av. C. I.	−0.14	+0.13	+0.62	+0.89	+1.20	....	+0.55
4′.3	Av. Pv. mag.	15.14	15.12	14.60	14.75	14.75	....	14.85
4′.4	No. stars	1	2	7	2	2	0	14
and	Av. C. I.	−0.18	+0.14	+0.66	+0.90	+1.35	....	+0.66
4′.5	Av. Pv. mag.	15.18	15.30	14.84	14.97	13.74	....	14.79
4′.6	No. stars	1	5	8	6	0	0	20
and	Av. C. I.	−0.13	+0.14	+0.70	+0.95	....	....	+0.59
4′.7	Av. Pv. mag.	15.31	15.25	15.16	14.59	....	....	15.02
4′.8	No. stars	2	1	5	0	0	0	8
and	Av. C. I.	−0.18	+0.33	+0.61	....	....	....	+0.38
4′.9	Av. Pv. mag.	15.28	15.11	14.75	....	....	....	14.93

B. *The frequency of color-class.*—The totals, combining all magnitudes, for the various parts of Table XII are brought together in Table XV. The tabulated quantities are numbers of stars. The successive lines of the table, with the exception of the next to last, are represented diagrammatically in Figs. 3, 4, and 5, with numbers of stars as ordinates and color-classes as abscissae. Fig. 3 shows the frequency of color for two inner regions of the cluster; Fig. 4 that for three exterior regions. Later it will be shown that

TABLE XI—*Continued*

DISTANCE	TABULATED QUANTITY	COLOR-CLASS						ALL COLORS
		b	*a*	*f*	*g*	*k*	*m*	
5′.0	No. stars.....	2	1	6	6	0	0	15
and	Av. C. I......	−0.14	+0.14	+0.71	+0.95	....	....	+0.65
5′.1	Av. Pv. mag..	15.33	15.56	14.85	14.10	....	....	14.66
5′.2	No. stars.....	1	0	5	5	0	0	11
and	Av. C. I......	−0.05	....	+0.60	+0.91	....	....	+0.68
5′.3	Av. Pv. mag...	15.11	....	14.96	13.76	....	....	14.43
5′.4	No. stars.....	0	0	4	7	0	0	11
and	Av. C. I......	....	....	+0.67	+0.92	....	....	+0.83
5′.5	Av. Pv. mag...	....	....	15.00	14.54	....	....	14.71
5′.6	No. stars.....	3	3	1	1	0	0	8
and	Av. C. I......	−0.10	+0.20	+0.70	+0.93	....	....	+0.24
5′.7	Av. Pv. mag...	15.18	14.89	15.34	12.82	....	....	14.80
5′.8	No. stars.....	0	1	2	1	0	0	4
and	Av. C. I......	....	+0.32	+0.42	+0.85	....	....	+0.50
5′.9	Av. Pv. mag..	....	14.56	13.95	14.95	....	....	14.35
6′.0	No. stars.....	0	0	7	4	1	0	12
to	Av. C. I......	....	....	+0.70	+0.92	+1.45	....	+0.84
6′.4	Av. Pv. mag...	....	....	14.66	14.76	13.69	....	14.61
6′.5	No. stars.....	3	1	4	2	0	0	10
to	Av. C. I......	−0.10	+0.35	+0.70	+1.00	....	....	+0.48
6′.9	Av. Pv. mag...	15.20	14.59	14.99	14.19	....	....	14.85
	No. stars.....	3	2	5	2	0	0	12
>6′.9	Av. C. I......	−0.11	+0.20	+0.69	+0.98	....	....	+0.46
	Av. Pv. mag...	15.12	13.75	14.64	14.46	....	....	14.58

the inner and outer regions must be considered separately, so that a curve for the whole cluster would have little meaning.

A single curve for the whole outer portion, Regions V–X inclusive, appears in Fig. 5 together with the frequency-curve of spectral types at large, the data for which are taken from Parkhurst's *Yerkes Actinometry*.[1] To plot Parkhurst's data the spectral types were interpreted directly as color-classes, and numbers given by him were divided by two. Though differing in detail, these curves are similar in two important characters, notwithstanding considerable lack of homogeneity in the Yerkes results as regards distance and absolute brightness—each curve has a maximum for

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[1] *Astrophysical Journal*, **36**, 218, 225, 1912.

TABLE XII

FREQUENCY OF COLORS AND MAGNITUDES

REGIONS I AND II—DISTANCE FROM CENTER 0′.0 TO 0′.9

LIMITS OF PHOTO-VISUAL MAGNITUDE	COLOR-CLASS												ALL COLORS
	*b*0 to *b*5	*b*5 to *a*0	*a*0 to *a*5	*a*5 to *f*0	*f*0 to *f*5	*f*5 to *g*0	*g*0 to *g*5	*g*5 to *k*0	*k*0 to *k*5	*k*5 to *m*0	*m*0 to *m*5	*m*5 to *m*9	
11.80–.99...	....	....	....	....	....	....	....	....	....	....	....	2	2
12.00–.19...	....	....	....	....	....	....	....	....	....	....	....	1	1
.20–.39...	....	....	....	....	....	....	....	....	....	2	....	1	3
.40–.59...	....	....	....	....	....	....	....	1	2	2	2	....	7
.60–.79...	....	....	....	....	....	....	1	1	2	3	2	....	9
.80–.99...	....	....	....	....	....	....	....	....	2	2	2	....	6
13.00–.19...	....	....	....	....	....	1	4	3	3	3	....	....	14
.20–.39...	....	....	....	....	....	....	....	1	2	....	3	....	6
.40–.59...	....	....	....	....	....	3	1	3	1	6	1	....	15
.60–.79...	....	....	....	1	....	1	4	4	3	1	1	....	15
.80–.99...	....	1	....	1	1	1	3	2	4	3	....	....	16
14.00–.19...	....	....	....	1	1	3	8	8	1	1	....	1	24
.20–.39...	....	....	....	....	2	2	11	5	4	1	1	....	26
.40–.59...	....	....	1	2	4	2	3	2	4	1	....	....	19
.60–.79...	....	....	1	7	3	5	4	1	1	....	....	....	22
.80–.99...	....	....	1	2	3	2	....	2	....	....	1	....	11
15.00–.19...	....	1	1	5	3	1	4	....	1	....	....	....	16
.20–.39...	....	....	3	....	....	2	2	....	....	....	....	....	7
.40–.59...	....	....	....	2	....	3	....	1	....	....	....	....	6
.60–.79...	....	....	1	....	....	....	....	....	....	....	....	....	1
.80–.99...	....	....	....	....	....	....	....	....	....	....	....	....	....
Total...	....	2	8	21	17	26	45	34	30	25	13	5	226
REGIONS III AND IV—DISTANCE FROM CENTER 1′.0–1′.9													
11.80–.99...	....	....	....	....	....	....	....	1	....	....	1	....	2
12.00–.19...	....	....	....	....	....	....	....	....	....	1	1	....	2
.20–.39...	....	....	....	....	....	....	....	....	2	....	....	....	2
.40–.59...	....	....	....	....	....	....	....	3	1	1	....	....	5
.60–.79...	....	....	....	....	1	....	....	5	1	1	....	....	8
.80–.99...	....	....	....	....	....	....	2	3	1	1	....	....	7
13.00–.19...	....	....	....	....	....	....	2	3	....	....	....	....	5
.20–.39...	....	....	....	....	1	1	2	3	....	....	....	....	7
.40–.59...	....	....	....	....	1	....	7	6	....	....	....	....	14
.60–.79...	....	....	....	2	....	....	5	2	....	....	....	....	9
.80–.99...	....	....	....	....	....	7	5	3	....	....	....	....	15
14.00–.19...	....	....	....	1	1	3	6	4	....	....	....	....	15
.20–.39...	1	....	1	....	2	9	19	3	....	....	....	....	35
.40–.59...	....	1	1	3	5	6	11	3	1	....	....	....	31
.60–.79...	1	1	10	2	4	8	7	3	....	....	....	....	36
.80–.99...	....	2	4	3	1	3	3	3	....	....	....	....	19
15.00–.19...	1	4	7	4	6	15	8	2	....	....	....	....	47
.20–.39...	1	4	7	5	3	10	4	....	....	....	....	....	34
.40–.59...	....	4	4	....	2	5	1	....	....	....	....	....	16
.60–.79...	2	3	1	2	5	....	....	....	....	....	....	....	13
.80–.99...	....	....	4	1	....	....	1	....	....	....	....	....	6
Total...	6	19	39	23	32	67	83	47	6	4	2	....	328

TABLE XII—*Continued*

REGIONS V AND VI—DISTANCE FROM CENTER 2′.0 TO 2′.9

Limits of Photo-visual Magnitude	Color-Class												All Colors
	bo to *b5*	*b5* to *ao*	*ao* to *a5*	*a5* to *fo*	*fo* to *f5*	*f5* to *go*	*go* to *g5*	*g5* to *ko*	*ko* to *k5*	*k5* to *mo*	*mo* to *m5*	*m5* to *m9*	
11.80–.99...	....	....	....	....	....	....	....	....	....	....	....	....	....
12.00–.19...	....	....	....	....	....	....	....	1	2	1	....	....	4
.20–.39...	....	....	....	....	....	....	....	....	1	....	....	....	1
.40–.59...	....	....	....	....	....	....	....	2	1	....	....	....	3
.60–.79...	....	....	....	1	....	....	1	1	....	....	....	....	3
.80–.99...	....	....	....	....	....	....	1	1	....	....	....	....	2
13.00–.19...	1	....	....	....	....	1	5	2	....	....	....	....	9
.20–.39...	....	....	....	....	....	....	3	1	1	....	....	....	5
.40–.59...	....	....	....	....	....	....	....	1	....	....	....	....	1
.60–.79...	....	....	....	....	....	2	2	1	....	....	....	....	5
.80–.99...	....	2	....	1	1	5	1	1	....	....	....	....	11
14.00–.19...	....	....	....	....	....	6	2	1	....	....	....	....	9
.20–.39...	....	....	....	....	....	3	3	1	....	....	....	....	7
.40–.59...	....	1	....	4	....	8	8	....	....	....	....	....	21
.60–.79...	....	....	1	2	1	2	7	1	....	....	....	....	14
.80–.99...	....	2	1	....	1	3	4	2	....	....	....	....	13
15.00–.19...	....	12	4	1	1	5	3	1	....	....	....	....	27
.20–.39...	4	8	2	3	2	10	7	....	....	....	....	....	36
.40–.59...	2	9	1	3	6	3	3	....	....	....	....	....	27
.60–.79...	....	5	1	1	8	2	....	....	....	....	....	....	17
.80–.99...	....	....	....	....	....	....	....	....	....	....	....	....	....
Total...	7	39	10	16	20	50	50	17	5	1	....	....	215

REGIONS VII AND VIII—DISTANCE FROM CENTER 3′.0 TO 4′.9

Limits of Photo-visual Magnitude	*bo* to *b5*	*b5* to *ao*	*ao* to *a5*	*a5* to *fo*	*fo* to *f5*	*f5* to *go*	*go* to *g5*	*g5* to *ko*	*ko* to *k5*	*k5* to *mo*	*mo* to *m5*	*m5* to *m9*	All Colors
11.80–.99...	....	....	....	....	....	....	....	....	....	....	....	....	....
12.00–.19...	....	....	....	....	....	....	....	....	1	1	....	....	2
.20–.39...	....	....	....	....	....	....	....	....	....	1	....	....	1
.40–.59...	....	....	....	....	....	....	....	1	1	....	....	....	2
.60–.79...	....	....	....	....	....	....	....	1	....	....	....	....	1
.80–.99...	....	....	....	....	....	....	1	1	....	....	....	....	2
13.00–.19...	....	....	....	....	....	....	3	....	....	....	....	....	3
.20–.39...	....	....	....	....	....	....	....	1	....	....	....	....	1
.40–.59...	....	....	....	....	....	1	3	....	....	....	....	....	4
.60–.79...	....	....	....	....	....	....	1	....	....	....	....	....	1
.80–.99...	....	1	....	....	5	7	4	....	....	....	....	....	17
14.00–.19...	....	....	....	....	1	1	1	1	....	....	....	....	4
.20–.39...	....	....	1	....	....	6	6	2	....	....	....	....	15
.40–.59...	....	....	1	....	2	10	6	....	....	....	....	....	19
.60–.79...	....	....	3	....	....	4	4	....	1	....	....	....	12
.80–.99...	....	2	2	....	....	1	1	....	....	....	....	....	6
15.00–.19...	....	6	5	2	1	12	4	1	....	....	....	....	31
.20–.39...	6	9	4	2	7	11	7	....	....	....	....	....	46
.40–.59...	1	2	3	1	5	4	2	1	1	....	....	....	20
.60–.79...	1	....	2	2	2	2	1	....	....	....	....	....	10
.80–.99...	....	....	....	....	....	....	....	....	....	....	....	....	....
Total...	8	20	21	7	23	59	44	9	4	2	....	....	197

TABLE XII—*Concluded*

REGIONS IX AND X—DISTANCE FROM CENTER ≧5′.0

LIMITS OF PHOTOGRAPHIC MAGNITUDE	COLOR-CLASS												ALL COLORS
	*b*0 to *b*5	*b*5 to *a*0	*a*0 to *a*5	*a*5 to *f*0	*f*0 to *f*5	*f*5 to *g*0	*g*0 to *g*5	*g*5 to *k*0	*k*0 to *k*5	*k*5 to *m*0	*m*0 to *m*5	*m*5 to *m*9	
11.80–.99	....	....	....	....	....	....	....	....	....	....	....	....	....
12.00–.19	....	....	....	....	....	....	....	....	....	....	....	....	....
.20–.39	....	....	....	....	....	....	....	....	....	....	....	....	....
.40–.59	....	....	....	....	....	....	....	....	....	....	....	....	....
.60–.79	....	....	....	....	1	....	....	....	....	....	....	....	1
.80–.99	....	....	....	....	....	....	2	....	....	....	....	....	2
13.00–.19	....	....	....	....	....	....	....	1	....	....	....	....	1
.20–.39	....	....	....	....	....	1	1	....	....	....	....	....	2
.40–.59	....	....	....	....	....	....	3	....	....	....	....	....	3
.60–.79	....	....	....	....	....	....	....	....	....	1	....	....	1
.80–.99	....	....	....	1	....	2	1	1	....	....	....	....	5
14.00–.19	....	....	....	....	....	2	....	....	....	....	....	....	2
.20–.39	....	....	....	....	....	1	4	1	....	....	....	....	6
.40–.59	....	....	....	3	1	6	2	4	....	....	....	....	16
.60–.79	....	....	2	....	....	....	1	....	....	....	....	....	3
.80–.99	....	1	....	....	1	1	3	....	....	....	....	....	6
15.00–.19	....	6	....	....	1	2	3	....	....	....	....	....	12
.20–.39	....	4	1	....	3	7	1	....	....	....	....	....	16
.40–.59	1	....	2	....	....	4	....	....	....	....	....	....	7
.60–.79	....	....	....	....	....	....	....	....	....	....	....	....	....
.80–.99	....	....	....	....	....	....	....	....	....	....	....	....	....
Total	1	11	5	4	7	26	21	7	....	1	....	....	83

REGIONS V TO X INCLUSIVE—DISTANCE FROM CENTER ≧2′.0

LIMITS OF PHOTOGRAPHIC MAGNITUDE	*b*0 to *b*5	*b*5 to *a*0	*a*0 to *a*5	*a*5 to *f*0	*f*0 to *f*5	*f*5 to *g*0	*g*0 to *g*5	*g*5 to *k*0	*k*0 to *k*5	*k*5 to *m*0	*m*0 to *m*5	*m*5 to *m*9	ALL COLORS
11.80–.99	....	....	....	....	....	....	....	....	....	....	....	....	....
12.00–.19	....	....	....	....	....	....	....	1	3	2	....	....	6
.20–.39	....	....	....	....	....	....	....	....	1	1	....	....	2
.40–.59	....	....	....	....	....	....	....	3	2	....	....	....	5
.60–.79	....	....	....	1	1	....	1	2	....	....	....	....	5
.80–.99	....	....	....	....	....	....	4	2	....	....	....	....	6
13.00–.19	1	....	....	....	....	1	8	3	....	....	....	....	13
.20–.39	....	....	....	....	....	1	4	2	1	....	....	....	8
.40–.59	....	....	....	....	....	1	6	1	....	....	....	....	8
.60–.79	....	....	....	....	....	2	3	1	....	1	....	....	7
.80–.99	....	3	....	2	6	14	6	2	....	....	....	....	33
14.00–.19	....	....	....	....	1	9	3	2	....	....	....	....	15
.20–.39	....	....	1	....	....	10	13	4	....	....	....	....	28
.40–.59	....	1	1	7	3	24	16	4	....	....	....	....	56
.60–.79	....	....	6	2	1	6	12	1	1	....	....	....	29
.80–.99	....	5	3	....	2	5	8	2	....	....	....	....	25
15.00–.19	....	24	9	3	3	19	10	2	....	....	....	....	70
.20–.39	10	21	7	5	12	28	15	....	....	....	....	....	98
.40–.59	4	11	6	4	11	11	5	1	1	....	....	....	54
.60–.79	1	5	3	3	10	4	1	....	....	....	....	....	27
.80–.99	....	....	....	....	....	....	....	....	....	....	....	....	....
Total	16	70	36	27	50	135	115	33	9	4	....	....	495

blue and for yellow stars, and the limits of color are practically the same for both.

C. *On the absorption of light in space.*—Probably the most remarkable feature of the distribution of color-classes is not the relative paucity of *a*'s and early *f*'s, nor the great extent of the range in color,[1] but rather the highly significant fact that there are any negative color-indices at all. Let us consider only that portion of the cluster, represented by the last line of Table XV, which is at a greater distance from the center than 1′.9. Of the

TABLE XIII

AREAL DIVISIONS OF THE CLUSTER

Region	Limits of Distance	Middle Distance	No. of Stars	Region	Limits of Distance	Middle Distance	No. of Stars
I.........	0′.00–0′.45	0′.22	61	VI.........	2′.46–2′.95	2′.7	95
II.........	0.46–0.95	0.7	165	VII.........	2.96–3.95	3.45	115
III.........	0.96–1.45	1.2	175	VIII.........	3.96–4.95	4.45	82
IV.........	1.46–1.95	1.7	153	IX.........	4.96–5.95	5.45	49
V.........	1.96–2.45	2.2	120	X.........	≧5.96	(6.8)	34

TABLE XIV

MAGNITUDE DIVISIONS OF THE CLUSTER

Interval	Limits of Pv. Mag.	Median Mag.	No. Stars	Interval	Limits of Pv. Mag.	Median Mag.	No. Stars
α.....	11.80–12.59	12.20	37	ε.....	14.40–14.99	14.70	248
β.....	12.60–13.19	12.90	73	ζ.....	15.00–15.59	15.30	348
γ.....	13.20–13.79	13.50	89	η.....	15.60–16.19	15.90	47
δ.....	13.80–14.39	14.10	207				

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[1] The detection of a wide range of color in Messier 13 by Barnard is the pioneer work along this line. His most recent paper on the subject (*Astrophysical Journal*, **40**, 173, 1914) gives the result of a comparison of the relative sizes of star-images on a plate made with the Potsdam photographic refractor and on one made with the Yerkes refractor using a yellow color-filter. The work is necessarily only qualitative, but the evidence for different types of color is conspicuous and conclusive. Barnard gives two lists, containing a total of 144 stars. The first contains those relatively yellow, the second those blue relative to the average of all the cluster stars. Using the color-indices in Section V, the average color-index for his two groups has been determined, first excluding a few stars that could not be positively identified, or were found to be double, or were suspected of variation. The result is as follows:

36 blue stars, Av. C. I. +0.05, largest value +0.49, smallest −0.52
96 yellow " +0.77 +1.20 +0.33

495 stars, more than 17 per cent have negative color-indices, and of these one-fifth are bluer than represented by color-class *b5*. Is this result attributable to a systematic error?

Suppose the Harvard Polar Sequence had been used instead of the Mount Wilson Polar Standards. A curve showing the relation between the two scales is given in the first paper of this series.[1] As the range of photographic magnitude considered in the cluster extends from 12.5 to 16.5, the average correction to reduce to the Harvard scale would be nearly three-tenths of a magnitude in the sense of increasing the brightness. The color-index, $Pg-Pv$, for all stars would therefore be decreased by this same amount, which would mean that in the outer region of the cluster there is scarcely a star redder than the sun. Thirty per cent of the stars would have negative color-indices, some with values less than −0.7. Negative color-indices of this magnitude have never been observed. No important modification of this result is obtained by using the Harvard scale as revised by Seares. A decrease of a quarter of a magnitude in all color-indices is still demanded, which would again call for unknown values of negative color-indices.

TABLE XV

FREQUENCY OF COLORS

REGION	COLOR-CLASS												ALL COLORS
	b0 to *b5*	*b5* to *a0*	*a0* to *a5*	*a5* to *f0*	*f0* to *f5*	*f5* to *g0*	*g0* to *g5*	*g5* to *k0*	*k0* to *k5*	*k5* to *m0*	*m0* to *m5*	*m5* to *m9*	
I and II	....	2	8	21	17	26	45	34	30	25	13	5	226
III and IV	6	19	39	23	32	67	83	47	6	4	2	....	328
V and VI	7	39	10	16	20	50	50	17	5	1	....	....	215
VII and VIII	8	20	21	7	23	59	44	9	4	2	....	....	197
IX and X	1	11	5	4	7	26	21	7	....	1	....	....	83
I to X	22	91	83	71	99	223	243	114	45	33	15	5	1049
V to X	16	70	36	27	50	135	115	33	9	4	....	....	495

The foregoing considerations throw light upon the relation of the Mount Wilson and Harvard photographic scales, but they fail to eliminate the class *b* colors from the cluster. We are left with the conviction that, so far as present data go, normal negative color-indices exist in the Hercules cluster in large numbers.

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[1] *Mt. Wilson Contr.*, No. 115, p. 16.

It is well known that a condition of this nature cannot exist if there is appreciable molecular scattering of light in space. Suppose we use the most recent value derived by Kapteyn for the change in color-index due to space absorption, namely, $d=+0.003$ mag. per unit of stellar distance ($\pi=0''.1$).[1] It will be shown later that the parallax of the Hercules cluster cannot reasonably be greater than $+0''.0001$, and it may be but one-tenth of that amount. The absorption, then, should be of the order of three magnitudes at least, and the minimum color-index $+2.5$. Suppose we use the very recent and smaller value,[2] $d=+0.0015$, determined by Van Rhyn from a consideration of the stars included in the *Yerkes Actinometry*. Even then all the color-indices in Messier 13 should exceed a magnitude.

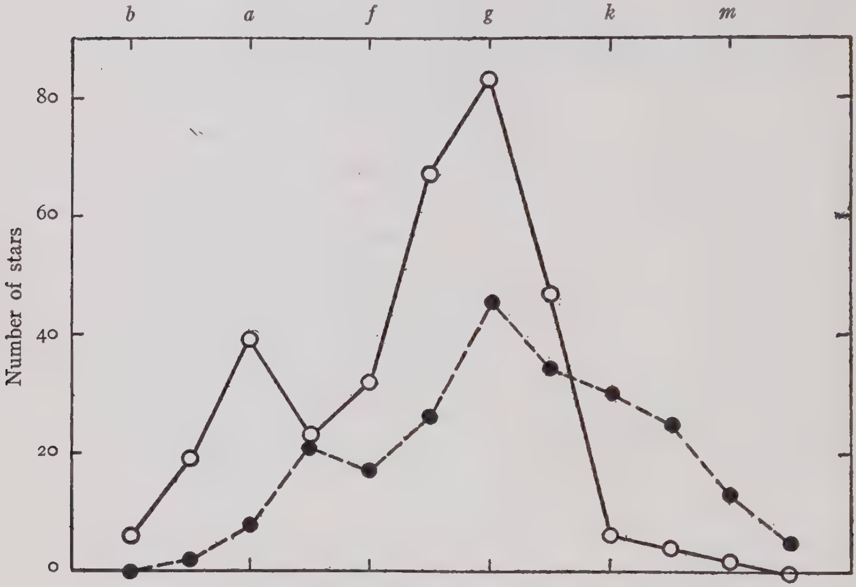

FIG. 3.—Frequency of colors in Regions I–II (dots) and III–IV (circles)

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[1] *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, 40, 187, 1914.

[2] *Derivation of the Change of Colour with Distance and Apparent Magnitude*, Groningen, 1915.

It seems necessary to conclude that in the direction of the Hercules cluster the selective extinction of light in space due to molecular scattering is negligible. If we grant a color-excess of +0.1 mag. in the cluster, due entirely to space absorption, the value of *d* cannot exceed +0.0001 mag. per unit of distance, an amount entirely inappreciable in dealing with the distances of the ordinary isolated stars. In the light of this result we are probably justified in assuming that the non-selective absorption in space (obstruction) is also negligible.

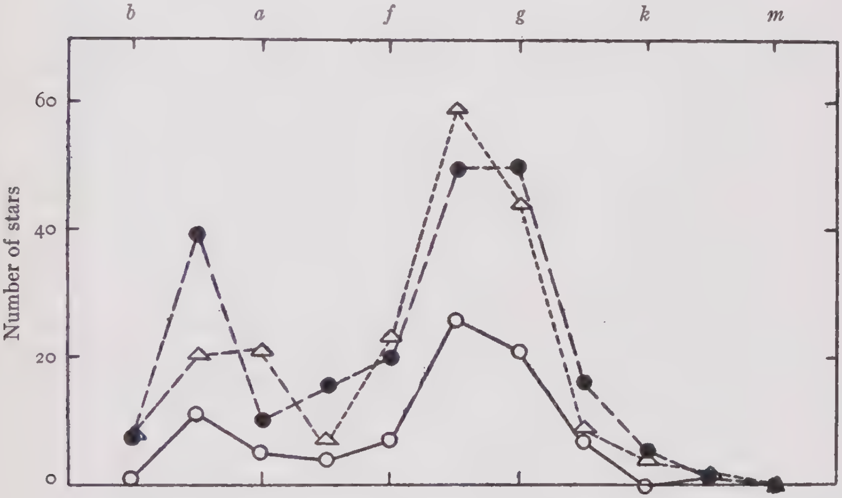

FIG. 4.—Frequency of colors in Regions V–VI (dots), VII–VIII (triangles), and IX–X (circles).

The interpretation of color-class directly as spectral type receives a valuable verification through the work by Pease on the spectra of the individual cluster stars, but owing to the provisional nature of the data the check cannot as yet be considered complete. So far as it goes, the material is as accordant with the previous findings as could be desired, both for the immediate problem of space absorption and for the later treatment of the central part of the cluster. The discussion of the spectra and the relevant color-indices will appear in a separate paper. For the present we only observe that for the four stars with distance greater than 1.5 the excess of observed color over that to be expected from the spectral types is but +0.08 mag. The error possible from the magnitudes and spectral classification of a single star may easily exceed a tenth of a magnitude.

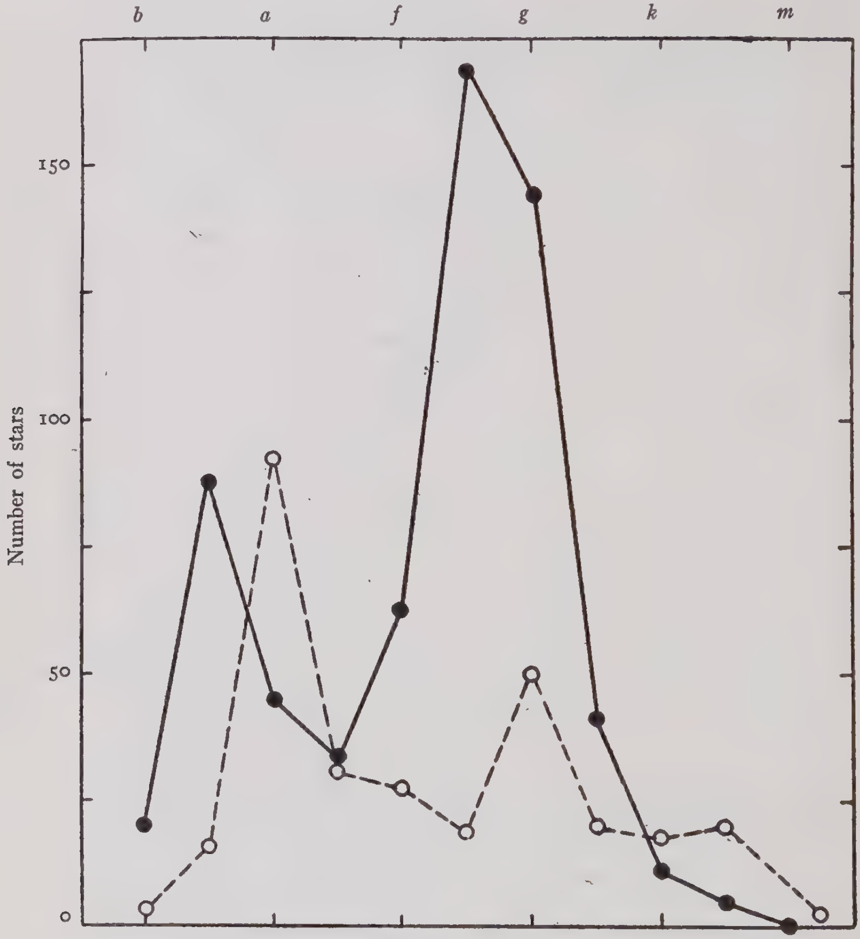

FIG. 5.—Frequency of color-classes in Messier 13 (full line), and in sky at large

In Van Rhyn's thesis, cited above, the observed increase of redness of stars with increasing distance is shown possibly to be dependent on spectral type; at least, it is vanishingly small or negative for B-type stars and the early A's.[1] This supports his alternative interpretation that the observed distance-effect is an influence rather of absolute luminosity on color than of space absorption. In that case the absolutely brighter stars would be redder; and for the stars considered by Van Rhyn it appears probable that, if an increase of five magnitudes in absolute brightness is assumed to correspond to about +0.05 mag. increase in color-index (apparent magnitudes and spectra being the same), the observed increase of color with distance could be accounted for. But, in contradiction to the increase of color with increasing absolute brightness, Van Rhyn presents data in his Table 30 that would suggest that the absolutely fainter stars are the redder. Similarly Adams[2] and Jones[3] present contradictory evidence relative to the effect of absolute magnitude on color.

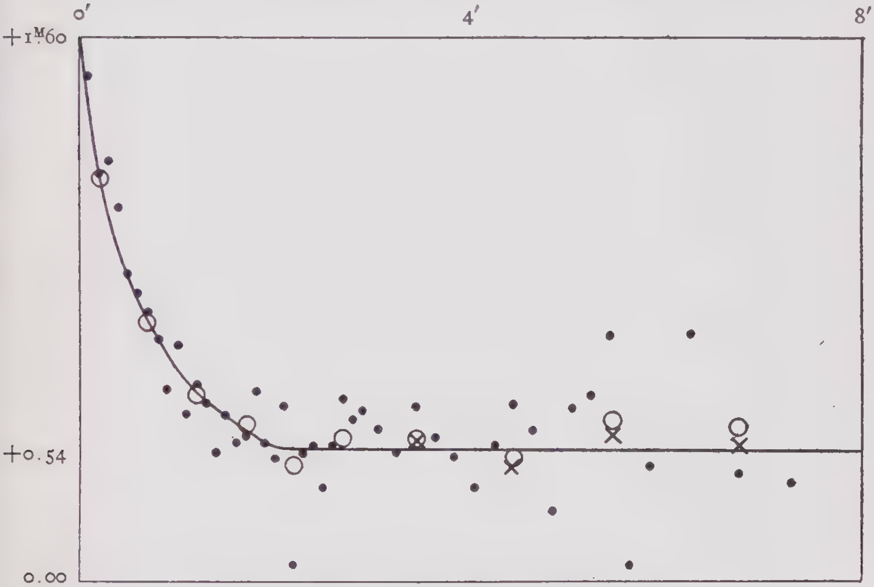

FIG. 6.—Average color and distance from center for all magnitudes combined

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[1] Adams found the effect of luminosity on spectral intensity inappreciable for first-type stars (*Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, **39**, 89, 1914).

[2] *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, **41**, 187, 1914.

[3] *Monthly Notices*, **75**, 4, 1914.

Whatever this effect of luminosity on redness may be, we can safely assume for the present that it is not sufficient to counteract or conceal the absorption of light in space if the previously found values of this quantity are adopted. It is also apparent that, over the small range in apparent magnitude (and, therefore, in absolute magnitude) observed in the cluster, the luminosity factor can have no serious influence on the observed color-index.

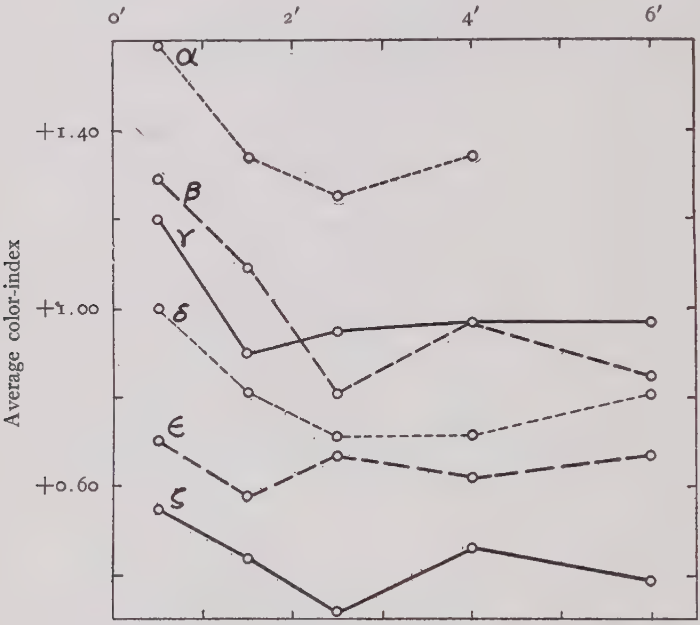

Fig. 7.—Average color and distance from center for successive magnitude intervals

VIII. DISTANCE FROM CENTER AND AVERAGE COLOR; THE EBERHARD EFFECT

The last column of Table XI contains, for each small interval of distance from the center, the average color-index of all stars included in the annular area defined in the first column of the table. These values of the average color are plotted against distance from the center as dots in Fig. 6. The large open circles represent means for each region, and the crosses are the mean values corrected in the outer part of the cluster for a supposed influence (on the average color-index) of the greater average redness of non-cluster stars.[1] The noticeably greater variations from the mean with increasing distance reflects the relatively lower weight (smaller number of stars) of those groups.

TABLE XVI

AVERAGE COLOR FOR DIFFERENT INTERVALS OF DISTANCE AND MAGNITUDE

MAGNITUDE INTERVAL	TABULATED QUANTITY	REGION AND MIDDLE DISTANCE					ALL REGIONS	REGIONS V TO X INCLUSIVE
		I and II 0′.5	III and IV 1′.5	V and VI 2′.5	VII and VIII 4′.0	IX and X 6′.0		
α	No. stars	13	11	8	5	0	37	13
	Av. C. I.	+1.59	+1.34	+1.25	+1.34	....	+1.41	+1.28
	Percentage I	35	30	22	13	....	100	35
	Percentage II	6	4	4	3	....	3.7	3
β	No. stars	29	20	14	6	4	73	24
	Av. C. I.	+1.29	+1.09	+0.81	+0.97	+0.85	+1.05	+0.86
	Percentage I	40	27	19	8	5	100	33
	Percentage II	13	7	7	3	5	7.3	5
γ	No. stars	36	30	11	6	6	89	23
	Av. C. I.	+1.20	+0.90	+0.95	+0.97	+0.97	+0.89	+0.96
	Percentage I	40	34	12	7	7	100	26
	Percentage II	16	10	6	3	7	8.9	5
δ	No. stars	66	65	27	36	13	207	76
	Av. C. I.	+1.00	+0.81	+0.71	+0.72	+0.81	+0.84	+0.73
	Percentage I	32	31	13	17	6	100	37
	Percentage II	29	21	14	19	16	20.7	16
ε	No. stars	52	86	48	37	25	248	110
	Av. C. I.	+0.70	+0.58	+0.67	+0.62	+0.67	+0.64	+0.65
	Percentage I	21	35	19	15	10	100	40
	Percentage II	23	28	24	20	30	24.7	24
ζ	No. stars	29	97	90	97	35	348	222
	Av. C. I.	+0.55	+0.44	+0.31	+0.46	+0.39	+0.40	+0.39
	Percentage I	8	28	26	28	10	100	64
	Percentage II	13	31	45	52	42	34.7	47
Total	No. stars*	226	328	215	197	83	1049	495
	Av. C. I.*	+0.97	+0.65	+0.56	+0.55	+0.62	+0.673	+0.55
	Percentage I*	21.6	31.2	20.5	18.8	7.9	100	47.2
	Percentage II	100	100	100	100	100	100	100

* Includes also the 47 stars of magnitude interval η.

The increase of color toward the center of the cluster is unmistakable. Its interpretation may be that the systematic errors noted in discussing the residuals are of more importance than supposed, or that the bluer stars are less concentrated than the redder ones, or that a selective scattering of light exists in the cluster itself though none is found in intervening space. The most probable explanation, however, as suggested in Section IV, is that a phenomenon analogous to the Eberhard effect has entered unequally for photographic and photo-visual magnitudes. Near the center the effect is progressive with distance and large in amount. Beyond distance 2′ there appears to be little change in the average color-index, which presumably means that the cause of excessive redness is absent from that region, or is too feeble to be effective.[1]

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[1] Cf. Section X, B.

The question naturally arises whether some systematic selection of data has entered to produce this remarkable relation. Tested from various angles the answer is uniformly negative. To show the independence of magnitude and color-excess, Table XVI and Fig. 7 have been prepared. It should first be noted that there has been in Ludendorff's catalogue (and therefore in the present one) an omission of the fainter stars in the center of the cluster, due probably to the hazy background. For the sake of homogeneity, then, in discussions involving magnitudes a slightly brighter limit is generally chosen than the faintest possible magnitude on the plate.

Table XVI, covering the first six magnitude intervals defined in Table XIV, gives for each magnitude interval and for each double region the number of stars, the average color-index, the percentage in each double region of all the stars of the magnitude interval, and the percentage in each magnitude interval of all the stars of the double region down to photo-visual magnitude 15.6. The last column contains the combined results for all the groups outside the zone of color-excess. Fig. 7, which represents the data for average colors in Table XVI, shows that the excess of color near the center of the cluster, illustrated for all the stars in Fig. 6, also affects each interval of magnitude separately.

It is to be noted that there are very few small or negative color-indices among the stars near the center of the cluster, in fact, none of color-class *b* or *a* are among the 61 objects within 0ʹ.5 of the center. Further, the very large positive color-indices (most of the stars of class *m*) are to be found in this central region. The spectral work by Pease does not, however, indicate any condensation of intrinsically red stars in that locality, and serves to confirm the conclusion expressed above that the presence of the *m*'s and absence of *b*'s from the central region is probably photographic in origin. A preliminary comparison of the spectral classifications of stars within distance 1ʹ.5 with the corresponding colors shows an excess of approximately +0.6 mag. Between distances 1ʹ.0 and 1ʹ.5 the average for five stars is +0.4 mag. The color-excess decreases on the average with distance, but for individual stars varies greatly, as might be expected whatever its cause.

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[1] Other evidence of the absence of the effect in the outer regions is presented in Section IV, C.

A certain number of the stars in the center of the cluster are there merely by projection. We should expect to find those in the outer regions on this side of the cluster, unaffected by the excess redness if it is to be interpreted as cluster absorption. The number thus unaffected will depend on the law of distribution for different spectral types, as well as the law for the density of the supposed absorbing matter. A preliminary computation shows that the number of superposed blue stars will be very small for the region where the color-excess is most pronounced.

IX. MAGNITUDE AND AVERAGE COLOR; BEARING ON THE ORDER OF STELLAR EVOLUTION

A. Hardly less striking than the apparent absence of light-scattering in space is the unexpected relation brought to light by plotting magnitude against color for any or all of the regions in the cluster. From Table XI the data are collected for the summary in Table XVII, which gives for successive regions the average photo-visual magnitude for each color-class. The number of stars involved in each case is in parentheses following the mean magnitude. (The last column of this table shows the central redness in a decisive manner.)

The successive lines of Table XVII show that for every region the average magnitude is brighter for the redder color-classes than for the blue. Not only are the *g*, *k*, and *m* colors associated in

TABLE XVII

AVERAGE MAGNITUDES FOR DIFFERENT REGIONS AND COLOR-CLASSES

REGION	COLOR-CLASS						ALL COLORS	AVERAGE COLOR-CLASS
	b	*a*	*f*	*g*	*k*	*m*		
I.	. . . .	. . . .	14.50 (9)	14.14 (16)	13.37 (24)	13.31 (12)	13.73 (61)	*k*1
II.	14.48 (2)	14.83 (29)	14.55 (34)	14.06 (63)	13.64 (31)	12.57 (6)	14.17 (165)	*g*1
III.	15.12 (10)	15.12 (33)	14.78 (48)	14.06 (76)	12.83 (7)	11.92 (1)	14.46 (175)	*f*7
IV.	15.26 (15)	14.98 (29)	14.74 (51)	14.12 (54)	13.19 (3)	12.08 (1)	14.57 (153)	*f*5
V.	15.18 (29)	15.12 (14)	14.88 (38)	14.36 (36)	12.11 (3)	. . . .	14.76 (120)	*f*2
VI.	15.22 (17)	14.75 (12)	14.89 (32)	14.03 (31)	12.74 (3)	. . . .	14.58 (95)	*f*4
VII.	15.16 (12)	15.14 (17)	14.81 (51)	14.18 (33)	12.19 (2)	. . . .	14.67 (115)	*f*4
VIII.	15.25 (16)	15.22 (11)	14.92 (31)	14.70 (20)	13.66 (4)	. . . .	14.91 (82)	*f*3
IX.	15.22 (6)	14.96 (5)	14.84 (18)	14.15 (20)	. . . .	. . . .	14.62 (49)	*f*5
X.	15.16 (6)	14.03 (3)	14.74 (16)	14.54 (8)	13.69 (1)	. . . .	14.67 (34)	*f*5
All of cluster.	15.19 (113)	14.99 (153)	14.78 (328)	14.15 (357)	13.34 (78)	12.96 (20)	14.50 (1049)	*f*6
Outside distance 2′.	15.20 (86)	15.01 (62)	14.85 (186)	14.28 (148)	12.74 (13)	. . . .	14.71 (495)	*f*3

the mean with the brighter stars of the cluster, but the decrease of redness with decreasing brightness is distinctly progressive, in fact, nearly linear in all parts of the cluster. The same result is also plainly shown by the numbers in all divisions of the frequency table, No. XII, and by Table XVI.

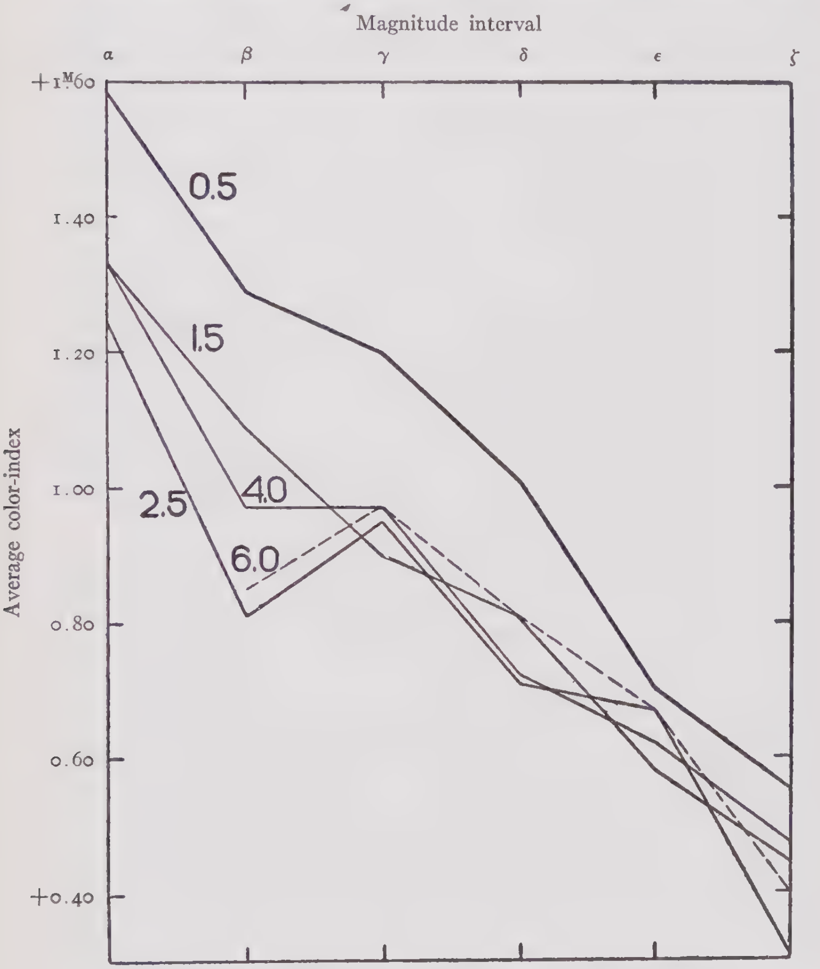

FIG. 8.—Decrease of color-index with decreasing brightness

The phenomenon is illustrated graphically in Fig. 8, which is based upon material given in Table XVI. In that table average color is tabulated for definite intervals of magnitude; Table XVII gives average magnitude for definite intervals (classes) of color. Both show that bright stars are red, faint stars are blue.

B. Fig. 8 illustrates several properties of the cluster. Each curve represents the double region which is indicated by the accompanying value of the middle distance (see the column headings in Table XVI).

1. That all the curves slope downward and to the right indicates that the color-magnitude relation is present throughout the whole cluster.

2. That the slope of all the curves is comparable indicates that change of color with magnitude is similar in all regions.

3. The separation of the curve for the center of the cluster (middle distance, $0'.5$) from the others indicates the presence of central redness, and shows that it exists for all magnitudes. The smaller separation for the fainter magnitudes may indicate the increasing influence of stars seen in the center only by projection.

4. The close coincidence of the four lower curves indicates that outside the central sphere the cluster stars are thoroughly mixed with respect to color for all magnitudes; that is, in the outer regions there is little dependence of magnitude or color upon distance. The same conclusion is to be drawn from the percentages in Table XVI when allowance is made for the systematic absence of faint stars (therefore, of blue stars) from the center.

5. The influence of the non-cluster stars on the mean color (already shown in Fig. 6) is probably indicated in the dotted line which represents regions IX and X.

6. The minimum in three of the curves at β, if real, indicates that between magnitudes 13.6 and 14.2 the stars average bluer than would be expected from the course of the remainder of each curve.

C. For open clusters such as Messier 67 and the Pleiades, and for the stars in the sky at large, the relation between apparent or absolute magnitude and color has been found always in the sense of increasing redness with decreasing brightness. It is not necessary to hasten the interpretation of the contrary result obtained for a globular cluster. Verification for other clusters is an obvious desideratum, but preliminary work on other globular systems has already verified the result provisionally. All the available material will be taken up in detail in the near future. For the present it will suffice to observe that we are now dealing with the three or four brightest magnitudes in a stellar system in which exists a range of absolute brightness of certainly more than 10 magnitudes. Doubtless we are discussing the giant stars and only the giants among a throng of hundreds of thousands. For the stars in the galactic system Russell has come to the tentative conclusion that the giants are of comparable absolute magnitude whatever their spectral type.[1] The present result suggests that in this globular cluster, at least, the giants are brightest when reddest. The relation between absolute brightness and color is therefore the inverse of that for dwarfs, where, without much doubt, the cooling stars are growing redder and smaller with age.

X. MISCELLANEOUS STATISTICAL RESULTS

A. *Preliminary remarks.*—The three results from the statistical examination that stand out as most definitely established have been treated separately in the preceding sections. A number of others are suggested, but for various reasons they can be treated to best advantage only provisionally in the present paper. A definitive discussion of some of them must await data from other clusters, and others demand more definite knowledge of the existence and amount of central redness. The present section gives first a solution for the superposed non-cluster stars, and then discussions of the relations between magnitude and distance, star-density and distance, and magnitude and star-numbers (luminosity-curves).

B. *The foreground stars.*—After determining the probable distance of the Hercules cluster it does not seem appropriate to term as "background stars" those members of the general population of the sky that happen to be in its direction. The chances are very great that any superposed stars are between us and the cluster. The actual sorting out of the non-cluster stars is probably impossible, but it is of some value to the statistical study to know about how many extraneous objects are included in the catalogue. The number of foreground stars per square minute has been determined in three partially independent ways: (1) from counts made by the writer on the Franklin-Adams charts for several square degrees in the vicinity of the cluster; (2) from computations of the number brighter than the observed limiting magnitude expected at galactic latitude 40° (based on the tables given by Chapman and Melotte);[1] (3) from counts on Potsdam plates by Scheiner, who decided that 27 non-cluster stars were probably included in his catalogue.[2]

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[1] *Popular Astronomy*, **22**, 275, 1914.

TABLE XVIII

Distribution of Foreground Stars

Region	$\frac{\text{Area}}{\pi}$	No. Foreground Stars	Total No. in Cluster	Density
	sq. min.			
I	0.2	0.1	61	96.9
II	0.7	0.3	165	75.0
III	1.2	0.5	175	46.3
IV	1.7	0.6	152	28.4
V	2.2	0.8	119	17.2
VI	2.7	1.0	94	11.1
VII	6.9	2.6	112	5.2
VIII	8.9	3.4	79	2.8
IX	10.9	4.1	45	1.3
X	13.6	5.1	29	0.7

The resulting estimates of the number of stars per square minute are 0.10, 0.12, and 0.19, with weights 4, 4, and 1, respectively. Scheiner's value is doubtless too large, for, underestimating the extent of the cluster, his counts probably include cluster stars. There is no evidence from the counts on the Franklin-Adams charts of a diminution of non-cluster stars in the vicinity of the cluster; moreover, such an effect is not expected. The adopted density of foreground stars is 0.12 per square minute.

Table XVIII contains the probable distribution of superposed non-cluster stars in the successive annular regions defined in Table XIII. The numbers of foreground stars in the third column are found by multiplying the relative areas in the second by 0.12π. The numbers of actual members of the cluster in the fourth column are the differences between the values of the third column of this table and the last of Table XIII. A ninth of the catalogued stars beyond distance 5′ and a seventh of those beyond distance 6′ apparently are not members of the cluster. The average numbers of actual cluster stars per square minute are given in the last column of Table XVIII.

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[1] *Memoirs of the Royal Astronomical Society*, **60**, Part IV, 1914.

[2] *Op. cit.*, p. 36.

For the stars near the North Pole and in the field of S Cygni in the Milky Way the average color-index at the sixteenth photographic magnitude is about +0.9, corresponding to color *g*3, and no small or negative indices are found.[1] The foreground stars superposed upon the Hercules cluster are not likely to be bluer than this on the average. Adopting +0.9 as the mean color-index, we can distribute the superposed stars of each region among the color-classes in such a manner that the average color approximates this mean value. The numbers of non-cluster stars for each region are then as given in Table XIX. None of the 18 foreground stars is assigned to color *b*, *a*, or *m*, and the average color class is *g*3. No differentiation of individual stars as cluster or non-cluster objects has been attempted, though such procedure might be possible, and, if proper motion determinations were more accurate, would also be of value in designating reference stars.

TABLE XIX

PROBABLE NUMBERS OF FOREGROUND STARS IN EACH COLOR-CLASS

Region	*f*	*g*	*k*	Total
I				0
II				0
III		½		½
IV		½		½
V	½	½		1
VI	½	½		1
VII	1	1	1	3
VIII	1	1	1	3
IX	2	2		4
X	2	2	1	5
Total	7	8	3	18

The combined numbers of foreground and cluster stars in each region for each color are in parentheses in Table XVII.

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[1] *Mt. Wilson Contr.*, No. 81; *Astrophysical Journal*, **39**, 368, 1914.

Subtracting the numbers given in Table XIX we obtain the distribution of actual cluster members, and these results, expressed as stellar densities, appear in Table XX.

The average color-index of all stars in the four outer regions of the cluster, according to Table XI, is about +0.60. These four regions contain several foreground stars (Tables XVIII and XIX) which presumably have a greater color-index than this. It is appropriate, therefore, to correct the average for the probably greater redness of the foreground. We obtain readily:

TABLE XX

Number of Cluster Stars per Square Minute

Region	*b*	*a*	*f*	*g*	*k*	*m*	All Colors
I.	0.0	0.0	14.3	25.4	38.1	19.1	96.9
II.	0.9	13.2	15.5	28.6	14.1	2.7	75.0
III.	2.6	8.7	12.7	20.1	1.9	0.3	46.3
IV.	2.8	5.4	9.5	9.9	0.6	0.2	28.4
V.	4.2	2.0	5.4	5.1	0.4	0.0	17.1
VI.	2.0	1.4	3.7	3.6	0.4	0.0	11.1
VII.	0.6	0.8	2.3	1.5	0.0	0.0	5.2
VIII.	0.5	0.4	1.1	0.7	0.1	0.0	2.8
IX.	0.2	0.1	0.5	0.5	0.0	0.0	1.3
X.	0.1	0.1	0.3	0.2	0.0	0.0	0.7
Mean density of whole cluster. .	2.6	3.6	7.5	8.2	1.8	0.5	24.2
Number of stars in cluster.	113	153	321	349	75	20	1031
Number of foreground stars. . .	0	0	7	8	3	0	18

Region VII, Observed av. C. I. = +0.57, Corrected av. C. I. = +0.56
" VIII, " " " = +0.52, " " " = +0.50
" IX, " " " = +0.62, " " " = +0.58
" X, " " " = +0.60, " " " = +0.55

For the inner regions the correction for the foreground is negligible. These revised values of the average color-index are plotted as crosses in the diagram showing change of observed color with distance, Fig. 6.

C. *Magnitude and distance from the center.*—Because of uncertainty regarding the necessity and amount of the correction to magnitude demanded by the observed central redness, the relation of magnitude to distance from the center cannot be very definite.

Fig. 9 presents without correction the relevant data from the last column of Table XI. Within the two central regions there is a definite increase of *average brightness*. This is to be attributed almost wholly to the absence of faint stars from Ludendorff's catalogue for the denser parts of the cluster. But when we take into account the probably progressive effect of color-excess, our tentative conclusion can well be that the brighter stars are slightly more concentrated toward the center than the fainter ones.[1] That outside of the very center there is little or no dependence of magnitude on distance is shown separately for all color-classes in the columns of Table XVII.

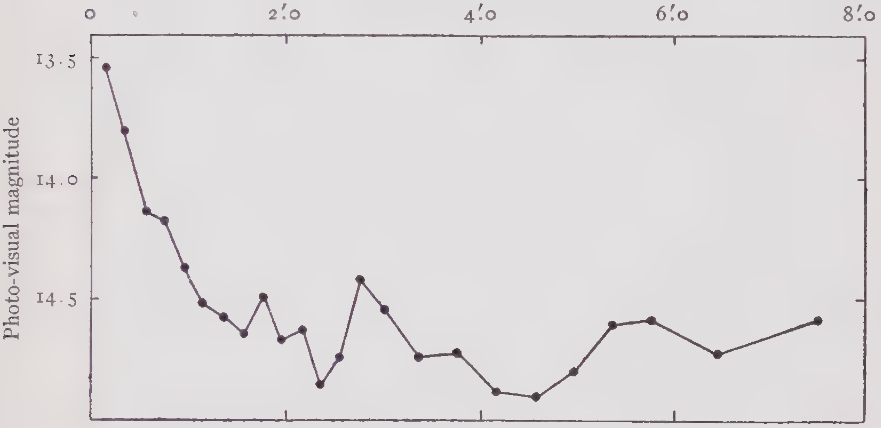

FIG. 9.—Average magnitude and distance from center

D. *Star-density for different colors.*—The distribution of stars is the chief subject considered in previous studies of clusters. It will not be advisable to press the investigation far in the present case for the same reason that the results of the preceding paragraph are tentative. Until we know definitely the relation of the Eberhard effect and similar phenomena to the magnitudes in the centers of condensed clusters all studies of density laws must be provisional or meaningless.

The observed mean density (number of stars per square minute) for the ten annular regions is given in Table XX, for each class of color and for all colors combined. The densities have been corrected for the foreground stars, assigning definite numbers to each color-class as described above.[1] The results for each color-class are shown diagrammatically in Fig. 10, while Fig. 11 gives the density for all colors, the lower part of this curve appearing also on an enlarged vertical scale (ordinates to right of diagram). The curves in both figures may be accepted as having some definite meaning up to about 2′ from the center of the cluster; within this distance they are affected both by anomalous color-excess and by discrimination against the faint stars in the center. But it must be remembered that in any case only stars of peculiarly high absolute brightness are involved and that the data afford nothing explicit as to the distribution of the great mass of faint stars which chiefly form the cluster.

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[1] Since bright stars average red, we may expect a slight fraction of the increase of redness toward the center to be attributable to this factor.

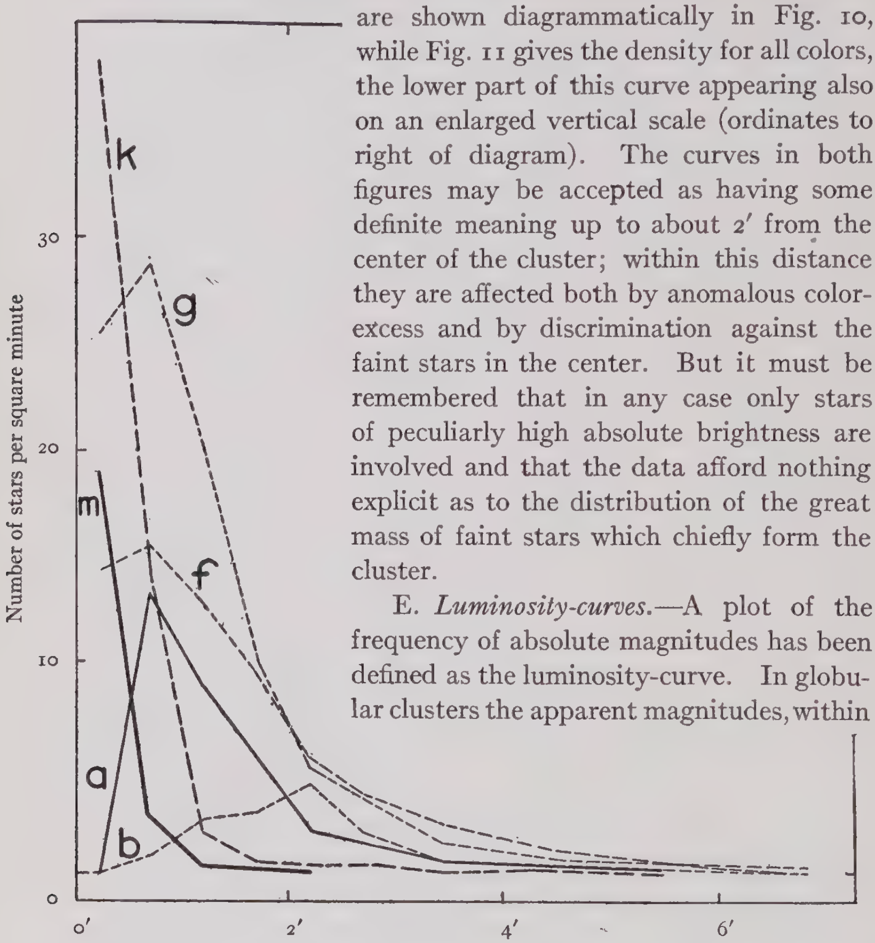

Fig. 10.—Density of stars in Messier 13 for different color-classes. Abscissae are distances from center.

E. *Luminosity-curves.*—A plot of the frequency of absolute magnitudes has been defined as the luminosity-curve. In globular clusters the apparent magnitudes, within easily permissible error, are equivalent to absolute magnitudes plus a constant.[2] In Messier 13, therefore, frequency-curves of apparent magnitudes for various colors should give us theoretically an appreciable part of the luminosity-curve for each class. Although all color-classes are represented with a total of more than a thousand stars, it happens that the luminosity-curves to be derived from Table XII are not very valuable, for we are evidently dealing with only the giant stars. Probably for not a single color-class is a sufficiently extensive portion of the luminosity-curve available to be of permanent value.

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[1] No correction has been made for eclipsed images.

[2] Cf. Section XIV.

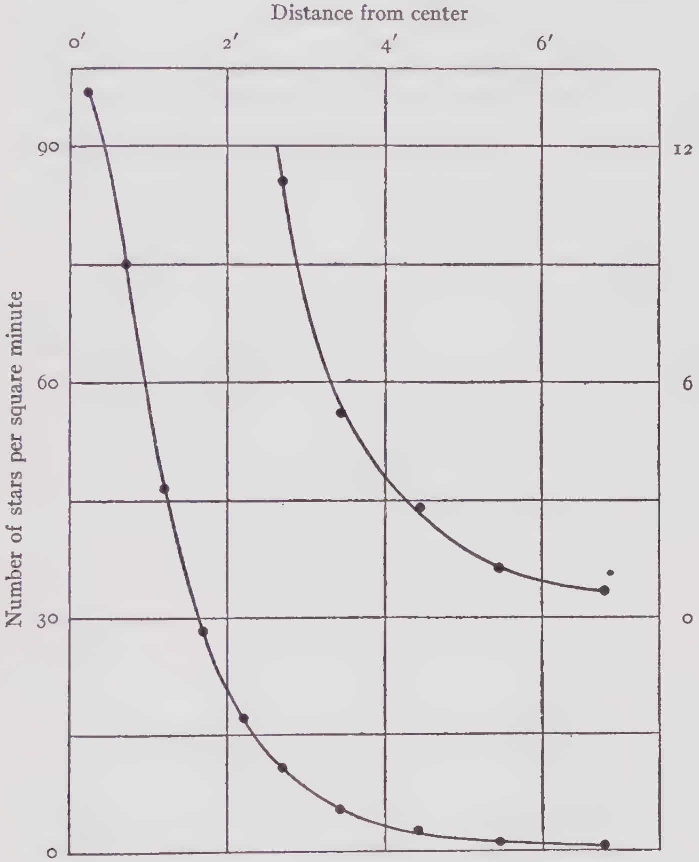

FIG. 11.—Density of stars for all colors combined

The frequency of magnitudes for color *b* is discussed further in Section XIII, but as to the rest of the data we will for the present only call attention to the numbers in Table XII, and reproduce in Fig. 12 the luminosity-curves for the whole cluster and for the region outside 2′. The curves show the rate at which numbers are increasing toward the limit of the photo-visual plates and may help to indicate roughly the total number of stars. The small value for the last point on each curve has no significance other than that the stars between magnitudes 15.3 and 15.8 are not wholly included.

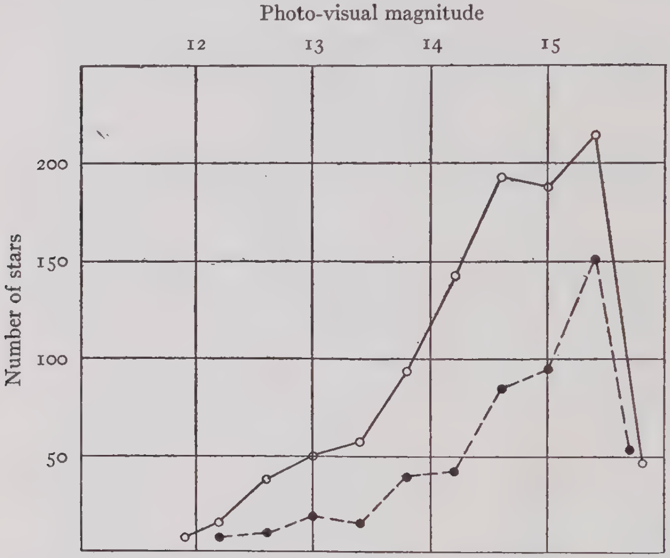

FIG. 12.—Luminosity-curves for the whole cluster (full line) and for the outer regions alone.

XI. ON THE FAINTER STARS IN THE CLUSTER

A. *The x-postscript stars.*—The standard magnitude plates have yielded complete and reliable photo-visual values only as far as 15.5. It is of obvious importance to extend the investigation to the fainter stars, but to do so by direct comparison with the Polar Standards is not expedient. Accordingly the scale was extended by the method of long and short exposures on the same plate.[1] This method is not without its dangers, but in practice has been found reliable, at least over the one-magnitude interval involved in the present extrapolations.

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[1] Seares, *Mt. Wilson Contr.*, No. 80; *Astrophysical Journal*, **39**, 311, 1914.

Magnitudes and colors for 32 faint stars were thus determined from Plates 2072 and 2073, described in Table I. The results are given in Table XXI. The designation indicates that the object is in the immediate vicinity of the catalogued star bearing the corresponding number; the positions are not more closely determined.

TABLE XXI

x-Postscript Stars

Designation	Distance from Center	Magnitude		Color	Dev. from Mean C. I.
		Pg.	Pv.		
58*x*	4′	16.52	16.02	*f2*	+0.17
115*x*	3.5	16.55:	15.47	*g7*:	....
y	3.5	16.89	16.71	*a4*	− .15
z	3.5	16.55	16.08	*f2*	+ .14
u	3.5	16.58	16.15	*f1*	+ .10
147*x*	3.5	16.80	16.34	*f2*	+ .13
y	3.5	16.93	16.75	*a4*	− .15
158*x*	3	15.74	15.04	*f8*	....
185*x*	2.5	16.45	15.97	*f2*	....
y	2.5	16.64	16.15	*f2*	+ .16
243*x*	3	16.69	16.41	*a7*	− .05
y	3	16.69	16.34	*a9*	+ .02
z	3	17.28	16.75	*f3*	+ .20
u	3	17.09	16.69	*f0*	+ .07
247*x*	2.5	17.05	16.79	*a6*	− .07
y	2.5	17.03	16.76	*a7*	− .06
265*x*	3	16.39	16.15	*a6*	− .09
434*x*	3	16.63	16.23	*f0*	+ .07
y	3	16.66	16.15	*f3*	+ .18
527*x*	3.5	16.76	16.38	*a9*	+ .05
534*x*	4	16.80	16.54	*a7*	− .07
y	4	16.92	16.69	*a6*	− .10
687*x*	5	16.61	16.21	*f0*	+ .07
765*x*	2.5	16.63	16.15	*f2*	+ .16
828*x*	2.5	16.64	16.56	*a2*	− .25
919*x*	3.5	16.97	16.73	*a6*	− .09
932*x*	3.5	17.11	16.81	*a8*	− .03
954*x*	4	16.63	16.64	*a0*	− .34
961*x*	3	16.53	15.72	*g0*	....
y	3	16.41	16.02	*f0*	+ .06
1011*x*	4	17.02	16.72	*a7*	− .03
y	4	16.74	16.56	*a5*	− .15

The "*x*-postscript stars" may be considered a fair sample of the stars between photo-visual magnitudes 16.0 and 16.8, outside the region of abnormal color.[1] They are not a selected group, as the stars treated in the next part of this section appear to be.[1] From Table XXI we derive:

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[1] Within two minutes of the center the confusion of the two exposures was too great to permit definite measures on Plates 2072 and 2073.

Average color-index, $+0.383 = a9.6$ } 32 stars
Average photo-visual magnitude, 16.34 }

Excluding	115*x*,	Pv. mag. 15.47,	color-index +1.08:
	158*x*	15.04	+0.70
	185*x*	15.97	+0.48
	961*x*	15.72	+0.81
	Mean of four	15.55	+0.77

because they are all brighter than the sixteenth photo-visual magnitude, and also because the first two at least are not free from suspicion of variability since they are not visible on the isochromatic plate No. 1920, we have the following results, based upon more homogeneous data:

Average color-index, $+0.328 \pm 0.114$ (av. dev.) $= a8 \pm 3$ } 28 stars
Average photo-visual magnitude, 16.54 ± 0.23 (av. dev.) }

The last column of Table XXI gives the individual deviations of the color-index from this mean for 28 stars. It is noteworthy that there is only one negative color-index; but this is probably a chance result, as allowance for but a small space absorption or color-excess would make the number of *b*'s more than normal. It is more significant that of these 28 sixteenth-magnitude stars the reddest has color-index +0.53. One is led to inquire if there are no stars of this magnitude in the cluster with colors *g* or *k*. If such is the case, it is a very important result—one in perfect keeping with the results for the brighter stars discussed in Section IX and shown in Fig. 8. A further consideration of the question is desirable.[2]

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[1] If any selection of the *x*-postscript stars exists, it is slightly in favor of redness; for the few "tf" stars, measured by chance in this indiscriminate choice, were excluded from Table XXI, and they are likely to be preferentially blue.

[2] A continuous relative divergence between photographic and photo-visual magnitude scales of 25 per cent could take into account the observed decrease of color with decreasing brightness, if all stars in the cluster are of the same color-type. But we assume, of course, that the divergence is negligible and that the observed phenomenon is real.

Outside the region of color-excess no dependence of color or magnitude on distance was found for the main body of bright stars (Section IX, B). Similarly, for the *x*-postscript stars we have Table XXII, from which the four fifteenth-magnitude stars have been excluded. Again magnitude and color appear independent of distance.

B. *The parentheses stars.*—All of the stars catalogued by Ludendorff that were too faint to be measured on the isochromatic plate No. 1920 were originally designated as "tf" stars. Later a number of the photo-visual magnitudes were determined from the long-exposure isochromatic plate; the results were placed in parentheses in the catalogue, and the group designated as "parentheses" stars. The colors are based on the photographic plate No. 1921 and the photo-visual plate No. 2073, Plate 2072 being used in some cases to check the photographic values. A study of these data gives some interesting information relative to faint cluster stars, substantiating in part the results derived above for the *x*-postscript stars.

TABLE XXII

x-POSTSCRIPT STARS

Mean Distance	No. Stars	Photo-visual Magnitude	Average Color
2'.5	5	16.48	*a*8
3	8	16.34	*a*9
3.5	8	16.49	*a*8
4+	7	16.48	*a*7
Mean.......		16.45	*a*8

A little consideration shows that the parentheses colors form a selected lot. They refer to stars visible on the Potsdam plate, but invisible on Plate 1920. Their photographic brightness, therefore, exceeds a certain lower limit, while the photo-visual brightness is less than a definite upper limit. These stars, then, are chosen for blueness, and cannot give a fair idea of all the stars in the magnitude interval covered; but they will be suitable to confirm the existence of the blue faint stars.

Collecting the data for the parentheses stars from the main catalogue of magnitudes,[1] we find:

	Color-Indices
Brighter than photo-visual magnitude 16.00	24 positive, 6 negative
Fainter than photo-visual magnitude 16.00	14 positive, 27 negative
Total	38 positive, 33 negative

The average color-index of all is +0.11.

The average magnitudes and colors for intervals of distance from the center are shown in Table XXIII separately for the stars brighter than and fainter than the sixteenth photo-visual magnitude. Of the forty-one stars fainter than 16.0, only three[2] have color-indices in excess of half a magnitude (+0.82, +0.51, and +0.54), a result to be compared with that for the *x*-postscript stars. The next to the last line of the table suggests that we may have picked up a foreground star or two (presumably red for magnitude 16), but beyond this there seems to be no dependence of magnitude or color-index on distance.

TABLE XXIII

PARENTHESES STARS

DISTANCE FROM CENTER	BRIGHTER THAN PV. MAG. 16.0			FAINTER THAN PV. MAG. 16.0		
	No. Stars	Av. Pv. Mag.	Av. C. I.	No. Stars	Av. Pv. Mag.	Av. C. I.
<2'.0	2	15.80	+0.14	4	16.24	+0.24
2.0–2.9	7	15.78	+ .37	4	16.40	− .13
3.0–3.9	5	15.81	+ .45	9	16.32	− .13
4.0–4.9	8	15.72	+ .37	12	16.40	− .18
5.0–5.9	3	15.77	+ .34	7	16.67	− .18
≧6.0	5	15.63	+0.51	5	16.47	+0.06
All of cluster	30	15.74	+0.39	41	16.42	−0.09

In Table XXIV the parentheses stars are grouped according to photo-visual magnitude intervals. The progression of average color, from *f*6 at 15.50 to *b*5 at 16.50, may be expected in part from the method of selection, but it is also evidence of the probably inherent decrease of color-index with increase of magnitude, as noted above for the *x*-postscript stars.

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[1] Six stars within 2′ of the center are included.

[2] The large color shown in the first line of the division of fainter stars is due mainly to the exceptional color-index of +0.82 for star No. 706. This star was remeasured and the color checked. On Plate 2072 the photographic magnitude is 17.12, differing by only 0.01 mag. from the value on Plate 1921.

TABLE XXIV

PARENTHESES STARS

Limits of Pv. Mag.	No. Stars	Average C. I.	Limits of Pv. Mag.	No. Stars	Average C. I.
15.38–15.49......	4	+0.74	16.10–16.19.....	4	−0.08
15.50–15.69......	5	+ .55	16.20–16.29.....	8	+ .09
15.70–15.79......	6	+ .20	16.30–16.49.....	6	− .25
15.80–15.89......	9	+ .26	16.50–16.59.....	8	− .22
15.90–15.99......	6	+ .41	16.60–16.69.....	4	− .10
16.00–16.09......	4	+0.14	16.70–16.92.....	7	−0.15

C. *The "tf" stars.*—Sixty-four of the catalogued stars that were too faint for measurement on Plate 1920, not being measured on the long-exposure isochromatic plate, retain the designation of "tf" stars. For them we know only a brighter limit to the photo-visual magnitudes, which must be fainter[1] than the limit of Plate 1920 and which are, therefore, bluer than a certain color defined by the difference between the observed photographic magnitude and the limit of the isochromatic plate. This limit varies, its change being directly related to the distance correction. It is easy to deduce the following average relations between distance from center and limit of readily measurable photo-visual magnitudes:

Distance 0′.0 to 2′.0,	Limit of Plate 1920, 15.75 mag.
2.0 to 4.0	15.65
4.0 to 6.0	15.50
>6.0	15.40

Table XXV summarizes all the available information relative to the "tf" stars. The upper limit of color-index now depends on photographic magnitude and distance from the center. In the first line, for instance, the five stars with photographic magnitudes between 15.6 and 15.8 have, according to their distances, photo-visual magnitudes fainter than 15.73, so that on the average their color-indices are less than −0.04. They may be any amount bluer than this limit so far as present data go; and even in the last line, where the stars may be as red as *g*5 (but no redder), the color-indices may be and quite likely are very small or negative.

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[1] Unless they are rapidly changing variables.

TABLE XXV

UPPER LIMITS OF COLOR FOR "*tf*" STARS

Interval of Pg. Mag.	Number of Stars	Mean Pg. Mag.	Mean Brighter Limit of Pv. Mag.	Upper Limit of Average C. I.
15.60–15.79	5	15.69	15.73	−0.04
15.80–15.99	5	15.94	15.64	+ .30
16.00–16.19	12	16.09	15.70	+ .39
16.20–16.29	15	16.24	15.66	+ .58
16.30–16.49	14	16.38	15.65	+ .73
16.50–16.99	13	16.74	15.63	+1.01

More than one-third of these 64 stars are necessarily bluer than *f*, and probably future investigation will show that most of the color-indices are very small, as found for the entirely analogous parentheses stars.

D. Combining the results for the *x*-postscript stars, the parentheses stars, and those regularly observed, we have a total of 592 measured indices outside the 2′ zone of abnormal color. Of this number more than 20 per cent are negative, and for stars fainter than magnitude 15 the percentage is nearly doubled. It would be of much importance to know the relative percentages for still fainter magnitudes. From theoretical reasons one is led to believe that larger color-indices appear again for fainter stars, and that at magnitude 20 or 22 the frequency of colors may be much the same as at magnitude 13. This matter will be referred to again in a subsequent treatment of the faint stars.

XII. VARIABLE STARS IN MESSIER 13

A. *New variables.*—Five variable stars have been found in the Hercules cluster during the course of the present study of magnitudes. Prior to this two were known, the first discovered by Bailey at Harvard,[1] and the second independently by Bailey and Barnard.[2] The periods of the two earlier variables are given by

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[1] *Harvard Annals*, **38,** 2, 1902.

[2] *Astrophysical Journal*, **12,** 182, 1900.

Barnard as 6.0 days for No. 1 and 5.10 days for No. 2. Ordinarily they appear to be faint, evidently belonging to the Cepheid type. The range of variation is approximately one visual magnitude. For the five new variables the periods are not known; indeed little is known of the light-variations beyond the fact that they appear definitely to exist.[1]

TABLE XXVI

OBSERVATIONS ON VARIABLE STARS

	Number of Variable						
	1	2	3	4	5	6	7
Catalogue No. Ludendorff..	816	306	135	322	806β	872	344
Catalogue No. Scheiner....	630	216	89	....	628β	667	245
Distance from center....	1′.0	1′.1	2′.4	1′.5	1′.0	2′.0	1′.5
Ludendorff magnitude...	13.9	14.1	15.6	15.1	....	13.9	14.9
Photographic magnitudes							
Plate 1820...........	13.7	12.9	16.0	15.4	14.6	14.0	14.8
1821...........	13.7	12.9	15.9	15.4	14.8	14.2	14.8
1901...........	14.23	12.85	16.11	15.33	14.74	14.74	14.96
1921...........	14.90	13.24	15.10	15.86	15.31	14.05	15.16
2072...........	14.7	13.8	15.2	15.7	15.0	14.4	15.1
2320...........	13.9	13.2	15.8	15.5	15.7	14.1	14.7
2322...........	13.8	13.4	15.7	15.4	15.8	14.2	14.7
Photo-visual magnitudes							
Plate 1900...........	13.74	12.63	....	....	....	14+	....
1920...........	14.09	12.73	15.18	15.26	14.35	13.80	14.54
2073...........	14.0	13.1	14.9	15.2	14.2	14.1	14.6
Color-Index							
Plate 1901–1900......	+0.49	+0.22	....	....	....	....	....
1921–1920......	+0.81	+0.51	−0.08	+0.60	+0.96	+0.25	+0.62
2072–2073......	+0.7	+0.7	+0.3	+0.5	+0.8	+0.3	+0.5

The magnitudes on the Mount Wilson plates[2] for all seven variables and other data relative to each star are given in Table XXVI. The Ludendorff magnitudes were obtained by reducing his estimates to the Mount Wilson scale by means of Table VIII. The evidence at the bottom of the table relative to changing color-index warrants serious consideration for Nos. 1 and 2 only. It should be noted, however, that in every case the smallest color-index is associated with the brightest photographic magnitude. This suggests that all seven stars are very probably Cepheid variables.

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[1] For data relative to some of these stars see the "Notes to the Catalogue."

[2] The relative values, therefore the evidence of variation, will be little changed by a revision of the magnitudes in the center; the absolute values for Nos. 1, 2, and 5 may be slightly altered.

Nos. 4 and 7 do not show a marked change in brightness, but for both the reference stars are so conveniently arranged that small variations can be detected with certainty. No. 7 was originally suspected by Barnard. He also classed No. 3 as a yellow star, though its color-index is −0.08. This probably can be taken as a verification of the variability, for the two plates upon which Barnard's color-estimate is based are not contemporary.

B. *Suspected variables.*—It seems very likely that a study of more plates would show other cases of variability among the fainter stars in the cluster. Those for which the light may be reasonably suspected of variation are probably included in the following groups:[1]

1. Stars suspected by Barnard, but not confirmed on Mount Wilson plates. See the papers cited in Section II of the First Paper, and "Notes to the Catalogue" relative to Stars 258 and 682. The former star has a color-index of +1.16, but is called blue by Barnard. It is probably variable in a long period.

2. Stars too faint for measurement on the photographic plate No. 1921. These objects were apparently seen by Ludendorff on the best Potsdam plate for which the lower limit is about half a magnitude brighter than that of No. 1921. The magnitudes in parentheses in the fourth column of the catalogue refer to the stars in question.

3. Stars remeasured because of large residuals on Plates 1901 and 1921, and for which the residuals in the fifth column of the catalogue are still large. These remeasured stars are indicated by Note 9 in the last column of the catalogue.

4. Stars for which both photographic and photo-visual residuals are large with deviations of the same sign for Plates 1900 and 1901. Such deviations indicate that on two different plates on the same night the magnitude was measured brighter (or fainter) than on the two plates of the following night.

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[1] Two or three of the notes referred to below have been omitted from Section V because the stars concerned are within the zone of color-excess.

5. Stars in Barnard's yellow list that have small color-indices; stars in Barnard's blue list that have large color-indices.[1]

6. The list of nine stars given by Ludendorff[2] for which his magnitude estimates differ by a magnitude or more from Scheiner's.

7. Bright *a*-postscript stars outside of the central region, such as 401*a*, 666*a*, etc.

8. The following stars, suspicion being based upon various grounds discussed in "Notes to the Catalogue": 426, 463*β*, 494, 516*β*, 527, 730, 816*a*, 1133.

Many of the stars in the above lists will not show variation, of course, but they should be considered first in any future survey for the purpose of detecting variables.

XIII. ON THE PROBABLE PARALLAX OF THE CLUSTER

If, admitting a high degree of comparability between our galactic system and globular clusters, we assume that the brightest stars in Messier 13 have exactly the same absolute magnitude as the most luminous stars in our galactic system, we can compute the distance of the cluster as soon as we obtain the apparent magnitudes of the brightest stars in both systems and an assurance of negligible space absorption. It would be better perhaps if we could assume equality of mean absolute magnitude for a number of stars of the same color-class in the two systems; or, if we could say, for instance, that the average B-type star has the same intrinsic luminosity wherever found. Then the determination of the distance of any cluster would await only measurements of magnitude.

In actual practice the derivation of the parallax of the Hercules cluster has not been simple or definite. The subject has been approached from many angles and the basic assumptions have been rather carefully examined. The complete discussion of the methods and results is deferred to a later paper, and for the present only a tabular summary will be presented.

The values in Table XXVII that are based on luminosity-curves fail of important weight because (1) of the provisional character of all theoretical luminosity-curves except for spectral Class B, (2) of the peculiar (perhaps chance) frequency of the brighter magnitudes among the early *b*-type stars in the cluster,[1] and (3) of the possible necessity of distinguishing sharply between dwarf and giant stars in deriving luminosity-curves. The derivation of the parallax from the variable stars follows the method employed by Hertzsprung[2] for the Small Magellanic Cloud.

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[1] *Astrophysical Journal*, **40**, 173, 1914.

[2] *Op. cit.*, p. 50.

TABLE XXVII

SUMMARY OF PARALLAX DETERMINATIONS FOR MESSIER 13

Method	π	No. Stars	Suggested Weight of $\frac{1}{\pi}$
I. From variable stars	0″.00008	2	4
II. From Kapteyn's luminosity-curves			
C. I. −0.39 to −0.20	.000005	17	1
" <−0.10	.000007	53	
" "	.000009	23	
(Pv. mag. <15.30)			
" −0.10 to −0.01	.00003	33	2
All colors	.00005	495	0
III. From Russell's data for absolute magnitude:			
C. I. <−0.10	.00005	53	4
All colors	0.00010	495	1
Provisionally adopted mean	0″.00003		

Incidental to the work on the Hercules cluster the following provisional parallaxes were obtained for similar objects:

Small Magellanic Cloud = 0″.00006
ω Centauri = 0.00027
Messier 3 = 0.0001

XIV. ON THE PROBABLE DIMENSIONS IN THE HERCULES CLUSTER

We conclude from the preceding considerations of the distance of the cluster that the parallax is certainly not in excess of +0″.0001. It may be but a tenth of this value, and some of the evidence would suggest still greater distance; but a parallax less than +0″.00001 is hard to reconcile with the magnitudes of the variable stars, if they are to be considered normal Cepheids. The provisionally adopted parallax, +0″.00003, is, however, more likely to be too large than too small. On the basis of this distance some computations relative to the probable dimensions of the cluster are summarized below.

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[1] See Section X, D.

[2] *Astronomische Nachrichten*, **196**, 205, 1913.

The angular diameter of that part of the cluster measured for the catalogue is less than 16′. It is readily seen, however, from the long-exposure photographs, as well as from the table of densities (Table XX), that the cluster extends far beyond this limit. Von Zeipel states,[1] on the basis of plates made especially for the purpose of fixing the limits, that the radius is not less than 17′. Accepting this result and the adopted value of the parallax, we find that the distance across the cluster is of the order of 1100 light-years. To an observer on the nearer edge of the cluster a star on the opposite side would have a parallax of +0″003, and if intrinsically one hundred times brighter than our sun, would still be nearly two magnitudes below the limit of visibility to the naked eye. To an observer on the earth, however, the difference in the apparent magnitude of two such stars, one on the nearer, the other on the farther side of the cluster, would be only 0.02 mag., and the difference in parallax relatively negligible.

At the distance corresponding to the adopted parallax, 100,000 light-years, the sun would appear fainter than the twenty-second magnitude; stars of the absolute brightness of Sirius would be fainter than the eighteenth magnitude. The absolute visual magnitude of the brightest star in the cluster would be −5.7. Several stars in the galactic system[2] of this absolute brightness are already on record; and of the giants so far known there naturally must be a selection in favor of the nearer ones. We know, in general, only the fainter limit for the absolute brightness of many if not most of the galactic giants. For instance, a second-magnitude star with a parallax not greater than 0″001 would have an absolute magnitude of −8 or brighter. Hence we could without seriously transgressing possibilities postulate absolute magnitudes of −10 or brighter. Since any such stars in the cluster do not exceed apparent magnitude 12.0, their distance must be nearly eight times greater than assumed above (perhaps of the order derived from the luminosity-curves of the early *b*'s) and the dimensions in the cluster would be octupled. But retaining the adopted value of the parallax, it is important to note that all the stars considered in the catalogues of Messier 13 have more than 200 times the solar brightness. In fact, probably no star as faint as the sun has yet been photographed in this cluster.

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[1] *Op. cit.*, p. 4.

[2] On fairly good evidence Walkey estimates the parallax of Canopus to be less than 0″007, and its absolute brightness, therefore, may be greater than represented by absolute magnitude −7 (*Knowledge*, **11**, 288, 1914).

The long-exposure plates by Pease[1] and by Ritchey[2] would indicate, on the basis of the assumed distance, that there are probably more than 50,000 cluster stars intrinsically brighter than the sun. If all stages of stellar development are represented in the cluster as we know them in our galactic system, we must believe that the total number of luminous stars is of the order of hundreds of thousands.[3]

The possibility of detecting relative proper motions in a system as distant as Messier 13 is very remote unless the observations extend over a great interval of time. Taking, for example, the upper limit of the parallax ($+0.''0001$), a relative cross-velocity of 50 km a second would correspond to an annual proper motion of only $0.''001$.

Taking into consideration the probable absolute magnitudes, the possibility of seeing nebulous matter in the cluster is similarly remote, unless the intrinsic brightness of such nebula is of a higher order than we believe such objects in our system to have. Nebulous material may be present, but to be visible in the greatest telescopes it must have an integrated brightness far in excess of the actual luminosity of Sirius.

Unless the distance is much greater than can be admitted at present, the condensation of stars near the center, notwithstanding the enormous dimensions of the cluster, must be considerably greater than the condensation believed to prevail in our galactic system near the sun. Since there are approximately 240 cluster stars brighter than photo-visual magnitude 15.5 within a central sphere of which the radius is 1′, the average distance separating these bright stars is about eight light-years. A star's nearer neighbors would appear brighter than Sirius appears to us. Multiply the distance and dimensions by eight, however, and there may be no more first-magnitude stars at any given point in the cluster than appear to us in our system. The number of faint stars in the central region is presumably very large, but we have no direct evidence that the increase in number with magnitude is the same there as for the outer parts.

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[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, **13**, 259, 1914.

[2] *Ibid.*, **9**, 166, 1910.

[3] Cf. references to von Zeipel's work in Section I.

Assigning for illustration a diameter of 10,000 light-years to the galactic system, it would appear from the Hercules cluster to have an angular diameter of about 5°, perhaps comparing closely in general appearance with the Greater Magellanic Cloud as seen from the earth. Our system, to the observer in the cluster, would also resemble the Magellanic Clouds in the remarkable characteristic of being rich in variables. From the cluster the thousand or so brightest apparent magnitudes would be our stars of greatest actual light, and among them, as in globular clusters and in the Magellanic Clouds, there would be a remarkably high percentage of variable stars. All types would be present, moreover, for we know they are all of high intrinsic luminosity; but those most readily discovered, for obvious reasons, would be the short-period Cepheids, that is, the cluster-type variables.

XV. SUGGESTIONS TOWARD SOME WORKING HYPOTHESES

Upon the basis of the foregoing computations and from the results of preceding sections of this paper we are led, for the guidance of future work on magnitudes in clusters, to suggest some working hypotheses relative to the form, extent, and distance of stellar systems. The proposed hypotheses, which are in part only assembled here from earlier pages, will be modified, no doubt, as the work progresses; their purpose now is chiefly to represent collectively and briefly those ideas concerning general stellar problems that the present study suggests.

The peculiar distribution of globular clusters and their great distances from the earth show that they do not form a part of the galactic stellar system.[1] Each globular cluster is a complete and separate system by itself. Apparently none of them is very near, compared to the outer limits of the galaxy. The stars of a few southern clusters, particularly of ω Centauri and 47 Tucanae, are brighter than those of Messier 13, and such systems may be much nearer than a hundred thousand light-years; but many clusters are considerably fainter and presumably more distant. In observing those similar to N.G.C. 4147 (if they are built on the same model as Messier 13) we may be seeing a million light-years into space. Our galactic system is distinctly out of the center of gravity of this higher order system of visible globular clusters.

Until contrary evidence is brought forward we may assume that all open clusters are parts of the galactic system.[2] This is suggested by their galactic concentration, by their motions so far as now measured, by the number of stars they contain, and more directly by the evidence of their comparatively small distances shown in the magnitudes and colors of their component stars. These galactic sub-groups are often if not generally real physical systems. The Hyades at a greater distance would resemble one of the galactic clusters, but because of the relative paucity of members could never be comparable to Messier 13, as is sometimes suggested.

Some of the globular clusters may be comparable to the galactic system in size, form, or at least in stellar constituency. For the Hercules cluster closely comparable dimensions cannot be claimed. A more appropriate comparison would be between our stellar system and the larger Magellanic Cloud. That the maximum radius of the Milky Way is probably not greater than ten thousand light-years and may be somewhat less has been deduced from many lines of evidence, the most important of which is the color of the faint stars. The shortest radius of the system is variously estimated from one-half to one-sixth of the radius in the galactic plane.

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[1] The asymmetric arrangement of globular clusters has led Bohlin to suppose that they form a system at the center of the galactic system and that the sun is eccentrically situated (*Kungl. Svenska Vetenskapsakademiens Handlingar*, **43**, No. 10, 1909). The relative distances of clusters and galactic stars precludes the possibility of such an hypothesis.

[2] The definite relationship of the planetary and extended nebulae to the galactic system has been recently established by Campbell and Moore, *Proceedings of the National Academy of Sciences*, **1**, 498, 1915.

Have globular clusters such planes of symmetry? From superficial appearances a negative answer is expected. But on closer consideration we conclude that the evidence at present is entirely indecisive. The discussion of planes of symmetry in globular clusters involves at least three important factors—the degree, orientation, and nature of the supposed oblateness. (1) The ratio of the shortest diameter to the longest may be near unity, so that only refined study from any position in space (in or out of the cluster) would be able to detect the oblateness. (2) The cluster may be so oriented in space that whatever the ratio of its axes of symmetry the discovery of oblateness is impossible. (3) The absolutely brighter stars may not show the oblate form, the property of condensation toward a plane being confined to the fainter stars. This latter consideration is very relevant. In our system the naked-eye stars of the second spectral type show practically no galactic concentration.[1] In globular clusters the brightest stars, which through their apparent arrangement have given the type-name to the systems, are those that are absolutely bright and red.

Under the most favorable conditions of oblateness and orientation, therefore, the analogy to the galactic system should make the elongated form apparent only for the faint stars in the cluster. The concentration may and probably does depend on spectral type as well as magnitude. The colors in the Hercules cluster have been examined for the purpose of testing possible oblate distribution of the blue stars, but no decisive result was obtained. The counts in other clusters have also as yet yielded no positive contribution to this matter, dealing as they mainly do with only the brightest stars. It should be noted, however, that a distinctly elongated or non-globular form has been noted for two or three of the brightest clusters where fainter stars are involved in the photographs, and counts have verified the superficial appearance of a plane of symmetry in ω Centauri.

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[1] These stars include absolutely bright and absolutely faint stars, both of which must be of sensibly globular distribution to avoid the necessity of assuming concentration of one or the other toward the galactic poles.

The conclusion is that globular clusters may very probably resemble our local stellar system in form, as well as in the properties of showing central condensation of stars, of containing all spectral types, including the giant red and yellow stars, and of having among their most luminous members a large quota of variables.

The base line used in the present case for the measurement of the amount of light-scattering in space is a very long one; those previously used have been relatively short. The hypothesis, based on the present result, that there is no appreciable space absorption is therefore fairly safe.[1] Since it is not certain as yet, however, whether or not molecular light-scattering exists in the central part of the cluster, we must make the reservation that there is a possibility of some measurable space absorption for lower galactic latitudes. And of course we may expect patches of absorbing material here and there in space, as, for example, Barnard's dark nebulae in the Milky Way. In any case absorption should be treated as a varying quantity[2] depending on the condensation of light-scattering material, and not as a quantity that is constant whether in the densest part of our galactic system or in the starless realm without.

The decision that molecular scattering of light is inappreciable in the direction of Messier 13 probably justifies the assumption that general non-selective space absorption (obstruction) is also of small importance. It also permits safer sounding of the depths of the

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[1] The coefficient of absorption can scarcely be one-tenth the most recent previous values, even with very generous allowance for error. That being the case, the result is intimately related to the discussion of the time-honored problem of the finiteness of the stellar universe. By universe in this sense we mean the system that is formed by stars of the kind that surround the sun and appear to us in all parts of the sky. Either the extent of the star-populated space is finite, or "the heavens would be a blazing glory of light," or there is an absorption of light in space, or there is a peculiar non-uniform distribution of stars and star-systems throughout space (Charlier), or, except in our immediate vicinity, the stars as light-giving bodies are dead or not yet born. The last two clauses are usually not included in the discussion. Omit them. Then, since the heavens are not a blazing glory, and since space absorption is of little moment throughout the distances concerned in our galactic system, it follows that the defined stellar system is finite. There is nothing novel in this conclusion. It has been the prevailing one for a long time, and has been very easily derived by making the assumption, in spite of prevailing values of the coefficient, that space absorption is really negligible.

[2] Seeliger has considered space absorption both as constant and as decreasing with distance (*Sitzungsberichte der mathematisch-physikalischen Klasse der Königlich-Bayerschen Akademie der Wissenschaften*, 1911, Heft II, p. 413).

Milky Way system in various directions, and, together with the absence of blue stars of fainter magnitudes, suggests that the limits have already been reached. There is a suggestion among the globular clusters that the stars at great distances from the center are intrinsically fainter than the nearer ones, because of either greater age, or smaller mass, or dependence of spectrum on distance. If such a condition exists in our galactic system we can better interpret the color-distribution of faint stars.

XVI. SUMMARY

All of the observations and measurements connected with the investigation of Messier 13 have been made by the writer. A great deal of assistance has been given by Mrs. Shapley in making and checking computations, in forming and manipulating the card catalogues that were used in the statistical discussions, and in the preparation of the paper for the press. A summary of the methods and purposes of the study of cluster magnitudes appears at the end of the preceding paper of this series. Summaries of the probable errors and of considerations of the parallax have been given in Sections IV and XIII. A synopsis of the results for the whole of the present paper is contained in the following paragraphs.

1. Former work on the Hercules cluster has been concerned chiefly with the measurement of positions, the observation of variables, and the theoretical discussion of the laws of stellar distribution. The present study involves the determination of star colors through the comparison of photographic and photovisual magnitudes.

2. Twelve plates, seven Seed "27" and five Cramer "Instantaneous Iso," made at the principal focus of the 60-inch reflector, were used for different phases of the work. The scales of magnitudes were transferred directly from the North Pole to the cluster on all plates used as standards or as controls. More than ten thousand estimates and measures of magnitudes are involved.

3. Small systematic errors related to distance correction, astigmatism, and length of exposure are found, but none is important

(see Section IV, A). The most serious difficulty involves the probable influence of the Eberhard effect, discussed in Sections IV, C, and VIII. The relative error in the zero points of the two magnitude scales does not exceed a few hundredths of a magnitude, and no appreciable scale-divergence is found.

4. The probable error of an average color-index is about six hundredths of a magnitude. This does not entirely include the systematic error allied to the color-excess in the center, but is valid for distance greater than 2′.

5. A catalogue of the photographic and photo-visual magnitudes of about 1300 stars has been prepared. Owing to the probable presence of systematic error in the magnitudes in the denser central regions, the lists published in Sections V and XI contain only about 650 stars; publication of the remainder awaits further investigation of suspected errors. Probably not more than 18 of the magnitudes refer to superposed non-cluster stars.

6. Discarding incomplete or doubtful results, 1049 magnitudes and color-indices are available for statistical investigation, of which 495 are outside the central region. The range of observed photo-visual magnitudes is from 11.9 to 16.8; but all of the stars are included only down to magnitude 15.5.

7. Five new variable stars have been discovered, making a total of seven in the cluster. The periods are probably short, and there is evidence that all are bluer at maximum than at minimum, suggesting the Cepheid type of variation. The two previously known variables are almost certainly Cepheids.

8. The values of the color-index in the cluster range from −0.52 to +1.96, with the greatest frequency about +0.90. Excluding the zone within 2′ of the center, the upper limit is +1.45, with greatest frequency near +0.70. More than 17 per cent are negative (outside distance 2′), and there appears to be no reason for attributing this result to error of any kind.

9. The size and distribution of the color-indices are what might be expected on the basis of no selective scattering of light in space. At least the absorption in the direction of the Hercules cluster must be less than +0.0001 mag. for each unit of distance ($\pi=0''.1$). The values derived by Kapteyn, Jones, Van Rhyn,

and some others are impossible in this case, even with the most favorable assumptions as to the distance of the cluster.

10. The average color-index at greater distances from the center than 2′ is +0.55, corresponding to spectrum F4. Toward the center of the cluster it increases to at least +1.3, and no negative values appear. The increase is present for all magnitudes (Figs. 6 and 7) and is probably to be interpreted as a systematic photographic error, perhaps the Eberhard effect. If the phenomenon is real, however, it would indicate an absorption within the cluster or a preferential concentration of red stars in that region.

11. The possible existence of "cluster absorption" suggests that for lower galactic latitudes there may be a measurable amount of space absorption; observations are being made for the purpose of testing this hypothesis.

12. The determination by Pease of spectra of individual stars in the cluster gives results confirmatory of the direct interpretation of color-class as spectral type, and verifies the excess of color in the center.

13. Of the four hundred faintest stars measured in the cluster, 85 per cent are bluer than color-class *g*0, which corresponds to solar spectral type; of the four hundred brightest stars 75 per cent are redder than *g*0. An analogous result holds for the outer regions alone.

14. The almost linear decrease of color-index with decreasing brightness is in evidence in all regions of the cluster (see Fig. 8). The phenomenon probably has an important significance in considerations of the order of development of giant stars.

15. Between photo-visual magnitudes 16.0 and 16.8 only four out of the special list of a hundred faint stars have color-indices greater than half a magnitude, and none is found redder than the sun (Section XI).

16. There is some evidence that the brighter stars are more strongly condensed in the center of the cluster than the faint ones; at least the average magnitude is brighter, even without allowing for the color-excess. Outside the central zone the cluster stars seem to be very thoroughly mixed with respect to magnitude and color.

17. The laws connecting star-density and distance cannot be determined until the phenomenon interpreted here as Eberhard effect (which also depends on distance from the center) has been thoroughly investigated; even neglecting this effect, the distribution laws ordinarily derived must be recognized as applying to only that very small percentage of all the stars which is represented by those of exceptionally high intrinsic brightness.

18. The probable distance of the Hercules cluster is estimated from considerations of its variable stars, its fragmentary luminosity-curves, and the average apparent brightness of certain color-groups. The parallax must be less than +0.″0001 and is probably greater than +0.″00001 (see Table XXVII).

19. At the provisionally adopted distance of a hundred thousand light-years, the cluster would be more than a thousand light-years in diameter. For a summary of further considerations of this nature see Section XIV.

20. There seems to be reasonably clear evidence that this and similar globular clusters are very distant systems, distinct from our galaxy and perhaps not greatly unlike it in size and form (Section XV). The open clusters, on the other hand, seem to be relatively small parts of the local system; but further observation is necessary to make certain of this conclusion.

To summarize in a single sentence the main results of the study of the Hercules cluster: Evidence is presented to show that in the direction of the cluster space absorption is entirely negligible, that over a range of three magnitudes at least the average color-index decreases with decreasing brightness, and that the distance of the cluster is so great that it and kindred objects should be considered as systems apart from our galaxy and of enormous dimensions.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

THIRD PAPER: A CATALOGUE OF 311 STARS IN MESSIER 67

By HARLOW SHAPLEY

The open cluster Messier 67 was discovered about a century ago at Milan by Oriani.[1] The early observations consisted mainly of general descriptions of the group by Messier and Sir John Herschel, and later of notes relative to the number and distribution of its members by Smyth[2] and Fenet.[3] More recently two photographic catalogues have been made of the positions of the individual stars, the first by Olsson at Stockholm in 1898,[4] the second eight years later by Fagerholm at Upsala.[5] In the positions of the 118 stars common to the two catalogues various systematic differences appear, but regarding the evidence of motion during the interval of eight years, Fagerholm concludes: "Im Hinblick auf die vielen Ursachen, welche diese zufälligen Abweichungen zu vergrössern streben, begreift man also, dass sich reelle Eigenbewegungen nicht aus dem Vergleich zwischen meiner Ausmessung von Messier 67 und derjenigen Olssons bestimmen lassen. Hierzu ist eine genaue Ausmessung nach einer bedeutend längeren Zeit als das knappe Dezennium erforderlich. . . . ."[6]

In the catalogues of positions the magnitudes are determined only approximately. The values derived by Olsson, evidently depending on rough estimates, are systematically too bright by more than two magnitudes. Fagerholm's magnitudes, on the other hand, are determined by careful measurements of diameters of stellar images; but they are based upon an extension of the visual scale of the *Potsdam Durchmusterung* and are systematically in error owing to the color of the stars.

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[1] Messier 67=G.C. 1712=N.G.C. 2682=h 531; $\alpha=8^h 45^m.8$, $\delta=+12°11'$, 1900.0; galactic longitude 183°, galactic latitude +34°. Position taken from *Harvard Annals*, **60**, 210, 1908.

[2] *Celestial Cycle*, p. 241.

[3] *Bulletin de la Société Astronomique de France*, **6**, 145, 1892.

[4] *Astronomiska Iakttagelser och Undersökningar anstälda på Stockholms Observatorium*, **6**, No. 4, 1898. Contains 148 stars.

[5] *Dissertation*, Upsala, 1906. Contains 295 stars.

[6] *Op. cit.*, p. 82.

In the present catalogue photographic magnitudes are given for all objects listed by Fagerholm, and for a few others which are within the zone covered by him but are missing from his table. Photo-visual magnitudes are determined for all but four of the stars within distance 12′ of the center. After discarding a few questionable values a total of 232 color-indices is available for statistical discussion. The observations used in the derivation of magnitudes are sufficiently described in Table I. Each plate involves a direct comparison with the Polar Standards. Because of uncertainty in the magnitudes of the brighter stars a larger number of isochromatic plates was made than usual.

TABLE I

LIST OF PLATES

No. Plate	Date, G.M.T.	Kind of Plate	Apertures	No. of Exposures	Duration of Each	Extinction Correction
2217....	1914, Nov. 20.925	Seed 27	60, 32	3	5^{m}	$+0^{M}.16$
2218....	20.929	Iso	60, 32	3	5	+ .09
2219....	20.969	Iso	60, 32	3	5	+ .11
2220*a*...	20.960	Seed 27	60	2	2	+ .21
2220*b*...	20.962	Seed 27	32	2	2	+ .21
2328....	1915, Mar. 16.637	Iso	60	3	5	+ .11
2329....	16.640	Seed 27	60	3	1	+ .21
2330....	16.662	Iso	60	3	3	+ .11
2366....	Apr. 16.786	Iso	32	3	15	.00
2444*a*...	June 6.679	Iso	60	2	3	− .09
2444*b*...	6.683	Iso	60	2	5	−0.12

The average errors are considered in some detail for representative groups of stars in Tables II and III for photographic and photo-visual magnitudes, respectively. The residuals are given in hundredths of a magnitude. For Plates 2217 and 2220*a* the appreciable systematic deviations, probably to a large degree due to the size and uncertainty of the extinction corrections, are applied to the individual magnitudes in computing residuals, and these corrected values in parentheses are employed in deriving average deviations. Such corrections are not made for the photo-visual magnitudes, and the relatively larger average deviations in Table III are to be ascribed in part to the inclusion of the systematic errors of individual plates in the final results. The errors for Messier 67 are larger than those for Messier 13,[1] owing chiefly to the average brighter magnitude and consequent difficulty of measurement. No evidence of progression of error with distance from the center is

TABLE II

PHOTOGRAPHIC RESIDUALS

Star	Mean Pg. Mag.	Residuals			
		2217	2220*a*	2220*b*	2329
84	12.14	− 1 (+ 6)	− 4 (−11)	+ 8	− 2
90α	12.04	− 5 (+ 2)	+19 (+12)	−10	− 4
90β	13.42	+ 3 (+10)	+15 (+ 8)	− 9	− 7
91	13.67	+ 1 (+ 8)	−13 (−20)	− 7	+18
94	13.62	−17 (−10)	+18 (+11)	+ 5	− 4
95	13.48	− 3 (+ 4)	+ 6 (− 1)	+ 2	− 5
98	13.49	− 7 (0)	+ 8 (+ 1)	+12	−12
102	13.50	−10 (− 3)	+ 4 (− 3)	+10	− 6
104	12.70	−13 (− 6)	+ 2 (− 5)	+ 6	+ 7
105	12.13	− 6 (+ 1)	+ 7 (0)	− 5	+ 5
106	13.95	−15 (− 8)	+24 (+17)	0	−10
107	14.58	−12 (− 5)	+14 (+ 7)	− 7	+ 4
108	11.68	− 7 (0)	+20 (+13)	−20	+ 7
111	13.58	−20 (−13)	− 4 (−11)	+ 2	+22
117	13.77	−15 (− 8)	+ 3 (− 4)	+ 5	+ 8
119	13.47	− 4 (+ 3)	+ 4 (− 3)	+14	−15
124	13.00	−24 (−17)	+27 (+20)	−16	+13
127	13.61	−18 (−11)	+ 4 (− 3)	+ 7	+ 9
130	13.55	− 1 (+ 6)	− 1 (− 8)	+13	− 9
131	12.05	− 2 (+ 5)	− 3 (−10)	+ 3	+ 1
136	12.30	+10 (+17)	− 4 (−11)	− 1	− 6
140	14.00	− 9 (− 2)	+22 (+15)	−15	+ 1
141	12.13	− 3 (+ 4)	− 9 (−16)	+ 6	+ 5
143	12.86	− 5 (+ 2)	+20 (+13)	− 7	− 6
145	13.74	−12 (− 5)	+23 (+16)	−14	+ 3
149	13.45	− 5 (+ 2)	+ 6 (− 1)	+13	−13
151	12.08	+ 5 (+12)	−13 (−20)	+ 7	+ 2
153	11.80	−10 (− 3)	+21 (+14)	−15	+ 4
155	11.42	− 4 (+ 3)	− 5 (−12)	+ 3	+ 6
156	11.45	+ 5 (+12)	− 3 (−10)	+ 1	− 4
Systematic deviation		− 7 (0)	+ 7 (0)	− 0	+ 0
Average deviation		(± 6)	(±10)	± 8	± 7

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[1] *Mt. Wilson Contr.*, No. 116.

apparent, and, in fact, since the cluster is not strongly condensed toward the center, the Eberhard effect or any analogous disturbance would be unexpected. The scales of the different plates deviate but slightly from each other, but it was found advisable to derive no results for the brightest or faintest stars on certain isochromatic plates because of the uncertainty at one or the other end of the curve of scale-reading against magnitude.

TABLE III

PHOTO-VISUAL RESIDUALS

Star	Mean Pv. Mag.	Residuals						
		2218	2219	2328	2330	2366	2444*a*	2444*b*
55...	11.55	−11	− 6	+ 7	....	+10	....	....
70...	11.68	− 8	−10	+16	....	+ 4	....	....
81...	10.23	− 1	....	....	−12	−13	+12	+14
84...	10.55	+10	....	....	....	−22	+ 6	+ 5
90*a*..	11.17	+ 7	+12	....	−21	− 5	0	+ 9
91...	12.72	....	− 7	+ 2	+ 4	....	....	....
95...	12.81	....	−10	− 5	+ 4	....	....	+11
100...	13.24	....	− 6	....	+13:	....	....	....
102...	12.44	....	− 5	....	−10	....	+ 1	+14
104...	11.35	−17	+ 9	+17	....	−10	+ 7	− 4
105...	10.38	+12	....	....	−26:	−11	+13	+ 1
106...	12.93	....	+ 1	+ 4	− 4	....	....	....
108...	9.90	+ 5	....	....	− 5:	−16:	0	+ 7
111...	12.61	....	− 7	+13	− 6	....	....	....
117...	12.64	....	− 8	+10	− 2	....	....	....
124...	12.25	−12	−10	+ 4	+ 1	+ 8	+ 8	+ 4
127...	12.72	....	−12	+ 5	+ 8	....	....	....
134...	12.34	+ 2:	−14	+11	+ 1	− 1	....	....
141...	10.45	+14	....	....	−10	− 7	+ 8	− 3
143...	11.63	− 3	....	+ 1	....	....	+10	− 8
145...	12.82	....	− 6	− 5	+10	....	....	....
149...	12.70	....	−11	−12	− 8	....	+11	+21
153...	11.50	− 6	0	+ 7	−35:	+15	....	....
155...	10.46	+28	....	....	−13	−15	+ 1	− 2
170...	9.85	+19	....	....	....	−32:	− 4:	− 1:
Systematic deviation.....		+ 3	− 5	+ 5	− 5	− 6	+ 6	+ 5
Average deviation.....		±11	± 8	± 8	± 9	±11	± 6	± 8

The mean magnitudes derived from all plates are given in Table IV. The number of the star in the first column is that

TABLE IV

Catalogue of Magnitudes in Messier 67

No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index	No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index
1	16	14.12	....	....	46	9	13.43	12.99	+0.44
2	15	14.05	....	....	47	14	12.44	11.48	+0.96
3	13	13.56	....	....	48	7	13.55	12.66	+0.89
4	14	13.97	....	....	49	11	13.47	12.58	+0.89
5	12	13.86	....	....	50	9	13.95	13.21	+0.74
6	11	13.47	....	....	51	6	13.34	12.68	+0.66
7	13	14.74	....	....	52	6	14.15	13.25	+0.90
8	12	13.40	....	....	53	6	13.95	13.21	+0.74
9	14	14.18	....	....	54	7	13.46	12.66	+0.80
10	15	13.15	....	....	55	7	12.08	11.55	+0.53
11	12	14.38	13.41	+0.97	56	10	13.91	13.17	+0.74
12	12	13.71	....	....	57	17	14.27	....	....
13	11	14.72	....	....	58	6	14.54	13.69	+0.85
14	10	14.52	13.76	+0.76	59	7	14.33	13.43	+0.90
15	10	14.16	13.41	+0.75	60	15	13.56	....	....
16	10	13.38	12.61	+0.77	61	6	14.12	....	....
17	11	14.72	13.75	+0.97	62	5	14.32	13.30	+1.02
18	15	13.51	....	....	63	6	13.97	13.14	+0.83
19	11	14.64	13.76	+0.88	64	7	14.72	13.77:	+0.95:
20	9	13.95	12.80	+1.15	65	7	13.37	12.85	+0.52
21	15	14.10	....	....	66	6	14.54	13.50	+1.04
22	12	13.46	....	....	67	14	14.77	....	....
23	9	13.36	12.41	+0.95	68	13	14.42	....	....
24	8	13.97	13.03	+0.94	69	13	14.19	....	....
25	12	14.28	....	....	70	5	12.40	11.68	+0.72
26	11	14.38	....	....	71	5	14.20	13.31	+0.89
27	9	14.25	13.31	+0.94	72	5	13.74	12.39	+1.35
28	11	13.93	12.76	+1.17	73	5	14.40	13.48	+0.92
29	9	14.00	13.24	+0.76	74	15	14.45	....	....
30	9	12.84	12.00	+0.84	75	5	14.14	13.20	+0.94
31	9	13.95	13.43	+0.52	76	4	14.70	13.81	+0.89
32	15	13.96	....	....	77	4	14.16	13.12	+1.04
33	8	14.67	13.90	+0.77	78α	11	14.10	13.29	+0.81
34	13	13.61	....	....	78β	11	14.45	13.65	+0.80
35	13	14.76	....	....	79	8	13.44	12.64	+0.80
36	10	13.83	13.18	+0.65	80	4	14.38	13.45	+0.93
37	8	13.85	12.72	+1.13	81	5	10.39	10.23	+0.16
38	15	14.79	....	....	82	4	14.66	13.51	+1.15
39	10	13.93	13.06	+0.87	83	4	14.02	12.98	+1.04
40	7	14.46	13.36	+1.10	84	5	12.14	10.55	+1.59
41	11	13.45	13.10	+0.35	85	4	14.32	13.32	+1.00
42	7	14.11	13.21	+0.90	86	4	14.64	13.28	+1.36
43	7	14.09	13.26	+0.83	87	4	14.19	13.28	+0.91
44	11	13.65	13.14	+0.51	88	4	13.21	12.36	+0.85
45	12	14.30	....	....	89	4	14.26	13.10	+1.16

TABLE IV—*Continued*

No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index	No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index
90α..	5	12.04	11.17	+0.87	132..	4	13.98	13.12	+0.86
90β..	5	13.42	12.48	+0.94	133..	9	14.28	13.40	+0.88
91...	3	13.67	12.72	+0.95	134..	3	13.35	12.34	+1.01
92...	10	14.28	13.76	+0.52	135..	3	13.15	11.51	+1.64
92*a*..	10	13.38	12.32	+1.06	136..	4	12.30	11.43	+0.87
93...	3	14.68	13.41	+1.27	137..	3	14.71	13.79	+0.92
94...	3	13.62	12.72	+0.90	137*a*.	3	14.04	13.11	+0.93
95...	3	13.48	12.81	+0.67	137*b*.	3	15.06	13.93	+1.13
96...	4	13.98	12.76	+1.22	138..	10	14.16	13.36	+0.80
97...	3	14.76	13.83	+0.93	139..	11	14.41	13.78	+0.63
98...	5	13.49	12.74	+0.75	139*a*.	11	14.58	13.89	+0.69
99...	6	14.10	13.27	+0.83	140..	2	14.00	13.04	+0.96
100...	5	14.20	13.24	+0.96	141..	1	12.13	10.45	+1.68
101...	3	14.00	12.94	+1.06	142..	5	14.68	13.96	+0.72
102...	4	13.50	12.44	+1.06	143..	1	12.86	11.63	+1.23
102*a*..	4	14.69	....	....	143*a*.	1	13.98	13.06	+0.92
103...	3	14.00	12.98	+1.02	143*b*.	1	13.48	12.82	+0.66
104...	3	12.70	11.35	+1.35	144..	2	13.91	12.86	+1.05
105...	2	12.13	10.38	+1.75	145..	1	13.74	12.82	+0.92
106...	3	13.95	12.93	+1.02	146α.	2	14.43	13.36	+1.07
107...	3	14.58	13.20	+1.38	146β.	2	14.46	13.25	+1.21
108...	4	11.68	9.90	+1.78	147..	1	14.08	13.03	+1.05
109...	3	14.28	13.16	+1.12	148..	4	13.93	13.05	+0.88
110...	9	14.20	13.31	+0.89	149..	1	13.45	12.70	+0.75
111...	4	13.58	12.61	+0.97	150..	1	14.08	13.03	+1.05
112...	6	14.03	12.95	+1.08	151..	5	12.08	10.62	+1.46
113...	7	14.70	13.85	+0.85	152..	1	14.16	13.26	+0.90
114...	6	14.13	13.12	+1.01	153..	5	11.80	11.50	+0.30
115...	2	13.34	12.70	+0.64	154..	10	14.16	13.40	+0.76
116...	3	14.08	12.88	+1.20	155..	0	11.42	10.46	+0.96
117...	3	13.77	12.64	+1.13	156..	3	11.45	11.19	+0.26
118...	8	14.52	13.84	+0.66	157..	5	13.40	12.72	+0.68
119...	2	13.47	12.79	+0.68	157*a*.	4	14.68	13.57	+1.11
120...	2	14.48	13.43	+1.05	158..	16	12.73	12.43:	+0.30:
121...	2	14.48	13.50	+0.98	159..	7	13.83	13.12	+0.71
122...	9	14.22	13.38	+0.84	160..	15	13.88	....	....
123...	4	14.64	13.55	+1.09	161..	1	13.40	12.77	+0.63
124...	2	13.00	12.25	+0.75	162..	4	13.42	12.77	+0.65
125...	4	14.40	13.52	+0.88	163..	3	13.44	12.68	+0.76
126...	2	14.71	13.52	+1.19	164..	2	12.06	10.75	+1.31
127...	3	13.61	12.72	+0.89	165..	6	13.37	12.72	+0.65
128...	3	13.91	12.99	+0.92	166..	3	14.00	12.79	+1.21
129...	3	13.94	13.08	+0.86	167..	10	14.32	....	....
130...	3	13.55	12.93	+0.62	168..	3	13.93	13.03	+0.90
131...	5	12.05	11.30	+0.75	169	3	14.68	13.58	+1.10

TABLE IV—*Continued*

No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index	No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index
170...	2	11.76	9.85	+1.91	213..	4	14.40	13.48	+0.92
171...	5	13.81	12.93	+0.88	214..	3	14.46	13.58	+0.88
172...	14	13.49	....	....	215..	7	13.40	12.70	+0.70
173α..	1	13.55	12.20	+1.35	216..	6	13.37	12.60	+0.77
173β..	1	13.97	13.05	+0.92	217..	5	12.67	11.43	+1.24
174...	1	13.46	12.68	+0.78	217a.	4	13.43	12.15	+1.28
175...	5	14.20	13.44	+0.76	218..	4	13.24	11.70	+1.54
176...	3	13.42	12.72	+0.70	219..	4	13.40	12.60	+0.80
177...	5	13.85	12.92	+0.93	220..	12	14.65	....	....
178...	3	14.76	13.64	+1.12	221..	4	13.28	12.56	+0.72
179...	7	14.05	13.45	+0.60	222..	13	14.42	....	....
180...	2	13.38	12.62	+0.76	223..	9	11.96	10.74	+1.22
181...	2	13.84	12.84	+1.00	224..	6	12.57	11.12	+1.45
182...	2	13.42	12.74	+0.68	225..	5	13.81	12.94	+0.87
183...	13	14.09	....	....	226..	5	13.81	12.72	+1.09
184...	2	13.02	12.29	+0.73	227..	5	14.03	12.72	+1.31
185...	1	11.84	11.28	+0.56	228..	8	13.84	12.90	+0.94
185a..	2	13.60	12.68	+0.92	229..	6	14.01	13.01	+1.00
186...	11	14.46	13.75	+0.71	229a.	6	13.42	12.78	+0.64
187...	3	14.06	12.99	+1.07	230..	10	13.95	13.21	+0.74
188...	3	14.31	13.41	+0.90	231..	5	13.10	11.61	+1.49
189	2	13.65	12.74	+0.91	232..	11	14.18	13.41	+0.77
190...	3	11.61	11.19	+0.42	233..	7	14.03	13.08	+0.95
191...	11	13.90	12.99	+0.91	234..	13	12.28	....	....
192...	10	13.40	12.77	+0.63	235..	5	14.00	13.01	+0.99
193...	5	13.52	12.27	+1.25	236..	6	13.40	12.35	+1.05
194...	2	14.39	13.38	+1.01	237..	10	14.05	12.70	+1.35
195...	3	13.77	12.68	+1.09	238..	5	11.54	11.19	+0.35
196...	9	14.52	13.83	+0.69	239..	8	14.62	13.73	+0.89
197...	10	14.40	13.62	+0.78	240..	17	13.46	....	....
198...	15	14.42	....	....	241..	6	13.44	12.68	+0.76
199...	7	13.50	13.03	+0.47	242..	7	9.30	8.03:	+1.27:
200...	11	11.96	11.03	+0.93	243..	6	13.42	12.66	+0.76
201...	3	14.67	13.58	+1.09	244..	6	12.12	11.03	+1.09
202...	3	13.42	12.66	+0.76	245α.	7	14.05	13.18	+0.87
203...	13	13.58	....	....	245β.	7	14.20	13.40	+0.80
204...	17	14.46	....	....	246..	17	13.56	....	....
205...	4	14.13	13.12	+1.01	247..	14	13.96	....	....
206...	11	13.57	12.68	+0.89	248..	11	13.42	12.34	+1.08
207...	12	13.00	....	....	249..	17	14.08	....	....
208...	7	14.40	....	....	250..	13	13.72	....	....
209...	3	14.44	13.35	+1.09	251..	14	14.73	....	....
210...	3	13.24	12.32	+0.92	252..	17	13.40	....	....
211...	5	14.42	13.51	+0.91	253..	16	14.46	....	....
212...	14	13.56	....	....	254..	10	13.93	13.24	+0.69

TABLE IV—*Continued*

No.	Dist.	Pg. Mag.	Pv. Mag.	Color-Index
255...	7	13.40	12.66	+0.74
256...	12	13.49	....	....
257...	11	12.32	11.64	+0.68
258...	11	14.65	13.83	+0.82
259...	7	14.60	13.70	+0.90
260...	15	14.73	....	....
261...	8	11.17	10.74	+0.43
262...	7	14.02	12.70	+1.32
263...	7	14.50	13.70	+0.80
264...	15	14.46	....	....
265...	8	13.10	12.46	+0.64
266...	10	11.70	10.68	+1.02
267...	9	13.43	12.68	+0.75
268...	9	14.14	13.12	+1.02
269...	9	13.74	13.05	+0.69
270...	13	14.59	....	....
271...	12	13.48	....	....
272...	14	13.55	....	....
273...	14	14.39	....	....
274...	10	14.14	13.16	+0.98
275...	10	14.02	13.10	+0.92
276...	10	14.38	13.65	+0.73
277...	12	13.94	....	....
278...	10	14.01	13.46	+0.55
279...	11	14.65	13.87	+0.78
280..	12	11.25	11.06	+0.19
281..	11	13.75	12.93	+0.82
282..	12	14.15	13.23	+0.92
283..	13	14.39	....	....
284..	14	14.31	....	....
285..	13	13.20	....	....
286..	13	12.11	10.74	+1.37
287..	14	13.53	....	....
288..	17	13.96	....	....
289..	14	13.53	....	....
290..	14	14.45	....	....
291..	16	13.61	....	....
292..	17	14.70	....	....
293..	16	14.33	....	....
294..	17	14.35	....	....
295..	17	14.27	....	....

NOTES TO THE CATALOGUE

Star
21 and 22. Plotted erroneously on Fagerholm's chart.
90α and 90β. Components so close that measures are somewhat uncertain.
92*a*=Olsson 53. Not seen by Fagerholm. No other indication of variability.
116. Owing to an error in the reductions this star is plotted incorrectly on Fagerholm's chart and an erroneous position is given in his final catalogue. The declination for 1906.0 should read +12° 12′ 24″ instead of +12° 7′ 6″.
135. There is some indication that this star is variable.
217*a*=Olsson 94. Not seen by Fagerholm. No other indication of variability.
229*a*=Olsson 105. Not seen by Fagerholm. No other indication of variability.
242. Too bright for accurate measurement on most of the plates.
246. Plotted erroneously on Fagerholm's chart.
248. This may be a variable star.

assigned by Fagerholm and progresses with right ascension. The postscript letters *a*, *b*, α, β, are employed in the way described in *Mt. Wilson Contr.*, No. 116, p. 23. The distances from the center of the cluster are counted from star No. 155 as origin.[1] Following Table IV are a few notes relative to individual stars and to the errors in the catalogue and chart printed by Fagerholm.

In the notes to the catalogue, three of the postscript stars are identified with objects observed by Olsson. For the 18 others, positions sufficiently approximate to enable ready identification are given in Table V. The co-ordinates were estimated directly from the negatives with the aid of a reseau on glass, each postscript star being referred to neighboring stars of Fagerholm's catalogue. Five objects previously measured as single stars are now seen to be close doubles.

TABLE V

POSITIONS OF POSTSCRIPT STARS

Star	α 1906.0	δ 1906.0	Star	α 1906.0	δ 1906.0
78α........	131°30′ 12″	+12° 20′ 19″	143*b*.......	131° 33′ 35″	+12° 9′ 56″
78β........	30 13	20 14	146α.......	33 53	8 16
90α........	31 28	6 5	146β.......	33 55	8 18
90β........	31 30	6 7	157*a*.......	34 43	14 9
102*a*........	32 6	6 47	173α.......	35 28	10 6
137*a*........	33 28	12 33	173β.......	35 31	10 1
137*b*........	32 51	12 7	185*a*.......	36 11	9 58
139*a*........	33 10	20 10	245α.......	40 28	6 6
143*a*........	33 36	10 18	245β.......	40 25	6 10

In determining magnitudes Fagerholm used the formula due to Charlier:

$$\text{Mag.} = a + b \log D,$$

where D is the diameter of the stellar image and a and b are constants to be derived from stars of known photographic magnitude. The data for the evaluation of the constants were obtained by measuring with a polarizing photometer the visual magnitudes of fifteen cluster stars with reference to neighboring stars of the *Potsdam Durchmusterung*. Excluding the values pertaining to the reddest two of these objects, the remaining 13 standard magnitudes were adopted without modification by Fagerholm as equivalent to photographic magnitudes and used to determine a and b. Table VI contains a comparison of his directly observed magnitudes with the photo-visual results of the present investigation. The two stars designated as red are in parentheses. Color-indices and the differences between the two determinations are also included in the table.

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[1] Its position at the epoch of Fagerholm's catalogue, 1906.0, is $\alpha = 8^{h}\,46^{m}\,18\overset{s}{.}34$, $\delta = +12^{\circ}\,9'\,46\overset{''}{.}59$.

TABLE VI

Comparison with Fagerholm's Magnitudes

Star	Color-Index	Magnitude		F.−Mt. W.
		Fagerholm	Mount Wilson	
54.....	+0.80	13.21	12.66	+0.55
55.....	+0.53	11.76	11.55	+ .21
81.....	+0.16	10.39	10.23	+ .16
90α....	+0.87	11.41	11.17	+ .24
(105.....	+1.75	10.16	10.38	− .22)
131.....	+0.75	11.54	11.30	+ .24
156.....	+0.26	11.25	11.19	+ .06
185.....	+0.56	11.26	11.28	− .02
190.....	+0.42	11.24	11.19	+ .05
216.....	+0.77	13.23	12.60	+ .63
224.....	+1.45	11.33	11.12	+ .21
231.....	+1.49	11.97	11.61	+ .36
238.....	+0.35	11.50	11.19	+ .31
(242.....	+1.27:	8.52	8.03:	+ .49:)
244.....	+1.09	11.13	11.03	+0.10
Mean of 13	+0.73			+0.24

The mean difference

$$\mathrm{F.}-\mathrm{Mt.W.}=+0\overset{\mathrm{M}}{.}24$$

holds for magnitude 11.5 and is in satisfactory agreement with the difference at the sixth magnitude found by Seares[1]

$$\mathrm{P.D.}-\mathrm{Mt.W.}=+0\overset{\mathrm{M}}{.}19$$

The discrepancy, therefore, between the new results and Fagerholm's is mainly one of zero-point difference between Mount Wilson and Potsdam. This correction, taken in connection with the large average color-index of the standard stars (+0.73), indicates that on the average Fagerholm's magnitudes are systematically half a magnitude too bright in comparison with Mount Wilson photographic results.

For the statistical discussion the results have been tabulated on the basis of photo-visual magnitude. The notation and arrangement closely follow that employed in the paper on Messier 13. As more than half of the stars have color-indices between +0.8 and +1.2, the color-class *g* has been divided into two equal parts in Tables VII and VIII. No negative color-indices were found and class *b* is omitted. The data in the catalogue, Table IV, for two stars, Nos. 64 and 242, have been excluded from the discussion because of uncertainty of the measurements of magnitude; and for six others, Nos. 11, 47, 158, 280, 282, and 286, the data have been excluded because the distance from the center equals or exceeds 12′.

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[1] *Mt. Wilson Contr.*, No. 97, p. 29; *Astrophysical Journal*, **41**, 234, 1915.

TABLE VII

Frequency of Colors and Magnitudes

Distance From Center	Limits of Pv. Mag.	Color-Class						All Colors
		a	*f*	*g* 0–4	*g* 5–9	*k*	*m*	
0′.0 to 4′.5....	<11.0	......	......	1	......	1	4	6
	11.0–11.5	1	2	1	......	1	......	5
	11.5–12.0	......	......	......	......	2	1	3
	12.0–12.5	......	2	2	2	2	......	8
	12.5–13.0	......	14	9	9	3	......	35
	13.0–13.5	......	......	13	12	4	......	29
	≥13.5	......	......	6	7	......	......	13
4.5 to 8.5....	<11.0	1	1	......	......	2	......	4
	11.0–11.5	1	1	1	1	2	......	6
	11.5–12.0	1	2	......	......	1	......	4
	12.0–12.5	......	1	2	1	2	......	6
	12.5–13.0	......	12	7	3	2	......	24
	13.0–13.5	......	6	14	3	......	......	23
	≥13.5	......	3	7	2	......	......	12
8.5 to 11.5...	<11.0	......	......	......	1	1	......	2
	11.0–11.5	......	......	1	......	......	......	1
	11.5–12.0	......	1	......	......	......	......	1
	12.0–12.5	......	......	1	2	......	......	3
	12.5–13.0	......	4	4	2	1	......	11
	13.0–13.5	1	12	8	1	......	......	22
	≥13.5	......	9	5	......	......	......	14
Whole cluster.	<11.0	1	1	1	1	4	4	12
	11.0–11.5	2	3	3	1	3	......	12
	11.5–12.0	1	3	......	......	3	1	8
	12.0–12.5	......	3	5	5	4	......	17
	12.5–13.0	......	30	20	14	6	......	70
	13.0–13.5	1	18	35	16	4	......	74
	≥13.5	......	12	18	9	......	......	39
	All magnitudes	5	70	82	46	24	5	232

Table VII, in which the tabulated quantities are numbers of stars, shows the relative frequency of color and magnitude in successive concentric regions. A very significant result, one in marked contrast to conditions in the globular cluster Messier 13, where there is a large percentage of negative and small color-indices, is that in this open galactic system there are no color-indices less than +0.16, and but 2 per cent of the stars are bluer than *f*. All of the *m*'s and most of the *k*'s are near the center and the former are chiefly among the brighter stars, but aside from this no definite relation exists between color, distance, and magnitude.

TABLE VIII

DISTANCE AND COLOR-CLASS

DISTANCE FROM CENTER	QUANTITY	COLOR-CLASS						ALL COLORS
		a	*f*	*g* 0–4	*g* 5–9	*k*	*m*	
0′0– 2′5	No. stars...	0	10	10	8	4	3	35
	Av. Pv. mag.	....	12.49	12.81	13.18	11.96	10.23	12.49
	Av. C. I....	....	+0.70	+0.93	+1.06	+1.28	+1.78	+1.00
2.5– 4.5	No. stars...	1	8	22	22	9	2	64
	Av. Pv. mag.	11.19	12.54	13.02	13.16	12.61	10.70	12.85
	Av. C. I....	+0.26	+0.66	+0.90	+1.08	+1.31	+1.71	+1.00
4.5– 6.5	No. stars...	3	15	15	8	8	0	49
	Av. Pv. mag.	10.97	12.70	13.04	12.75	11.59	....	12.52
	Av. C. I....	+0.27	+0.71	+0.90	+1.05	+1.39	....	+0.91
6.5– 8.5	No. stars...	0	11	16	2	1	0	30
	Av. Pv. mag.	....	12.80	13.21	13.04	12.70	....	13.03
	Av. C. I....	....	+0.62	+0.87	+1.12	+1.32	....	+0.81
8.5–10.5	No. stars...	0	19	9	4	2	0	34
	Av. Pv. mag.	....	13.29	13.26	12.23	11.72	....	13.06
	Av. C. I....	....	+0.68	+0.90	+1.06	+1.28	....	+0.82
10.5–11.5	No. stars...	1	7	10	2	0	0	20
	Av. Pv. mag.	13.10	13.35	13.05	12.60	....	....	13.11
	Av. C. I....	+0.35	+0.68	+0.87	+1.12	....	....	+0.80
All of cluster..	No. stars...	5	70	82	46	24	5	232
	Av. Pv. mag.	11.44	12.89	13.07	12.98	12.09	10.42	12.80
	Av. C. I....	+0.29	+0.68	+0.89	+1.07	+1.33	+1.75	+0.91

Table VIII gives for each color-class and each interval of distance from the center the number of stars and the average magnitude and color-index. The variation of average color or average magnitude with distance is but roughly indicated for each color separately because of the small number of stars in some of the groups. For all colors combined the last column sums the material and suggests immediately that the three inner regions are to be distinguished from the remainder.

This matter is treated more fully in Table IX. The limits of the successive annular regions, the number of stars each contains, and the area of each in square minutes are given in the first three columns. The average density of stars in each region is given in the fourth column, followed by the average magnitude and color-index taken from the preceding table. It is very striking that the quantities in all three of these columns are sensibly constant for distances greater than 6′.5. This result can hardly be accidental, for considerable numbers of stars are involved, and its tentative interpretation must be that the diameter of the cluster does not exceed 13′. The stars in this nucleus are four times as numerous as in the sky immediately surrounding it, and average slightly brighter and redder than the extra-cluster stars.

TABLE IX

Distance from Center	No. Stars	Area	No. Stars per Square Minute	Av. Pv. Mag.	Av. C. I.	Number of Stars	
						Background	Cluster
		sq. min.					
0′.0– 2′.5...	35	19.6	1.78	12.49	+1.00	6	29
2.5– 4.5...	64	44.0	1.45	12.85	+1.00	13	51
4.5– 6.5...	49	69.1	0.71	12.52	+0.91	21	28
6.5– 8.5...	30	94.3	0.32	13.03	+0.81	28	2
8.5–10.5...	34	119.5	0.28	13.06	+0.82	36	(−2)
10.5–11.5...	20	69.1	0.29	13.11	+0.80	21	(−1)
Total....	232					125	107

From Table IX we may deduce closer details of the average conditions in the cluster itself. First, we assume that the background in which the cluster is situated has a density of 0.30 stars per square minute, and we derive at once the last two columns of Table IX, showing that background stars exceed cluster stars in number and that even within 6′.5 of the center 27 per cent are not members of the system. In the second place, assuming from Table IX that the average magnitude (to the limit of the plates used) is 13.07 for the background stars in the vicinity of the cluster, we may obtain the average magnitude of the cluster stars alone, for we know the number of each kind. For example, in the first region

$$\text{Av. Pv. Mag.} = (35 \times 12.49 - 6 \times 13.07) \div 29.$$

Finally we may derive in a similar manner the average color-index of the cluster stars alone; for example, in the first region

$$\text{Av. C. I.} = (35 \times 1.00 - 6 \times 0.81) \div 29.$$

The results for the cluster alone in the three inner regions are, then, as follows:

1. Cluster stars per sq. min. 1.48, Av. Pv. Mag. 12.37, Av. C. I. +1.04
2. “ “ “ “ “ 1.16 “ “ “ 12.79 “ “ +1.05
3. “ “ “ “ “ 0.40 “ “ “ 12.11 “ “ +0.98

The evidence of Table IX relative to the limits of the cluster seems conclusive. The assumed average background density of 0.30, however, is equivalent to more than one thousand stars per square degree, which is much in excess of the number (320 per square degree) predicted for this limiting magnitude and galactic latitude by the tables of Chapman and Melotte.[1] To investigate the matter further, counts were made of all stars within a degree of the cluster on the Palisa-Wolf photographic chart No. 40. The outcome of this count, which is given in detail in Table X, indicates that the cluster extends far beyond the limits of the plates used for magnitudes. Small squares (10′ on the side) were counted separately and the means of the rows and columns are given at the side and bottom of the table. It appears that the diameter of the cluster may be a degree of arc, or even more, though beyond 15′ or 20′ the ratio of background stars to cluster stars probably becomes very large. When allowance is made for the different magnitude limits of the Palisa-Wolf chart and the present catalogue, the average density at the edges of Table X is found to be but one-sixth of that assumed for the background from Table IX and one-half of that predicted from the Chapman-Melotte tables.

[1] *Memoirs of the Royal Astronomical Society*, **60**, Part IV, 1914.

The star group of thirteen minutes diameter, shown so clearly in Table IX, evidently represents a secondary condensation, perhaps a nucleus of the brighter or more massive stars. If the region studied in that table could have been extended to 20′ or 30′, further change in the density would doubtless have been found and the constancy of average color and magnitude might no longer have been maintained. The limit of the Palisa-Wolf chart is approximately a magnitude fainter than the plates used in this paper. A second but less accurate count was made on the chart, including only those stars brighter than the limit of the magnitude catalogue. The result relative to the extension of the cluster is practically the same as in Table X, indicating that the phenomenon is not wholly connected with the fainter stars.

TABLE X

COUNTS OF STARS NEAR M 67 ON PALISA-WOLF CHART

	$8^h 40^m$						$8^h 44^m$						$8^h 48^m$	
+11°	6	10	8	12	8	7	7	13	12	7	7	15	14	9.7
	8	9	19	11	15	10	10	5	9	12	12	12	9	10.8
	12	11	12	10	10	10	10	6	12	7	8	13	10	10.1
	7	10	10	15	15	10	6	13	14	13	6	10	9	10.6
	9	12	19	16	13	17	11	8	9	14	16	8	12	12.6
	12	14	17	14	14	20	23	16	16	15	11	9	10	14.7
+12°	8	11	11	6	14	38+	40+	42+	9	15	8	10	5	16.2
	11	11	15	18	13	29	80+	80+	36	13	6	6	8	25.1
	13	8	9	12	23	36	45+	50+	21	10	9	6	5	19.0
	9	12	7	11	9	24	24	19	17	15	14	11	6	13.7
	12	15	7	9	14	19	8	13	11	10	22	10	6	12.0
	7	14	6	13	14	12	9	17	7	6	10	10	15	10.8
+13°	18	9	13	11	13	12	15	5	13	3	9	6	11	10.6
	10.2	11.2	11.8	12.2	13.5	18.8	22.2	22.1	14.3	10.8	10.6	9.7	9.2	

SUMMARY

1. A catalogue of the magnitudes of 311 stars in the open cluster Messier 67 has been prepared from the partial or complete measurement of eleven plates. Color-indices are obtained for 240 stars. The average probable error of a color-index is about $\pm 0^{M}.1$.

2. The average color-index is +0.91, with a strong condensation of color in classes *f* and *g*. No negative indices and but five less than +0.4 were obtained.

3. From a study of these colors alone the cluster appears to consist of but a hundred stars and to be of small extent (Table IX), but an examination of the large star charts shows that its influence among the background stars may extend to a distance of nearly a degree (Table X).

4. There is much in common between the stars of the immediate background and those of the cluster—average color, magnitude, and condensation not appearing greatly unlike. In this respect, as well as in others, there is a wide difference between Messier 67 and the condensed globular system Messier 13, discussed in a preceding paper.

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PRELIMINARY EVIDENCE OF INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 101

PHOTOGRAPHS BY
G. W. RITCHEY, J. E. KEELER, C. D. PERRINE, AND H. D. CURTIS
MEASURED AND DISCUSSED BY
A. VAN MAANEN

INTRODUCTION

Although the nebulae have been under observation for many years, it is only within the last quarter-century that their motions have been detected. In 1887 Dreyer[1] stated that "so far we do not possess any clear evidence of change of form or change of place." Several of the older observations seemed to indicate such changes, but Dreyer's careful investigation of all known cases led him to the conclusion that these motions must be illusory "unless we are to believe that nebulae in the good old days moved about as they liked, but have been on their good behavior since 1861 and kept quiet."[2]

Motions of nebulae in the line of sight were first observed with the spectroscope by Keeler in 1890,[3] and since that time the radial velocities of many of them have been determined by Campbell, Slipher, and others; but we remained in ignorance as to their cross-motions for another twenty-five years. Curtis, from a comparison of plates taken with the Crossley reflector of the Lick Observatory by Keeler, Palmer, Perrine, and himself, published in 1915 the proper motions of about one hundred nebulae.[4]

In recent years evidence of internal motion has also been discovered. Here too the radial motions were detected first. In 1914, Buisson, Fabry, and Bourget demonstrated the existence of internal motions in the Orion nebula,[5] a result confirmed by Frost and Maney.[1] In 1914 V. M. Slipher[2] published the "Detection of Nebular Rotation," relating to the Virgo nebula, which is of the "spindle" type, and Wolf[3] detected the rotation of Messier 81, while in 1915 Campbell and Moore[4] announced the rotation of the planetary nebula N.G.C. 7009. Also Pease[5] discovered a possible internal motion in Messier 33.

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[1] *Monthly Notices*, **47**, 413, 1887. [2] *Ibid.*, **47**, 418, 1887.

[3] *Publications of the Lick Observatory*, **3**, 217, 228.

[4] *Publications of the Astronomical Society of the Pacific*, **27**, 214, 1915.

[5] *Comptes Rendus*, **158**, 1017, 1914; *Astrophysical Journal*, **40**, 241, 1914.

In December 1915 Mr. Ritchey suggested that the two plates of Messier 101 taken by him with an interval of about five years be placed in the stereocomparator to see if they would show motion. Since no motion whatever was revealed in that way, I then proposed that the plates be placed at my disposal for measurement, to which Mr. Ritchey kindly consented. As sixteen nebulous points chosen for this purpose showed some evidence of internal motion, I at once decided to take up the work in a more extended way, measuring many more points. Feeling also the necessity of measuring at least one other pair of plates, I asked Dr. Curtis if he would be willing to place at my disposal the Crossley reflector plates of this nebula. This request was very kindly granted, and Dr. Curtis sent me three plates, one by Keeler (1899), one by Perrine (1908), and one by himself (1914). The measures on these three plates and on the two made by Mr. Ritchey are discussed in the following pages. My thanks are due to Dr. Campbell, director of the Lick Observatory, and to Dr. Curtis for the use of the Lick plates and for permission to publish my measures of them, and also to Mr. Ritchey for the use of the two plates taken by him.

MEASURES AND DISCUSSION

The measures were made with the "blink" arrangement of the stereocomparator in the manner described in "The Photographic Determination of Stellar Parallaxes with the 60-Inch Reflector."[6] The plates were measured in four pairs, combined and arranged in the stereocomparator as shown in Table I.

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[1] *Popular Astronomy*, **23**, 485, 1915.

[2] *Lowell Observatory Bulletin*, No. 62.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 162, 1914.

[4] *Harvard College Observatory Bulletin*, No. 591.

[5] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

[6] *Mt. Wilson Contr.*, No. 111, 4 ff.

The exposure times of the Lick Observatory plates were as follows: 1899, June 8, 4^h; 1908, July 28, $1^h\ 45^m$; and 1914, March 20, 2^h; the last plate being under-exposed on account of clouds. The exposure times of the Mount Wilson plates were: 1910, March 10 and 11, $7^h\ 30^m$ (aperture 54 inches); 1915, May 14, 15, and 16, $8^h\ 37^m$ (aperture 50 inches).

TABLE I

Pair	Left Plate-Carrier	Right Plate-Carrier	Observer	Remarks
R I..........	1910	1915	Ritchey, Ritchey	....
R II..........	1915	1910	Ritchey, Ritchey	....
L I...........	1899	1908	Keeler, Perrine	1899 through glass
L II..........	1914	1899	Curtis, Keeler	1914 through glass

As the 1899 plate of the Lick Observatory was taken with the early arrangement of the Crossley reflector, involving the use of a Newtonian flat, while the later photographs were made without this auxiliary mirror, the field of the 1899 plate is inverted with respect to that of the later photographs. In using the stereo-comparator it was thus necessary to measure through the glass of one of the plates of the Lick pairs.

The interchange of the new and old plates in the plate-carriers should in the mean eliminate any relative displacement of the images due to the measuring instrument.

Thirty-two stars, as near to the nebula as seemed safe, were used for comparison purposes. It is true, of course, that some of these objects may really belong to the nebula, but the results show that the choice of comparison stars, on the whole, was a fortunate one. To prevent the appearance of a magnitude error it was necessary to avoid a great range in brightness, and in order to increase as much as possible the weight of the plate-constants the comparison stars had to be distributed as uniformly as possible around the nebula. The co-ordinates of the comparison stars relative to the center of the nebula are given in the second and third columns of Table II, + indicating east and north for x and y, respectively.

In the choice of points in the nebula which were to be measured I tried to avoid stars. A priori no certain distinction can be made, but in general I avoided those points which through total absence of surrounding nebulosity give the impression of stars rather than of parts of the nebula. This is a hazardous procedure, and undoubtedly some of the points do not belong to the nebula. From countings on the outer parts of the plates the conclusion was reached that the area containing the 87 measured points should include about 20 measurable stars, but several of these must have been omitted in choosing the points in the nebula. Further, the points selected must be as symmetrical in appearance as possible to avoid difficulties in bisecting their images. This requirement, too, could not be rigorously fulfilled.

TABLE II

Co-ordinates of the Comparison Stars

	x	*y*		*x*	*y*
a	−10′.9	+ 4′.9	*q*	+10′.0	+ 2′.4
b	−10.3	− 0.1	*r*	+10.2	+ 4.7
c	− 6.4	+ 3.4	*s*	+ 8.4	+ 9.1
d	− 6.6	− 8.1	*t*	+ 4.9	+ 5.9
e	− 6.2	− 6.8	*u*	+ 3.3	+ 5.3
f	− 3.8	−10.9	*v*	+ 9.0	+12.0
g	− 3.4	− 8.6	*w*	+ 6.3	+12.0
h	− 0.3	−10.7	*x*	+ 2.3	+11.0
i	+ 1.4	− 5.4	*y*	+ 1.0	+12.6
j	+ 1.8	− 7.1	*z*	− 0.2	+ 8.9
k	+ 2.3	− 9.4	α	− 0.6	+ 6.7
l	+ 7.3	−10.9	β	− 3.8	+ 7.8
m	+ 7.5	− 7.5	γ	− 2.0	+11.5
n	+ 4.7	− 4.7	δ	− 5.9	+11.1
o	+ 6.5	− 2.0	ε	−10.5	+ 8.4
p	+ 8.0	+ 0.1	η	− 5.2	+ 5.9

On R I and R II, 87 nebular points were measured; on L I, 69; and on L II, 46. The co-ordinates of these points are given in the second and third columns of Table VI.

Each pair of plates was measured in four positions with east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. In the reductions the two measures of each pair in right ascension were combined into one set; the same was done for the measures in declination.

As the measurement of a pair of plates in one position required several hours, the temperature of the room was read during every set of measures. In no case did the range during the measures in one position exceed 1° C. Disturbances from this source are therefore negligible. Moreover, several measures were duplicated to see if any shift could be detected, but no positive evidence was found.

The mean of the measured differences of every object for the two positions of the plate which differ by 180° may be called D. From the values of D the proper motions in α and δ were derived by a method analogous to that described in my paper on stellar parallaxes.[1] It was found from a preliminary reduction, however, that the quadratic terms of the co-ordinates could not here be neglected. The equations of condition therefore have the form

$$D=\mu_{\alpha}+a+bx+cy+dx^2+exy+fy^2$$

with a similar equation for declination.

Supposing μ_{α} and μ_{δ} of the comparison stars to be zero, we can solve the 32 equations for these stars to find the 6 plate-constants a, b, c, d, e, and f. This was done by the regular least-squares method. The constants found for each pair of plates were then substituted into the equations of condition for the stars and for the points in the nebula, thus giving μ_{α} and μ_{δ}, the proper motions in α and δ relative to the mean motion of the comparison stars.

These values of μ_{α} and μ_{δ}, still expressed in two-thousandths of a revolution of the micrometer screw, must be multiplied by 1.04, 1.04, 0.84, and 0.52 for R I, R II, L I, and L II, respectively, to give the annual proper motion expressed with 0ʺ001 as a unit. The resulting values are entered in Table III. The first column contains the designation a, b, ... α, β, ... for the comparison stars, and the numbers 1, 2, 3, ... 87, for the nebular points.

The weight to be given to each pair of plates depends on the scale (which for the Mount Wilson plates is 1 mm=27ʺ20, and for the Lick plates 1 mm=38ʺ73), their quality, and the interval in years. As the influence of quality is difficult to determine in

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[1] *Mt. Wilson Contr.*, No. 111, 7 ff.

TABLE III

Total Annual Proper Motions

(Unit=0″001)

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_δ
a	+20	−19	+ 8	+ 2	+24	−26	+24	+21	+ 3	+11
b	+14	− 8	−53	+ 5	−10	−17	−40	+10	−11	−14
c	−28	+ 9	−13	−16	−17	+16	− 4	+16	−12	+ 3
d	+33	+14	+46	+42	−19	+ 4	− 5	−18	+34	−10
e	−10	+18	+ 8	− 2	− 1	+ 9	− 2	+33	+ 4	+10
f	−18	−27	−23	−18	+15	− 4	+ 4	− 7	−22	+ 2
g	+ 5	+22	+18	− 2	−16	−14	−12	− 6	+11	−12
h	− 8	− 7	+ 5	−10	0	+ 8	+18	−18	− 5	+ 2
i	−35	−19	−27	−32	+41	+25	+30	+13	−28	+27
j	−12	− 4	− 3	−18	+10	+14	+10	− 5	− 9	+ 7
k	+ 1	+12	+14	+ 9	− 5	−20	+20	−14	+ 9	− 5
l	−19	−15	−44	− 9	+ 9	+17	+24	+17	−22	+17
m	+52	+31	+34	+39	+ 8	−48	−60	+ 9	+39	−23
n	+26	+ 7	+ 7	+20	−28	+27	−52	+26	+15	− 7
o	− 3	−11	−13	−29	−38	+ 3	+21	−27	−14	−10
p	+ 5	+16	− 5	+ 1	− 3	−10	−10	+ 2	+ 4	− 5
q	+10	− 9	+41	+ 1	+11	+14	+23	−15	+11	+ 8
r	−50	+12	− 6	+ 2	−32	− 5	+ 5	+ 9	−11	− 6
s	−21	−43	+16	+13	+33	+18	+ 1	+ 8	− 9	+15
t	+14	+ 5	− 4	− 2	+15	−13	+ 3	−17	+ 3	− 3
u	+24	+11	+ 2	+ 8	− 6	−18	− 1	−22	+11	−12
v	− 6	−10	−40	−36	+25	+31	+10	−12	−23	+14
w	+33	+22	+21	+20	−21	−25	−16	+14	+24	−12
x	−11	−12	−13	− 5	−14	+19	+26	− 2	−10	+ 7
y	+18	+ 7	0	+ 6	−26	−29	−18	+ 9	+ 8	−16
z	+26	+ 8	+ 6	− 6	−51	−43	− 8	+12	+ 9	−23
α	0	− 4	+ 3	+ 8	+52	−10	+17	+ 6	+ 2	+16
β	−56	− 6	−46	−11	+46	+64	+21	+14	−30	+36
γ	− 1	+ 6	+13	+12	+14	−23	−35	+13	+ 8	− 8
δ	−10	− 7	+22	+10	−16	+40	− 3	−14	+ 4	+ 2
ε	−10	+15	+16	−21	−12	+ 4	+18	−30	0	− 5
η	+25	− 1	+22	+ 8	+12	−14	+ 5	−32	+14	− 7
1	+43	+32	......	......	− 1	+12	......	......	+38	+ 6
2	−20	−23	−55	− 9	+ 8	+16	0	+12	−27	+ 9
3	+14	− 4	......	......	−16	+50	......	......	+ 5	+17
4	+16	− 3	+45	+ 3	+50	+50	+24	− 5	+15	+30
5	+ 7	− 4	+ 5	+ 3	+ 6	+42	− 2	+21	+ 3	+17
6	+ 7	+ 1	+68	......	+40	+24	+87	......	+25	+50
7	+26	+17	......	......	+ 4	+58	......	......	+22	+31
8	+14	+ 1	+15	−15	+18	+48	+15	+ 7	+ 4	+22
9	+35	+23	+45	+37	−18	− 7	−34	−49	+35	−27
10	+14	−10	+35	+15	+ 6	+33	+34	+ 6	+14	+20
11	− 6	+34	+ 3	......	+ 4	+56	−24	......	+10	+12
12	−22	+19	+61	......	−30	+14	− 6	......	+19	− 7
13	− 2	+ 4	+ 1	− 9	+10	+20	+ 5	+15	− 2	+13
14	+14	+ 4	+ 8	+ 3	− 2	+15	+23	+ 4	+ 7	+10
15	+38	+17	+23	+ 2	− 2	+35	+21	+28	+20	+21
16	+47	+25	+42	+12	− 1	+12	+34	+26	+32	+18

TABLE III—*Continued*

No.	μ_α				μ_δ				MEAN	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_δ
17........	+7	+25	......	......	−1	+9	......	......	+16	+4
18........	+43	+33	+91	......	−10	−2	+60	......	+56	+16
19........	+9	+16	−12	......	−24	+6	+19	......	+4	0
20........	−21	+5	+29	......	−42	+17	+22	......	+4	−1
21........	−12	−43	+18	+26	+22	+2	+58	+23	−3	+26
22........	+7	−6	−8	+3	+21	+22	−8	+4	−1	+10
23........	+14	−1	......	......	+4	−12	......	......	+7	−4
24........	+21	+4	......	......	+18	+54	......	......	+13	+36
25........	+5	+10	−9	......	+20	+9	+1	......	+2	+10
26........	−19	−36	......	......	−29	−8	......	......	−28	−19
27........	+11	−15	−26	−16	−40	−2	+6	+20	−12	−4
28........	+19	+15	+15	......	−75	−1	+30	......	+16	−15
29........	+52	−2	−55	......	+4	−31	−81	......	−2	−36
30........	−6	−47	......	......	−10	−31	......	......	−27	−21
31........	+45	−8	+23	......	+23	−27	−15	......	+20	−6
32........	+4	−49	+29	−27	−24	+16	+28	+28	−11	+12
33........	−20	−28	−22	......	+16	+2	+9	......	−23	+9
34........	−11	−44	+15	−13	−22	+12	+66	+16	−13	+18
35........	+27	−21	−32	−32	−1	−10	+28	+16	−15	+8
36........	+9	−30	−12	−21	+5	+24	−8	−9	−14	+3
37........	−23	−12	......	......	−9	+27	......	......	−18	+9
38........	+23	−23	......	......	+19	+4	......	......	0	+12
39........	−25	−36	+13	−7	+5	+6	−8	+27	−14	+8
40........	−44	−16	......	......	−21	−18	......	......	−30	−20
41........	+24	+16	......	......	−12	−19	......	......	+20	−16
42........	−48	−44	−19	......	−16	−27	−29	......	−37	−24
43........	+58	−35	......	......	−22	−45	......	......	+12	−34
44........	−3	+11	−15	......	−44	−60	+19	......	−2	−28
45........	+14	−28	+64	......	+8	+1	−36	......	+17	−9
46........	−11	−29	−36	−20	+29	−16	−12	−38	−24	−9
47........	−38	−9	−15	−7	+20	−26	−30	−28	−17	−16
48........	+6	−29	−18	−14	−15	−25	+30	−24	−14	−9
49........	−9	−57	+30	−3	−10	−42	−3	−37	−10	−23
50........	+5	−43	+37	−16	+5	−46	−37	−20	−4	−25
51........	+30	−40	+46	......	−11	−49	−34	......	+12	−31
52........	−69	−83	+5	......	−6	−38	−60	......	−49	−35
53........	+50	−17	+4	−1	−14	−28	−39	−12	+9	−23
54........	−15	−34	−32	−45	−32	−73	−18	−21	−32	−36
55........	−49	−77	−19	−28	−36	−11	+21	+21	−43	−1
56........	+47	+1	+51	−1	−45	−66	−19	−27	+25	−39
57........	+15	−11	+18	−4	−49	−57	+11	−3	+5	−25
58........	−5	+10	+59	......	−51	−52	−44	......	+21	−49
59........	+33	−6	+24	−9	−73	−60	−89	−38	+11	−65
60........	+25	+9	−14	......	−15	−54	−34	......	+7	−34
61........	−14	−35	−10	+3	−46	−41	−66	−2	−14	−39
62........	+7	+1	+44	−20	−52	−51	−19	−16	+8	−35
63........	+21	−20	−7	−9	−74	−60	−46	−21	−4	−50
64........	+15	−57	+76	−28	−77	−11	+41	−5	+2	−13
65........	+28	−29	+19	−7	−58	−38	−23	−31	+3	−38
66........	+60	−28	+23	−27	−30	−22	−44	−41	+7	−34
67........	+58	−4	+8	−6	−48	−31	−31	−48	+14	−40

TABLE III—*Concluded*

No.	μ_α				μ_δ				MEAN	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_α
68........	+ 1	−10	+69	......	−59	−69	−66	......	+20	−65
69........	+ 7	−28	......	......	−46	−29	......	......	−11	−38
70........	+15	+24	+24	+10	−70	−55	−25	−27	+18	−44
71........	−11	−21	−23	+11	+12	−28	−39	+10	−11	−11
72........	+21	−15	+18	− 3	−42	−46	−13	−12	+ 5	−28
73........	+38	− 1	+33	......	−59	−72	−14	......	+23	−48
74........	+11	− 4	......	......	−107	−55	......	......	+ 3	−81
75........	+12	+23	+ 5	......	−35	−18	−33	......	+13	−29
76........	+41	+27	......	......	−38	−34	......	......	+34	−36
77........	+29	+37	+115	......	−17	−57	−10	......	+60	−28
78........	+37	+33	+13	+13	+ 1	−33	−33	−36	+24	−25
79........	+41	+ 9	+24	− 3	−28	−68	−71	−38	+18	−51
80........	+14	+18	+37	− 3	−21	−41	− 6	−35	+17	−26
81........	+ 2	−19	+34	− 5	+ 5	−61	+24	−29	+ 3	−15
82........	+ 4	+46	+42	− 5	−34	−75	−14	−24	+22	−37
83........	+31	+ 5	+20	+ 6	−80	−63	−31	+ 4	+16	−43
84........	+81	+28	+56	......	+ 2	+11	+71	......	+55	+28
85........	+16	+34	......	......	−17	+ 3	......	......	+25	− 7
86........	−17	+20	+47	+16	−11	+36	− 7	+18	+17	+ 9
87........	+43	+31	......	......	−33	+31	......	......	+37	− 1

advance, all pairs were provisionally given the same weight and combined to form mean motions for the 32 comparison stars and the 46 nebular points which were measured on all four pairs.

The average deviation from these means for each pair of plates was as shown in Table IV (unit = 0.″001). Since the mean deviations

TABLE IV

Pair	R I	R II	L I	L II
Comparison stars..........	12	11	11	13
Nebular points...........	16	16	17	14

are sensibly the same for the different pairs, it seems permissible to use the same weights for all pairs, the quality very nearly counterbalancing the influence due to scale and time interval. The adopted mean μ_α and μ_δ are accordingly those given in the last two columns of Table III. There is of course quite a range in the agreement for the different points, but the mean probable error of a mean μ_α or μ_δ is only 0.″008.

The results found may be due partly to a translation of the nebula as a whole, and partly to internal motions. To derive the motion of translation of the nebula three methods were employed.

1. The mean of the proper motions μ_α and μ_δ for all the nebular points was found.

This gives $\mu_\alpha=+0''.005$, $\mu_\delta=-0''.012$. The Mount Wilson plates treated separately give: $\mu_\alpha=+0''.002$, $\mu_\delta=-0''.016$, while the Lick plates alone give $\mu_\alpha=+0''.007$, $\mu_\delta=-0''.008$.

2. Whatever the character of the internal motions of a nebula like Messier 101, we may expect them to be nearly symmetrical with respect to the center of the nebula. If, therefore, we divide the points into four groups corresponding to the four quadrants, the two pairs of opposite quadrants should give a good agreement. Using all the 87 points for this purpose, the results are as summarized in Diagram I, which gives for each quadrant the mean values of μ_α and μ_δ, and the number of points measured.

DIAGRAM I

	N	
I		II
$+0''.002$		$-0''.010$
$+0''.008$		$-0''.023$
31		18
E		W
$+0''.018$		$+0''.013$
$-0''.001$		$-0''.040$
16		22
IV		III
	S	

Mean of I and III $\begin{cases}\mu_\alpha=+0''.007^5\\ \mu_\delta=-0''.016\end{cases}$

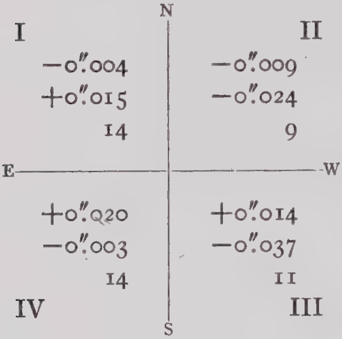
Mean of II and IV $\begin{cases}\mu_\alpha=+0''.004\\ \mu_\delta=-0''.012\end{cases}$

3. As, however, the distribution of the points measured is far from uniform in the four quadrants, it seemed worth while also to derive analogous values for the central parts of the nebula, within a radius of $5'.0$, where the distribution of the points is more symmetrical. The results may be seen in Diagram II.

The results of the three methods are given in Table V. The agreement throughout is as good as could be expected. For the final motion of translation the values $\mu_\alpha=+0''.005$, $\mu_\delta=-0''.013$

have been used. No great weight, however, should be given to this motion of translation, as the hour angles and the parallax factors of the different plates may have vitiated the results. With the material at hand, however, it is the best we can do. Subtracting the adopted μ_α and μ_δ from the total motions in Table III, the results are what may be called the internal motions. They are given in the fourth and fifth columns of Table VI.

DIAGRAM II



$$\text{Mean of I and III}\begin{cases}\mu_\alpha = +0''.005\\ \mu_\delta = -0''.011\end{cases}$$

$$\text{Mean of II and IV}\begin{cases}\mu_\alpha = +0''.005^5\\ \mu_\delta = -0''.013^5\end{cases}$$

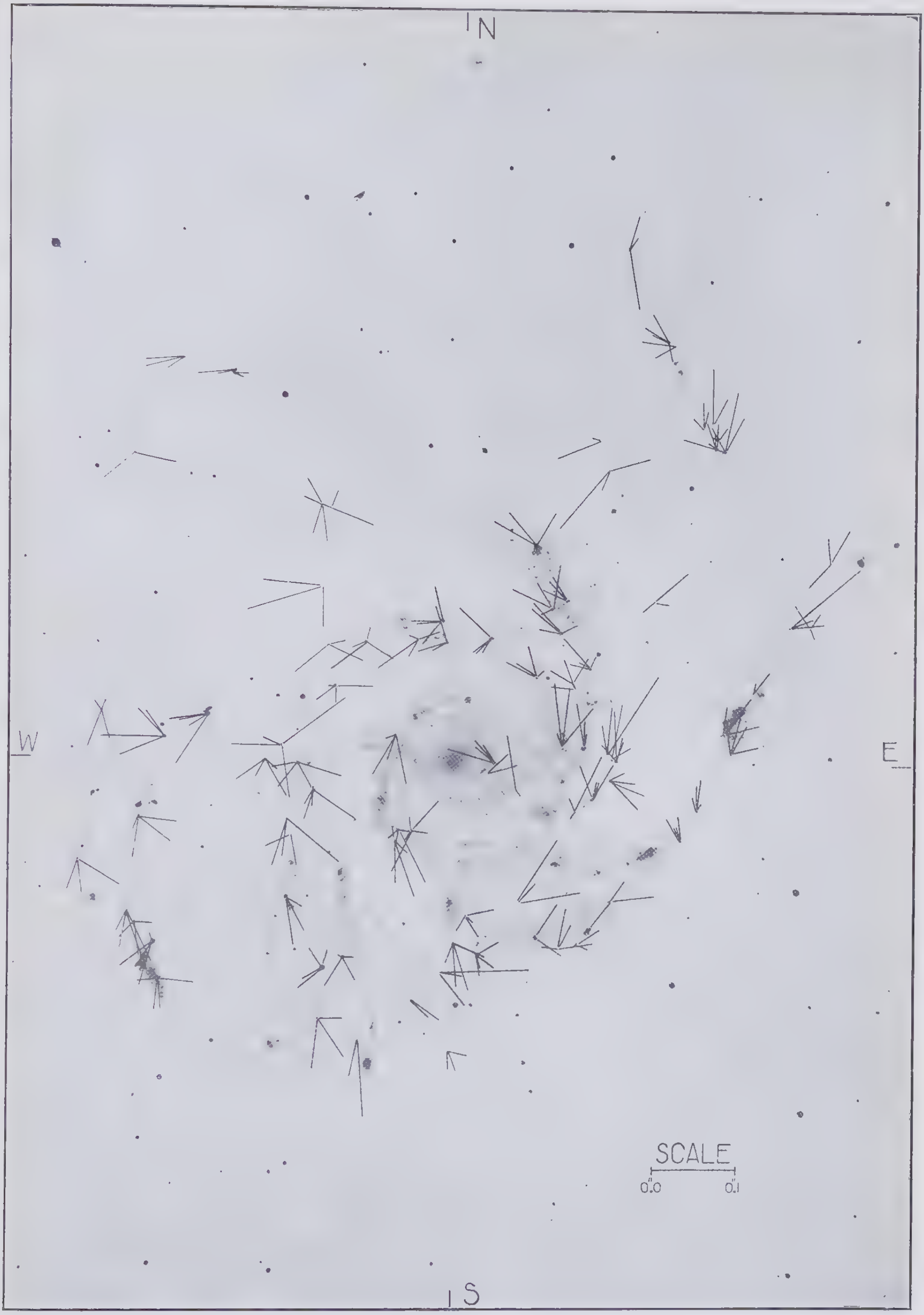
TABLE V

Method	μ_α	μ_δ
1	+0″.005	−0″.012
2	+0.006	−0.014
3	+0.005	−0.012

The accompanying illustrations show these motions, freed from the motion of translation of the whole nebula. The density of the center of the prints used for the engravings was reduced in order to show the motions more clearly. Plate VIII indicates the motions given by each pair of plates; they show in general a good agreement, although there are several large discrepancies. Plate IX shows the mean annual proper motion for each of the 87 points, and also for the comparison stars, which are surrounded by circles. The scale for the annual motions is indicated on the plates.

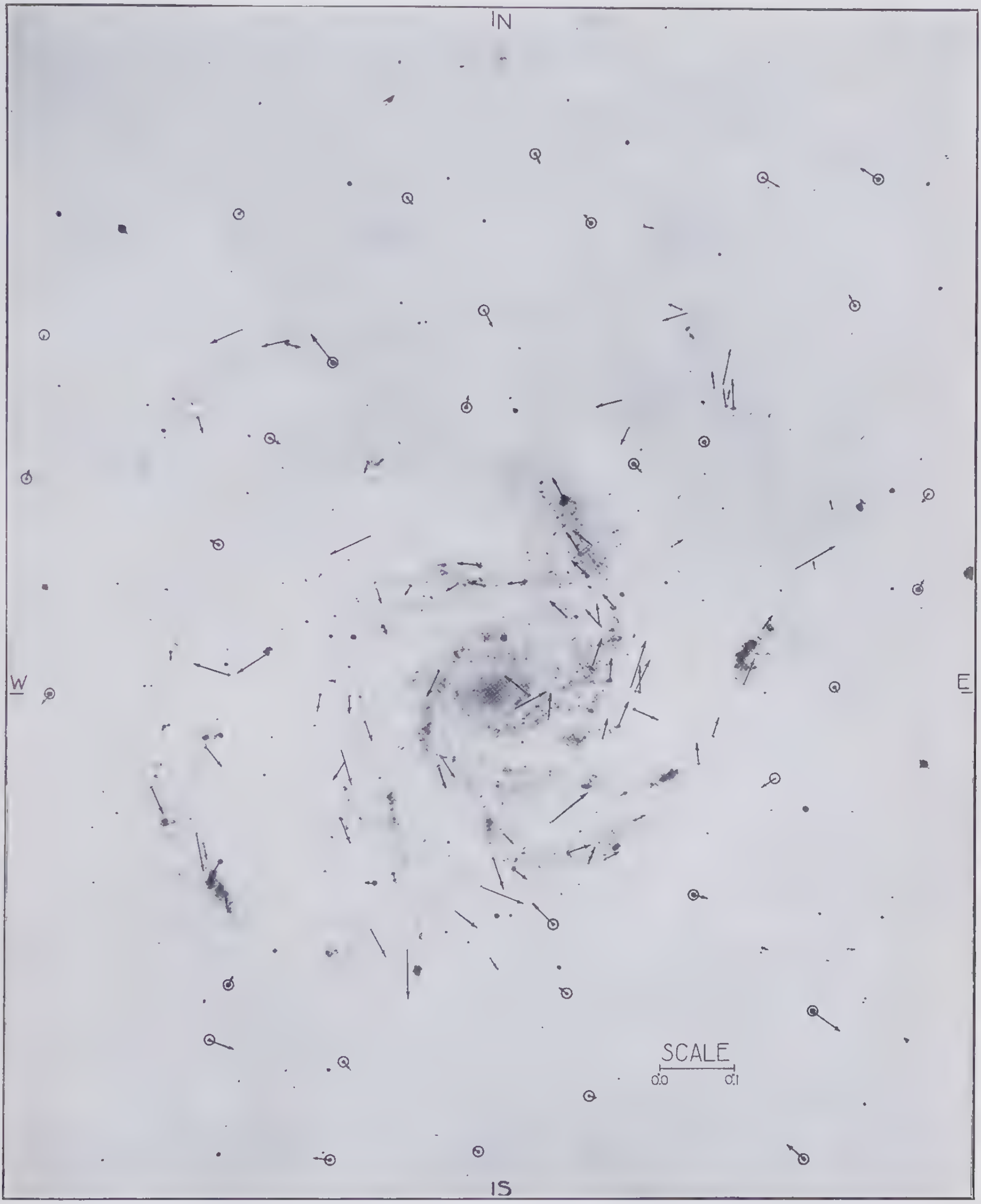
If the results as illustrated in Plate IX could be taken at their face value, they would certainly seem to indicate a motion of

PLATE VIII



Internal Motions in Messier 101

The lines indicate the direction and magnitude of the annual motions of the nebulous points as derived from the different pairs of plates. Their scale (0″.1) is indicated on the illustration. The scale of the nebula is 1 mm = 9″.1.



INTERNAL MOTIONS IN MESSIER 101

The arrows indicate the direction and magnitude of the mean annual motions. Their scale (0.″1) is indicated on the illustration. The scale of the nebula is 1 mm = 10.″5. The comparison stars are enclosed in circles.

rotation or possibly motion along the arms of the spiral. Without expressing a final opinion as to the character of the motion, which must be determined by future work, it may be of interest to examine the evidence afforded by the existing material.

To discuss the internal motions from the standpoint of rotation, they were analyzed into components along and perpendicular to the radius; the latter for convenience will be spoken of as the rotational component. The results are given in the last four columns of Table VI. In view of the uncertainty of the measures it seems well to treat the available material in two ways: first, by taking all points measured, and, secondly, by excluding those points which, because of internal disagreement of the measures on the four sets, seem to be uncertain in their final motion. As the probable error in μ_α or μ_δ derived from a single pair of plates is a little over $0\overset{''}{.}013$, I have thought it well to exclude in the latter case points whose deviation in μ_α or μ_δ for one or more of the pairs of plates is $>0\overset{''}{.}040$. In this way the following 25 points were excluded: Nos. 6, 11, 12, 18, 20, 28, 29, 34, 35, 43, 44, 45, 49, 50, 51, 52, 53, 64, 66, 67, 68, 77, 81, 83, and 84. This procedure seems the more reasonable as 18 of these 25 points are involved asymmetrically in the nebulosity; the measures in such cases must be very uncertain if the plates compared are not of exactly the same density.

The results are as follows, numbers in parentheses relating to the restricted group: 78 (55) points have a left-handed motion,[1] only 9 (7) moving right-handedly; 58 (44) points appear to be moving outward, while 28 (17) show motion inward. The rotational motion is the larger in the majority of cases, viz., for 63 (45) points. The mean rotational motion is $0\overset{''}{.}022$ ($0\overset{''}{.}021$) left-handed; the mean radial motion $0\overset{''}{.}007$ ($0\overset{''}{.}009$) outward.

The probable reality of the result is indicated by the satisfactory agreement of the pairs of plates as shown in Table VII, where the + sign indicates left-handed and outward motions, respectively. For this comparison only the 46 objects common to all plates were used.

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[1] The rotation directions refer to the illustrations. See Plate IX, noting the orientation.

TABLE VI

Co-ordinates and Internal Motions

Unit = 0″.001

No.	x	y	μ_α	μ_δ	$\mu_{\text{rad.}}$		$\mu_{\text{rot.}}$	
					Out	In	L.H.	R.H.
1	+0′.7	−0′.3	+33	+19	21	....	32	....
2	+0.9	−0.1	−32	+22	....	34	20	....
3	+1.3	−0.6	0	+30	....	13	27	....
4	+2.3	+0.6	+10	+43	20	....	39	....
5	+2.8	+0.3	− 2	+30	1	....	30	....
6	+3.2	+0.1	+20	+63	22	....	62	....
7	+3.4	0.0	+17	+44	17	....	44	....
8	+3.5	0.0	− 1	+35	....	1	35	....
9	+3.3	−0.4	+30	−14	31	....	....	11
10	+3.0	−0.8	+ 9	+33	0	0	34	....
11	+2.6	−1.1	+ 5	+25	....	5	25	....
12	+3.4	−3.0	+14	+ 6	7	....	14	....
13	+4.8	−1.7	− 7	+26	....	15	22	....
14	+5.2	−1.0	+ 2	+23	....	2	23	....
15	+5.9	+0.2	+15	+34	17	....	33	....
16	+5.7	+0.6	+27	+31	30	....	28	....
17	+6.3	+1.5	+11	+17	15	....	14	....
18	+7.1	+2.9	+51	+29	58	....	8	....
19	+7.6	+2.9	− 1	+13	4	....	12	....
20	+7.9	+4.3	− 1	+12	5	....	11	....
21	+5.7	+6.7	− 8	+39	25	....	31	....
22	+5.4	+6.7	− 6	+23	14	....	19	....
23	+5.5	+6.9	+ 2	+ 9	8	....	4	....
24	+5.4	+7.2	+ 8	+49	44	....	24	....
25	+5.2	+7.1	− 3	+23	17	....	16	....
26	+4.6	+8.9	−33	− 6	....	20	17	....
27	+4.4	+9.0	−17	+ 9	1	....	19	....
28	+3.5	+10.9	+11	− 2	2	....	....	11
29	+3.1	+6.2	− 7	−23	....	24	....	4
30	+3.0	+6.8	−32	− 8	....	20	26	....
31	+4.2	+3.4	+15	+ 7	16	....	....	4
32	+1.6	+4.6	−16	+25	19	....	24	....
33	+2.5	+3.4	−28	+22	1	....	36	....
34	+2.0	+3.2	−18	+31	17	....	32	....
35	+2.2	+2.8	−20	+21	3	....	29	....
36	+2.8	+2.0	−19	+16	....	7	24	....
37	+2.5	+1.5	−23	+22	....	8	31	....
38	+2.5	+1.6	− 5	+25	9	....	23	....
39	+1.7	+1.8	−19	+21	2	....	28	....
40	−4.8	+8.2	−35	− 7	12	....	34	....
41	−4.9	+8.1	+15	− 3	....	10	....	11
42	−5.9	+8.4	−42	−11	15	....	40	....
43	−6.9	+6.5	+ 7	−21	....	19	11	....
44	−2.9	+5.4	− 7	−15	....	10	14	....
45	−2.7	+5.3	+12	+ 4	....	2	....	13
46	−0.3	+3.0	−29	+ 4	7	....	28	....
47	+0.7	+2.6	−22	− 3	....	9	20	....
48	−0.2	+2.5	−19	+ 4	5	....	18	....

TABLE VI—*Continued*

No.	x	y	μ_α	μ_δ	$\mu_{\text{rad.}}$		$\mu_{\text{rot.}}$	
					Out	In	L.H.	R.H.
49.....	−0.′9	+2.′5	−15	−10	....	5	17	....
50.....	−1.9	+2.5	− 9	−12	....	4	15	....
51.....	−2.8	+2.4	+ 7	−18	....	17	9	....
52.....	−2.9	+3.6	−54	−22	16	....	56	....
53.....	−2.6	+1.6	+ 4	−10	....	9	6	....
54.....	−5.2	+0.9	−37	−23	33	....	29	....
55.....	−6.2	+0.4	−48	+12	48	....	....	8
56.....	−6.7	−1.3	+20	−26	....	15	29	....
57.....	−7.5	+0.9	0	−12	....	2	12	....
58.....	−8.0	−2.2	+16	−36	....	6	39	....
59.....	−6.9	−3.3	+ 6	−52	16	....	49	....
60.....	−6.8	−3.6	+ 2	−21	8	....	19	....
61.....	−6.4	−4.0	−19	−26	30	....	12	....
62.....	−6.2	−4.7	+ 3	−22	11	....	19	....
63.....	−1.3	+0.5	− 9	−37	....	5	38	....
64.....	−3.7	+0.3	− 3	0	3	....	0	0
65.....	−4.1	−0.1	− 2	−25	3	....	25	....
66.....	−3.4	−0.1	+ 2	−21	....	1	21	....
67.....	−3.0	−0.7	+ 9	−27	....	3	28	....
68.....	−3.5	−1.3	+15	−52	4	....	54	....
69.....	−3.4	−1.6	−15	−25	24	....	16	....
70.....	−3.5	−3.0	+13	−31	11	....	32	....
71.....	−2.7	−4.5	−16	+ 2	7	....	....	15
72.....	−2.3	−4.3	0	−15	13	....	7	....
73.....	−2.8	−5.5	+18	−35	23	....	32	....
74.....	−2.0	−6.0	− 2	−68	65	....	19	....
75.....	−0.1	−6.2	+ 8	−16	16	....	8	....
76.....	−0.9	−5.1	+29	−23	17	....	33	....
77.....	−0.2	−4.6	+55	−15	12	....	56	....
78.....	+0.5	−4.1	+19	−12	14	....	18	....
79.....	0.0	−3.9	+13	−38	38	....	13	....
80.....	+0.3	−3.3	+12	−13	14	....	11	....
81.....	−0.9	−1.5	− 2	− 2	3	....	....	1
82.....	−1.2	−1.5	+17	−24	8	....	28	....
83.....	−1.3	−1.7	+11	−30	17	....	27	....
84.....	+1.4	−3.1	+50	+41	....	17	63	....
85.....	+2.6	−3.9	+20	+ 6	6	....	20	....
86.....	+2.2	−4.0	+12	+22	....	13	21	....
87.....	+1.8	−3.8	+32	+12	2	....	34	....

TABLE VII

	$\mu_{\text{rot.}}$	$\mu_{\text{rad.}}$
R I..............	+0.″021	+0.″004
R II..............	+0.032	+0.012
L I	+0.017	+0.006
L II..............	+0.012	+0.007

Of the comparison stars, 13 have left-handed and 19 right-handed motion; 15 move outward and 15 inward; the mean rotational motion, as well as the mean radial motion, is 0".000; a glance at Plate IX shows that very few of the stars can share in the motions of the nebula.

The measures indicate a small and hardly trustworthy decrease of rotational motion with increasing distance from the center, as shown by Table VIII.

TABLE VIII

Distance	Mean Distance	Rotational Component	Number of Points
<3'.1	2'.2	0".024 (0".026)	19 (14)
3'.1 to 5.0.....	3.9	0.028 (0.024)	29 (16)
5.1 to 7.0.....	5.9	0.014 (0.015)	18 (15)
>7.0	8.9	0.019 (0.021)	21 (17)

The change in the rotational component with increasing distance from the center is of interest in connection with the assumption that the nebular points are moving about the nucleus in elliptical orbits. For different points the observed orbital motion should then be inversely proportional to the square root of the mean distance from the center of the nebula. It does not seem safe to attempt final conclusions on the basis of the material now available.

The annual rotational component of 0".022 at the mean distance from the center of 5' corresponds to a rotation period of about 85,000 years. If we knew the parallax of the nebula, and if we could assume that the motions and the distances of the points from the center are mean values for elliptical orbits, the central mass could be calculated. The parallax is unknown and the assumption concerning distances and motions is probably far from the truth. Nevertheless, such an assumption, combined with more or less probable values for the parallax, may give a notion as to whether we are dealing with masses of the order of those of the stars or with much greater aggregations of material. It goes without saying that the universality of the Gaussian gravitation constant must also be assumed.

The relation between the parallax and a central mass which would cause a particle to move in an elliptical orbit at a mean distance of 5′ with a period of 85,000 years can be derived from Kepler's third law; it is $M=0.0037\,\pi^{-3}$. We thus find the corresponding values of parallax and mass given in Table IX.

TABLE IX

π	Mass (Sun=1)	Mean Distance (Astron. Units)	Rotational Compt. km/sec.
0″.0001....	3,700,000,000	3,000,000	1,038
0.0004....	57,812,500	750,000	260
0.0016....	903,320	187,500	65
0.0064....	14,114	46,875	16
0.0256....	220	11,719	4
0.1024....	3.4	2,930	1

The order of the parallax of Messier 101 is suggested by the following considerations: Curtis gives 0″.033 as the average annual motion of 66 large spiral nebulae.[1] Could we assume that these objects are comparable with the stars in the matter of proper motion and distance, we should have, supposing them to be of the twelfth magnitude, 0″.005 for their mean parallax. Again, we may compare the cross-motion of 0″.033 with the observed radial velocities. Only a few of the latter have been determined as yet, but those found indicate high speeds. Assuming them to be distributed at random in space, Curtis finds the distance to be of the order of 10,000 light-years,[2] corresponding to a parallax of 0″.0003. Various objections to the acceptance of these results, even as rough guesses, immediately suggest themselves, but they are the best we have. At the moment, therefore, this method of discussion suggests large values for the masses involved.

An inspection of Plate IX conveys the impression that the general drift of the motion is outward along the branches. The direction of the branches in the vicinity of 52 of the points can be specified with fair accuracy, and a comparison with the observed motions shows a mean divergence for the latter of 7° ± 4° toward the *concave* side of the spiral.

[1] *Publications of the Astronomical Society of the Pacific*, **27**, 217, 1915.

[2] *Ibid.*, 218, 1915.

It would be of interest to consider the further development of a nebular mass in which, as apparently is the case with Messier 101, the direction of motion is so closely coincident with the branches of the spirals, and to examine the evidence afforded by other nebulae as to the probability of the continued existence of such motion. For example, continued outward motion should result in forms in which the distribution of the nebular material is widely different from that in Messier 101. Interesting cases such as Messier 77, Messier 81, Messier 101, Messier 100, and possibly N.G.C. 7293 immediately suggest themselves in this connection; in these we see a gradual transition from a nebula with the bulk of the material strongly concentrated in the center, while the spirals contain relatively little material, to the other extreme, where the spirals, containing most of the material, seem to be connected hardly at all with the central body.

All such questions as those touched on above, which naturally are highly speculative, are here considered because of the part played by elliptical motion in the Chamberlin-Moulton hypothesis as to the origin of spiral nebulae. The explanation which they have given, at least for the typical case, requires that the motion of a particle belonging to one of the branches should be inclined at a considerable angle toward the convex side of that branch. The departure of Messier 101 from the typical case imagined by Chamberlin and Moulton is sufficient to excite comment. Professor Chamberlin, however, kindly allows me to quote parts of his manuscript of a book, *The Origin of the Earth*, bearing on this question. He states:

> . . . . that the paths pursued by the projectiles are not identical with the spiral chain of projectiles into which they are forced to arrange themselves. The divergence between the paths and the chain of nebulous matter may vary widely. A much closer approach to coincidence between the paths of the projectiles and the chain of projectiles is assignable in certain cases of closer approach and more violent projection.

And also:

> . . . . the relative amount of this forward or tangential pull is a critical factor; its value is obviously dependent on the relative distance to which the bolt was projected.

The results can therefore perhaps be reconciled with the Chamberlin-Moulton hypothesis, by assigning sufficiently high values to the "disruptive" forces as compared with the central attractions; but all such questions can well await the accumulation of further observational data. In the meantime my measures of two photographs of Messier 81 (made by Mr. Ritchey in 1910 and 1916) give preliminary evidence of internal motion similar to that revealed in the case of Messier 101.

For the precision of such results the time factor is of course the most important, but when the photographs are to be used for the study of internal motions, various precautions will also contribute in no small degree:

1. Several photographs of each object should be taken; 4 or 5 old plates compared with 4 or 5 new ones should enable us to derive the motions satisfactorily.

2. Care should be taken that both hour angles and parallax-factors are similarly distributed in both the old and the new series of plates. A neglect of this precaution undoubtedly will introduce errors difficult or impossible of elimination; some of the discrepancies in the measures on Messier 101 are probably due to this cause.

3. As it is impossible to measure points near the center, if the exposures are long enough to show the fainter parts of a nebula, two sets of plates of the same object are desirable, one with long, the other with short, exposures.

4. The plates should be taken with a telescope of the greatest possible focal length, in order to increase the scale.

5. It is advisable to insert a star-trail on at least one plate for purposes of orientation. This of course is not necessary in deriving possible internal motions, but for the motion of translation it will afford more accuracy.

6. For a determination of the parallaxes of spiral nebulae, the distribution of the plates, if feasible, should be as outlined in my paper on stellar parallaxes.[1] The short exposures used for the proper motions might be utilized in part for this work.

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[1] *Mt. Wilson Contr.*, No. 111.

7. On account of asymmetry in the nebulous points, the plates should be measured in a purely differential manner, so that the settings for any point can be made on both plates in quick succession. The chance of choosing different parts of the image for the settings on the two plates is thereby greatly decreased. The stereocomparator is an admirable apparatus for such measures, but if not available, two miscroscopes may be used on an ordinary measuring machine, adjusted in such a way that they point to the same part of the field on the two plates, which are mounted side by side on the plate-carriage.

To test the possibility that the results found might be a consequence of systematic personal or instrumental errors, it was important that the plates should be measured with another machine and, if possible, by another measurer. I am glad to acknowledge the great service rendered by Mr. Seth B. Nicholson, who willingly spent a large amount of time in remeasuring parts of the Mount Wilson plates with two different machines. We first mounted a second microscope alongside the first, on one of our regular instruments used for measuring spectra. The two plates were mounted on the plate-carriage moved by the micrometer screw in such a way that the microscopes were directed toward nearly identical points on the two plates. In this way we were able to bisect corresponding points one after the other. As the instrument allowed only a part of a plate to be measured without readjustment on the carriage, 8 or 9 points were selected on each side of the center of the nebula, and their relative shift in declination was derived with respect to a dozen comparison stars. Mr. Nicholson found an annual difference of motion in declination of 0ʺ.083 (the east side of the nebula moving northward as compared with the west side), while my measures with the stereocomparator had given for the same points a relative shift in the same direction of 0ʺ.063. The agreement is satisfactory.

After having thus proved that the motion found was not due to any defect of the stereocomparator, Mr. Nicholson kindly undertook with this instrument a set of measures of 53 points of the nebula, using all the 32 comparison stars for reference. The plates were measured in all four positions and reduced in exactly the same way as the original measures. Of the 53 points measured, 11 are points mentioned as difficult on p. 11. The results for all 53 points, and also for the 42 remaining after excluding these 11 difficult points, are summarized in Table X.

TABLE X

Annual Motions	53 Points	42 Points	Agreement of Sign with Van Maanen	
			53 Points	42 Points
μ_α of translation	+0.″003	+0.″002	72%	76%
μ_δ of translation	−0.013	−0.011	75	83
μ_{rad}.	−0.003	−0.002	68	69
μ_{rot}.	+0.009	+0.010	75	76

The agreement of the motion of translation with that given on p. 9 is very satisfactory. Although Mr. Nicholson's value of the rotational component is smaller than that found above, a comparison with the data on p. 13 shows that the difference is within the uncertainty of the determination.

I wish to express my thanks to Mr. Hale and Mr. Seares for many suggestions they have made during the work on the plates discussed here, and to Miss Helen Davis for much assistance in the computations.

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INTENSITY OF THE CONTINUOUS SPECTRUM OF STARS AND ITS RELATION TO ABSOLUTE MAGNITUDE

By GEORGE S. MONK

Some time ago a comparison[1] was made by Mr. Adams of the intensity of the continuous spectrum of pairs of stars of large and small proper motion photographed upon the same plate, which showed that stars of small proper motion are relatively weaker in the more refrangible portion of the spectrum. Later, a large number of spectrograms obtained for radial velocity were compared and the results tabulated.[2] This comparison showed (1) that in types F0 to K4 stars of small proper motion have spectra which are weaker in the violet than stars of large proper motion; (2) that this difference increases with advancing type from F to K.

Accepting proper motion as an indication of distance, the first result might be interpreted as a consequence of the scattering of light in space; but the second result points to the fact that the absorption in the violet portion of the spectrum is due, at least in part, to physical conditions in the stars rather than to scattering.

In continuation of this method of comparison, the density of the continuous spectrum of most of the stars on the radial-velocity program has been estimated. The method used has been fully described in the second of the articles previously referred to, the only difference being that a negative with five successive exposures of Arcturus has been used as a standard for comparison instead of the plate of α Tauri previously used. Briefly, the method consists of comparing the photographic densities of stellar spectra with the standard plate of Arcturus. The standard plate is then measured under a microphotometer and the densities obtained at the points of comparison. From the estimated relation between the densities of the standard plate and that of any other star, the actual densities of the spectrum of the star may be deduced. In order to check the accuracy of this method, twenty spectra selected at random were measured under a microphotometer; the results were found to be in good agreement with those obtained by comparison with the standard plate of Arcturus. The densities on about 1200 plates have been estimated and reduced by this method.

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[1] *Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, **39**, 89, 1914.

[2] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, **40**, 85, 1914.

It was necessary to reject many plates for the following reasons: (*a*) presence of clouds or haze during exposure; (*b*) great zenith distances; the average zenith distances for groups of stars compared have been made to agree closely; (*c*) photographic blemishes on negatives; (*d*) extremely strong or weak negatives; the average intensities of any two groups compared were made to agree.

By the use of these restrictions, it is believed that the disturbing effects which must be present in a mass of material obtained under such widely differing conditions have been eliminated to a considerable extent. The plate numbers which were used in tabulating the measures show that no one group of stars was photographed within a short range of time, so that variations in color-sensitiveness of the emulsions should not affect the results materially. In most cases two or three plates of the same star have been measured and the mean values of the densities used.

These measures were tabulated for each type, the stars being grouped by large and small proper motion. The results appear in Table I. Slight corrections have been applied to the densities to make the values at λ 4930 equal for each spectrum type, thus affording a direct comparison at λ 4250.

These results are in the general direction of those of Adams and Kohlschütter, but show the connection of proper motion with absorption in the violet and the increase of this effect with spectrum type to a less extent. Soon after Table I had been prepared, Mr. Adams completed his determination of the absolute magnitudes of certain F5–K9 stars on the basis of differences in spectrum lines.

A comparison of the results of Table I with these new data gave Table II, in which "absorption in violet" is the amount of density by which the part of the spectrum about λ 4250 is weaker than that about λ 4930.

This is suggestive of a relationship between absorption in the violet and absolute magnitude—a result which substantiates that obtained by Adams and Kohlschütter, who compared stars of average apparent magnitude 6.0 having small proper motion with stars of fainter average apparent magnitude and large proper motion.[1] Consequently their differences in absolute magnitude were greater than those for the stars of Tables I and II, which were based on proper motion alone, apparent magnitude being disregarded.

TABLE I

Average Type	Average μ	Density at λ 4250	Density at λ 4930
F1	0.″020	0.41	0.35
F3	0.330	0.39	0.35
F6	0.015	0.40	0.36
F7	0.370	0.42	0.36
G3	0.018	0.27	0.38
G2	0.600	0.33	0.38
G7	0.012	0.29	0.43
G7	0.660	0.33	0.43
K1	0.017	0.23	0.39
K1	0.600	0.28	0.39

TABLE II

Average Type	Absorption in Violet Large μ − Small μ	Differences of Absolute Magnitude Large μ − Small μ
F6–7	+0.02	+1.8
G3–2	0.06	4.0
G7	0.04	2.7
K1	+0.05	+4.4

All of the density measures which could be so used were then tabulated according to absolute magnitudes, giving, as a final result, Table III. The average deviation of a single star from the mean "absorption in violet" in the last column of the table is ±0.09.

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[1] *Mt. Wilson Contr.*, No. 89, p. 1; *Astrophysical Journal*, **40**, 385, 1914.

Few stars were available having A-type spectra and large proper motion. A comparison of this scanty material shows very little difference in absorption, the spectra of stars which were used having large proper motion being slightly weaker in the violet.

TABLE III

Type	No. of Stars	Average Type	Average μ	Average M	Absorption in Violet
F3–F9	18 33	F6.1 F6.0	0″.010 0.500	+1.7 +4.8	+0.01 −0.05
G0–G4	17 28	G2.7 G2.0	0.015 0.660	+0.6 +5.2	+0.12 +0.02
G5–G9	51 22	G7.0 G7.2	0.011 0.690	+0.9 +6.0	+0.20 +0.07
K0–K4	26 18	K1.4 K2.5	0.017 0.680	+1.5 +6.6	+0.32 +0.16
K5–K9	10	K7.0	0.700	+7.7	+0.22

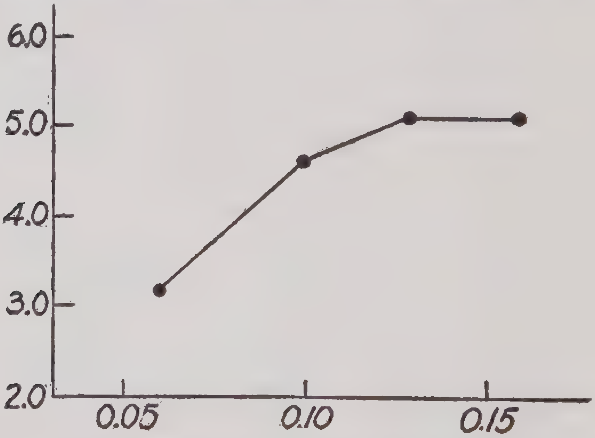

FIG. 1.—Abscissae: Differences of photographic density in the violet for stars of large and small absolute magnitude.
Ordinates: Differences in absolute magnitude.

This result, together with those expressed by Table III, is similar to that obtained by Van Rhijn.[1]

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[1] *Derivation of the Change of Colour with Distance and Apparent Magnitude* (dissertation, Groningen, 1915), p. 73.

Plotting the differences in *M* for each type against the differences in absorption in the violet, the curve in Fig. 1 is obtained. The relationship which seems to exist between absolute magnitude and the relative weakness in the violet of the stars having small proper motion leads to the conclusion that at least the greater part of this effect is due to differences in absolute magnitude rather than to absorption of light in space.

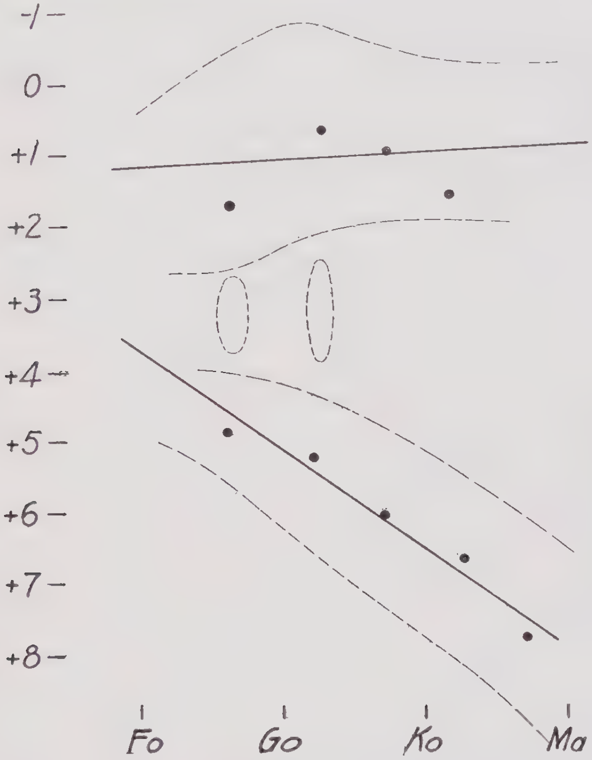

Fig. 2.—Variation of absolute magnitude with spectrum type

No stars of large absolute magnitude between K5 and K9 were originally available. Seventy-six additional stars of small average proper motion and about 1.5 magnitudes fainter in apparent magnitude than the stars having small proper motion used above have been measured. Mr. Adams has found their average absolute magnitude to be +2.1, about one magnitude fainter than those used in Table III. Although the values for the violet absorption fall between the two groups compared, these stars have been omitted, in part because of uncertainty in their proper motions, and in part because many of the photographs are of inferior quality.

An interesting relationship is shown in Fig. 2, in which the average absolute magnitudes of the stars used are plotted against spectrum type. The faint dotted lines show the range of magnitude within each group. The dotted ovals represent seven F and four G stars measured, but not used in this discussion. It will be seen that there is an absence of intermediate magnitudes for the latest types, a result in agreement with the hypothesis of giant and dwarf stars as discussed by Hertzsprung and Russell. It is probable that the greater irregularity of the curve in the case of the stars of small proper motion is due mainly to the relative uncertainty in the determination of their absolute magnitudes.

The photographs of spectra used in this discussion were obtained under a wide variety of conditions, and the results, accordingly, are to be considered mainly from a qualitative point of view. It seems reasonable to conclude from them, however, that with the aid of photographs taken with this purpose directly in view, the relative intensity of the violet portion of the spectrum, together with spectrum type, might be employed to provide values of absolute magnitude of a fair degree of accuracy.

I am indebted to Mr. Adams for much valuable criticism during the progress of the work and in the preparation of this paper.

Mount Wilson Solar Observatory
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THE ACCURACY OBTAINABLE IN THE MEASURED SEPARATION OF CLOSE SOLAR LINES; SYSTEMATIC ERRORS IN THE ROWLAND TABLE FOR SUCH LINES

By CHARLES E. ST. JOHN and L. W. WARE

I. INTRODUCTION

Close pairs of solar lines play such a rôle in the search for evidence of mutual influence between neighboring lines in the solar spectrum—a deduction from the anomalous-dispersion hypothesis—that it is important to have an idea of the accuracy obtainable in their measurement, and hence of the relation that the errors bear to the probable magnitude of the suggested effect. The investigation thus necessitated is also preliminary to a determination of the wave-lengths of the solar lines in international units, and in particular is an effort to develop a method of measuring the wave-lengths of such as have lines closely adjacent.

In investigations on the resolving power of spectroscopes, a complex line is considered to be resolved when the intensity at the midpoint of the overlap between two lines of equal intensity is 0.81 of the maximum for the single lines. For precision in the measurement of close doubles in the solar spectrum the important consideration is the lower limit for the difference in wave-length between lines of a specified intensity and character, that can be measured to a given accuracy with the spectrographs and photographic plates now available, rather than the resolving power which shows the line as double. Though the foregoing standard of resolution is sufficient for determining the complex character of a solar doublet, the accuracy of the wave-length determination for the separation implied by it has not previously been investigated. Moreover, since the conditions presented by the unequal intensity of the background on the two sides of each component are such as may introduce systematic personal errors, it has seemed desirable to examine this question in detail; and, in fact, the results of the examination form an important portion of this contribution.

2. APPARATUS

For the present investigation an excellent grating by Anderson has been at our disposal through the kindness of the Physical Laboratory of the Johns Hopkins University. The ruled surface is 114×160 mm and the number of lines is 95,000. The definition is all that can be asked, even when the whole surface is used. As determined from the iodine absorption lines, the excellent performance is retained up to and including the fifth order. It has been used as a Littrow spectrograph, focus 30 feet (9.14 m), in connection with the 60-foot tower telescope. For comparing the results given by large-scale spectrograms obtained by long focus and high resolving power, respectively, a series of plates was taken with a large grating by Michelson, number of lines 77,500, used in the 75-foot spectrograph of the 150-foot tower equipment. The slit-width was usually four-normal, but for the very difficult separations a two-normal slit was employed, giving 94.3 per cent of the resolution for an indefinitely narrow slit[1] with monochromatic light. Commercial photographic plates have been used as follows: for the blue-violet, Seed Process plates; for the green, Cramer's X-ray for the most part, but in special cases where fineness of grain was particularly important, Process plates sensitized with pinaverdol; for the yellow-red, Wratten "M" plates and Seed Process sensitized according to the Wallace formula.[2] Backed plates have been used with advantage for some of the most difficult separations.

Upon a plate of an intensity suitable for the general run of lines, the measurement of the separation and, hence, of the wave-lengths of the components of a close pair, in which one or both components are strong lines, is difficult; but with exposure and development adapted to the case, somewhat too strong for weak lines, the difficulties and uncertainties are decreased with a corresponding increase of confidence in the results. The exposure times and the intensity of development have been chosen to obtain the best conditions for the particular separation under consideration.

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[1] Schuster, *Astrophysical Journal*, **21**, 207, 1905; Zeeman, *K. Akad. Amsterdam Proceedings*, **18**, 412–415, 1915.

[2] *Astrophysical Journal*, **26**, 317, 1907.

For measuring the spectrograms the usual filar-micrometer instruments have been employed; in addition a registering microphotometer of the Koch form has been used for purposes of checking, and also for some absolute determinations where it appeared especially difficult to eliminate the personal equation.

3. METHODS OF MEASUREMENT

In the measurement of the separation of close pairs of solar lines some means of determining whether the results, barring accidental errors, represent the real separation or are affected by systematic errors is an important desideratum. That concordant values are obtained by repeated measurements of the same or of other similar plates by a single observer has little bearing upon the question, as possible systematic errors introduced by the personal equation of the observer are not thereby eliminated. This comes into play in estimating the position of the maximum intensity of a line, one of whose edges is seen against a strong continuous spectrum background, and the other against a background—the region of overlap—whose intensity may differ but little from that of the line itself. In order to obtain a check upon the measurements for ordinarily close pairs, all determinations have been made by at least two observers, and in order to vary the conditions widely they have been made upon a series of six spectrograms, representing the first five orders of the 30-foot and the first order of the 75-foot spectrograph. The appearance of a close pair of lines differs greatly under these conditions, as the scale of the spectrograms ranges from 1.8 to 0.25 A per mm and the resolving power changes by fivefold. Under such widely different circumstances any personal error depending upon the proximity and character of the adjacent lines will not remain constant, and concordance among the results can be taken to indicate the practical elimination of such error. For selecting the data to be used in forming the final mean separation for a given pair of lines the criterion is, therefore, the degree of concordance between the orders. Beginning with the highest, those orders are included for which the agreement between the orders is comparable to that between the measurements within the orders.

For pairs which with the highest resolving power and the largest scale employed are not completely separated, but yet appear accurately measurable with the filar micrometer, curves have been drawn by the registering microphotometer. Separations so obtained appear to be least affected by errors, either systematic or accidental, and to represent most nearly the true separation of the components.

For doublets at or very near the limit of resolution, whose components cannot be sufficiently separated for micrometric settings upon their maxima to be made without influence by the neighboring line, and are not reproduced as discrete curves by the registering microphotometer, the following method has been used to obtain the separation and furnish a check upon the attempted micrometer measurements. In such cases it is generally possible to make intensity classifications for the components as precisely as for the free-standing lines. An inspection of the plate enables one to select the free-standing lines that correspond most nearly to the components of the complex line. Microphotometer curves are obtained for the complex line and the selected list of isolated lines, the sensitiveness of the instrument being held constant during the run over the plate. From the curves of the isolated lines one selects those that conform to the violet and red branches of the compound curve, superposes the negatives so that the corresponding branches of the three curves are coincident, and measures the distance between the axes of the constituent curves. The scale of the spectrograms used for this purpose, 1 mm=0.30 A, is increased fifty fold by the microphotometer, so that the scale of the curves is 1 mm=0.006 A. The measurements are made with sufficient accuracy with a fifth-millimeter scale. The operation is an analysis of the doublet, since one determines not only the separation but also the form and intensity of the components.

A less satisfactory procedure is to measure upon the original spectrogram the widths of a large number of isolated lines of the same nominal intensities as the components of the doublet, by setting upon their edges as in the measurement of star-images. Similar settings are made upon the free edges of the complex line. Its width decreased by the half-widths of the two sets of free-standing lines furnishes a measure of the separation of the components. The weak point of the method lies in the fact that lines of the same nominal intensity vary greatly, even in the same spectral region, as shown by the curves of widely different heights and characters which they yield; but by including several reference lines of each intensity the results are in fair agreement with those given by the microphotometer.

4. ILLUSTRATIONS OF THE METHODS

a) *Filar micrometer.*—A number of doublets in the violet region were measured for separations upon a fifth-order plate of excellent definition, scale 1 mm=0.29 A. One exposure was measured by an observer five times at intervals extending over two months; three exposures were measured by another observer, once each. Both observers had had experience in the measurement of solar lines. The results are shown in Table I. In the first column between the Rowland wave-lengths, an indication is given of the appearance of the doublets. The symbol $s<<<<c$ means that the space between the maxima is very much less dark than the continuous background bounding the free edges of the components, i.e., the doublet is barely resolved; while for the pair at λ 4457, $s=c$, there is practically complete resolution. In the eighth and ninth columns are the means of the series of 5 and 3 measures, and in the tenth column the differences between the Rowland and the mean Mount Wilson separation. These differences are all positive and indicate an average systematic error of 0.006 A, either in the Rowland tables or in the Mount Wilson measures, carrying with it a corresponding error in the wave-lengths of the constituent lines. In the last column are given the separations obtained by the same observers upon a third-order spectrogram of excellent quality. The agreement with the fifth-order plate is mutually corroborative. The systematic character of these discrepancies between the Rowland and Mount Wilson results lends them a greater significance than their magnitude alone would imply. Further evidence of

their reality and the reasons for considering them actual errors in the Rowland values will be given in later sections of the paper.

b) *Registering microphotometer.*—The Mg-Fe pair at λ 5167 and the Fe pair at λ 5169 furnish examples of the unrecognized difficulty and the unsuspected systematic errors that occur in filar-micrometer determinations of separations apparently within the range of ordinary methods. In these cases the resolution under high power is so nearly complete that it has not been easy to convince one's self that the micrometer settings upon the components are subject to the systematic errors that appear when the results are compared with those obtained from the registered curves. In Plate X (*a*) a spectrum of the fourth order is reproduced, magni-

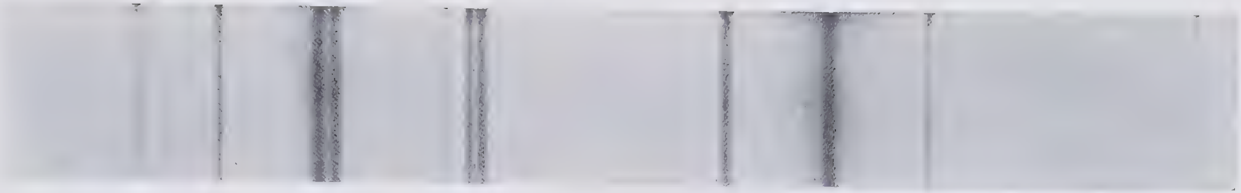
TABLE I

MICROMETRIC MEASURES ON CLOSE PAIRS OF SOLAR LINES

Pairs	Int.	Order V							Rowland *minus* Mt. Wilson	Order III
		1	2	3	4	5	S (5)	W (3)		
4455.980 $s<<<<c$ 6.064	2 3	0.067	0.078	0.072	0.072	0.075	0.073	0.078	+0.009	0.074
4457.600 $s=c$ .712	2 2	.108	.107	.105	.106	.104	.106	.106	+ .006	.108
4459.199 $s<<<c$ .301	2 3	.093	.098	.093	.094	.096	.095	.093	+ .008	.091
4464.844 $s<c$ .938	2 1	.088	.086	.088	.088	.088	.088	.090	+ .005	.091
4472.884 $s<<<c$ .967	1 0	.079	.080	.068	.078	.082	.077	.082	+ .004	.076
4476.185 $s<<<<c$ .253	4 3	.071	.065	.067	.064	.064	.066	.068	+ .001	.070
4482.338 $s<<<<c$ .438	5 3	0.091	0.091	0.089	0.096	0.092	0.092	0.097	+0.006	0.094
Mean.....	.....	.....	.....	.....	.....	.....	.....	.....	+0.006	......

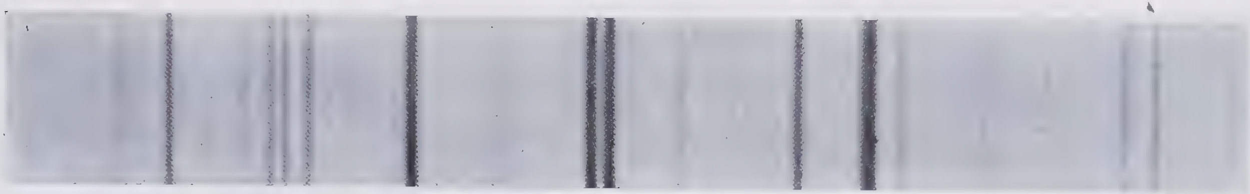
PLATE X

a



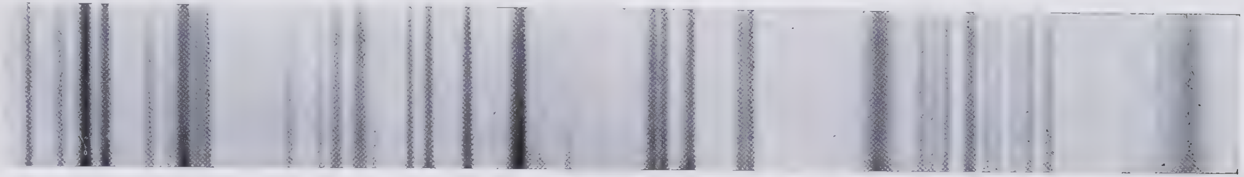
Fourth Order. Enlargement 4.7

b



Fourth Order. Enlargement 4.7

c



Fourth Order. Enlargement 4.7

Separation:		λ3918	λ3919
	Rowland	0.099	0.101
	Mount Wilson	0.099	0.093

fication 4.7, and in Figs. 1 and 2 are the corresponding curves drawn by the registering photometer.

Under the title "An Adaptation of the Koch Registering Microphotometer to the Measurement of the Sharpness of Photographic Images"[1] Tugman shows that the upper slit of the microphotometer should not cover more than one-fifth of the total width between maximum and minimum density, i.e., the half-width of the spectrum line; but that with slit-widths less than 0.75 mm the pinhole effect begins to enter, which falsifies the density-gradient by widening the base of the curve. Such an effect, while distorting the curve and lowering the resolution by the basal widening, does not shift the axis of the curve for an isolated line; and the free portions

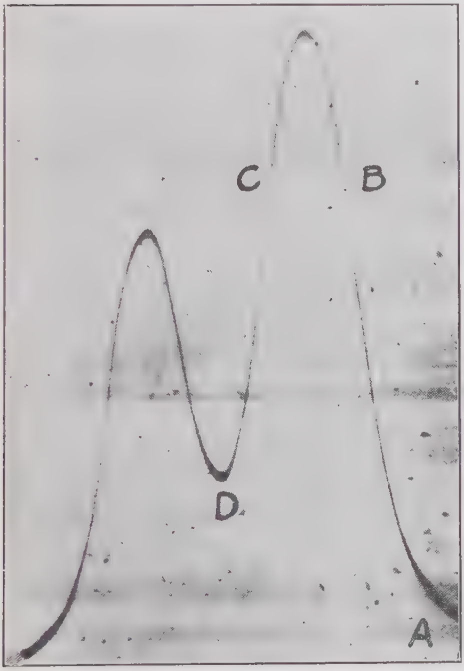


FIG. 1.—Microphotometer curve of Mg–Fe pair λ 5167.

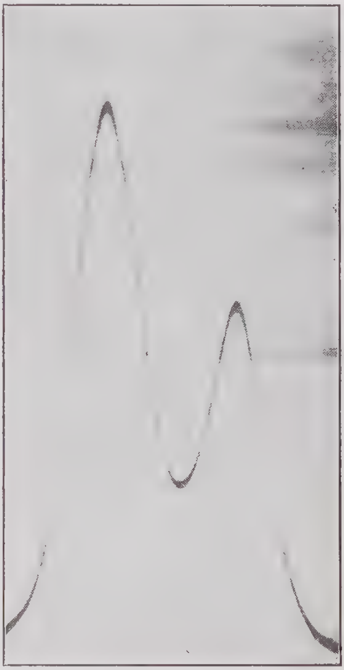


FIG. 2.—Microphotometer curve of Fe pair λ 5169.

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[1] *Astrophysical Journal*, 42, 321, 1915.

of the components of a nearly separated doublet, such as the pair at λ 5167 or at λ 5169, appear to be unaffected.

The shortest distance between maximum and minimum intensity occurs between the lowest point in the region of overlap and the peak of the smaller curve. Upon the high-dispersion spectrogram used the horizontal distance is 200 μ for λ 5167. The objective magnifies fivefold; with a slit-width of 100 μ one-tenth of the image of the half-line is covered. The symmetrical character of the free portions of the two curves strikes one at a glance. The distance between their axes is 430 μ=0.158 A. Three observers with long experience in the measurement of solar lines by the filar micrometer found separations of 0.175, 0.169, and 0.172 A, respectively. That of the Rowland table is 0.181 A.

Third- and fifth-order spectrograms yield separations consistent with those of the fourth order, showing similar discrepancies between micrometer measures and the values derived from the registered curves. If the Rowland value represents the true separation, there must be a relative instrumental displacement of the axes of the curves of 3 to 4 mm, which seems inadmissible. The two methods show similar results for the pair at λ 5169, namely, 0.122, 0.136, and 0.151 A for the registering microphotometer, filar micrometer, and Rowland, respectively. To ascribe such discrepancies to systematic errors in the filar-micrometer settings, as seems the probable explanation, is a decided shock to all the observers, since they thought in advance that the settings could be made without systematic errors. The resolution upon the original spectrogram is more nearly complete than the curves indicate, as the pinhole effect has accentuated the overlap.

c) *Analysis.*—For pairs with components of intensities 3 and 4 on the Rowland scale the limit of resolution measurable by micrometer appears to be in the neighborhood of 0.10 to 0.15 A, increasing with the wave-length. In Table II are given the separations for very close pairs determined by micrometric settings upon the maxima of the components, upon the edges of the doublets and the reference lines, and by analysis of the curves registered by the microphotometer. The spectrograms were taken in the fifth order, the scale varying from 0.23 to 0.30 A per millimeter in passing from the green to the violet. The seventh column in Table II contains the unweighted means of the three methods of determination, and the last the Rowland *minus* Mount Wilson differences, which again are all positive with a mean magnitude of 0.012 A. The registered curve for the pair at λ 4219, Rowland intensities 3 and 4, is reproduced in Fig. 3, with curves of the isolated lines λ 4201, intensity 3, coincident with the red branch, and λ 4196, intensity 4, coincident with the violet branch. The separation derived from these component curves is 0.051 A, the Rowland value being 0.064 A. Although the gradation-curve corresponding to the free edge of the doublet does not

TABLE II

COMPARISON OF METHODS FOR THE SEPARATION OF PAIRS AT THE LIMIT OF RESOLUTION

ROWLAND			FILAR MICROMETER		ANALYSIS	MEAN Δλ	R−MT. W
Wave-Lengths	Int.	Δλ	On Max.	On Edges			
4204.101 *s*<<<<<*c* .163	3 4	0.062	0.046	0.040	0.041	0.042	+0.020
4219.516 *s*<<<<<*c* .580	4 3	.064	.053	.045	.049	.049	+ .015
4476.185 *s*<<<<*c* .253	4 3	.068	.068	.063	.068	.066	+ .002
4482.338 *s*<<<<*c* .438	5 3	.100	.094	.081	.087	.087	+ .013
4878.313 *s*<<*c* .407	3 4	.094	.093	.086	.086	.088	+ .006
5264.329 *s*<<<<*c* .415	4 3	.086	.079	.068	.075	.074	+ .012
5270.438 *s*<<<*c* .558	3 4	0.120	0.104	0.108	0.098	0.103	+0.017
Mean R−Mt. W							+0.012 A

represent the actual distribution of energy in the line, it does represent the slope and height of a curve due to a free-standing line which in intensity and character is identical with the corresponding component of the doublet. Any errors, due to the pinhole effect, in reproducing the density-gradient of the free edges of the components occur also in the curves of the isolated lines of the same form and intensity. A simple compounding of the curves would yield too high a resultant for the center. This apparent discrepancy is referable to the fact that the areas of the registered curve, as Tugman shows, do not measure the energy of the spectral lines.

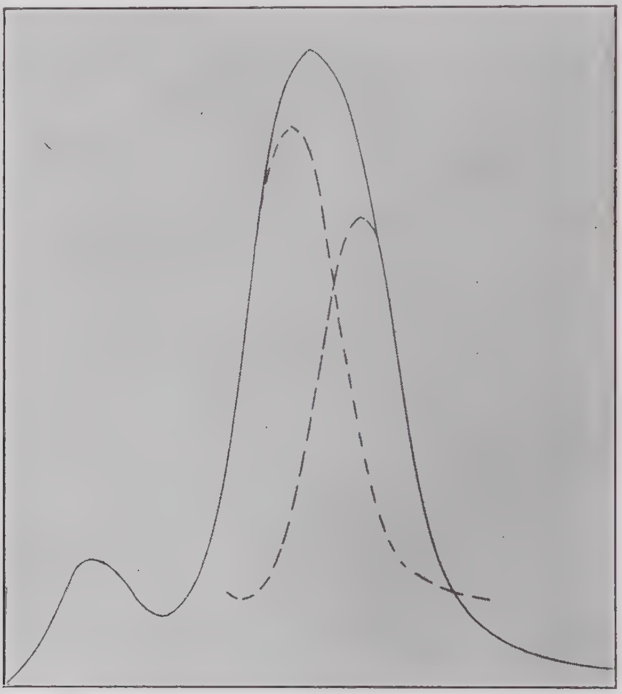

FIG. 3.—Microphotometer curve of solar pair λ4219, intensity 3 and 4, with curves of isolated lines λ4196 and λ4201, intensities 4 and 3, superimposed.

5. DEFINITIVE RESULTS

Pairs consisting of lines of intensities 3 and 4 occur more frequently than any others and offer the most favorable basis for a comparative study. To obtain an extended series of observations, thirty pairs with components of these Rowland intensities were taken for the measurement of the separations. These are the pairs between λ 3900 and λ 5270, the separation of whose components is under 0.35 A. As the conditions upon which the discrepancies between the Rowland and the Mount Wilson values depend vary with the spectral region—the width of the lines of the same nominal intensity increasing with the wave-length—separate consideration is given to the pairs to the red of λ 4000.

In Table III are assembled the data relative to these thirty pairs, which, as far as possible, were measured on each of the six series of plates. Vacant spaces in the table mean that the resolution was not sufficient for the present purposes of measurement. The parentheses indicate that, though the measurement seemed possible at the time, the results were not considered in obtaining the means because of the lack of agreement with the more concordant results for the higher orders. Whether or not they should be included is a matter of judgment; their inclusion, however, would increase the discrepancy between the Rowland and the Mount Wilson values. The pairs are arranged in the order of decreasing separation; therefore, as one passes down the table, the first-order measures soon become uncertain, then impossible, a course followed by the other orders in succession. This furnishes the means for fixing a minimum separation measurable to the indicated degree of precision with the given resolving powers and dispersions. This is influenced, however, by the wave-length and the character and intensities of the lines. The results for the 75-foot spectrograph recorded in the twelfth column were not used in obtaining the means, as the purpose was to compare the results for the two instruments. As far as the measures are dependable they are in excellent agreement.

The magnitude of the errors in the Rowland separation depends upon the proximity of the components. For the first 6 pairs, mean separation 0.274, it is +0.003; for the following 8 pairs, mean separation 0.145, it is +0.008; and for the last 8 pairs, mean separation 0.075, it is +0.013 A, based upon the mean of the three methods used.

TABLE III

FILAR-MICROMETER SEPARATION OF CLOSELY ADJACENT SOLAR LINES

PART I. PAIRS TO THE RED OF λ 4000

ROWLAND		ORDER, *nm* AND I MM IN A FOR 30-FOOT					MEAN Δλ	WT.	MEAN DEV.	ROWLAND *minus* MT. W	75-FOOT	REMARKS
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
4461.818 2.165	4 3	0.342	0.344	0.342	0.341	0.340	0.342	31	0.001	+0.005	0.340	
4489.911 90.253	4 3	.339	.338	.341	.340	.340	.340	31	.002	+ .002	.339	
4339.617 .882	4 3	.267	.265	.264	.264	.262	.264	12	.002	+ .001	.263	
5079.158 .409	3 4	.254	.250	.248	.249	.246	.249	17	.003	+ .002	.248	
4556.063 .306	3 4	.238	.240	.240	.241	.238	.239	21	.002	+ .004	.241	
5139.427 .644	4 4	.209	.212	.212	.212	.217	.212	15	.002	+ .005	.210	Poor quality
5107.619 .823	4 4	.190	.193	.192	.194	.192	.192	27	.002	+ .012	.191	Excellent quality
5041.069 .255	3 4	.180	.182	.185	.189	.188	.183	18	.005	+ .003	.175	Violet component double
4024.726 .881	3 4	.154	.157	.156	.154	.156	.156	21	.001	− .001	.153	
5169.069 .220	3 4	....	.133	.137	.137	.139	.136	19	.003	+ .015	(.124)	Difficult
4171.068 .213	4 4	(.129)	.134	.136	.136	.136	.135	14	.002	+ .010	.132	

4667.626 .768	4 3	(.125)	.131	.131	.133	.133	.132	16	.001	+ .010	(125)	
4315.138 .262	3 4	(0.099)	.111	.114	.119	.114	.115	19	.003	+ .009	(.097)	
4078.515 .631	4 3	....	.107	.106	.110	.112	.109	32	.003	+ .007	(.103)	
4581.575 .693	4 4	....	(0.102)	.109	.107	.106	.107	10	.003	+ .011	(.090)	
5270.438 .558	3 4	....	....	.098	.104	.113	.104	15	.006	+ .016	....	Very difficult
4878.313 .407	3 4	....	....	.092	.094	.093	.093	17	.005	+ .001	....	Very difficult
5264.329 .415	4 3	....	....	.076	.080	.083	.079	11	.004	+ .007	....	Very difficult
4476.185 .253	4 3	....	....	0.069	0.069	.067	.068	2	.003	.000	....	Very difficult
4219.516 .580	4 3	....	....	....	....	.053	.053	1	....	+ .011	....	Very difficult
4204.101 .163	3 4	....	....	....	....	.046	.046	3	.002	+ .016	....	Very difficult
4210.494 .561	4 3	....	....	....	....	0.046	0.046	2	0.002	+0.021	....	Very difficult

TABLE III—*Continued*

PART II. PAIRS TO THE VIOLET OF λ 4000

ROWLAND		ORDER, *nm* AND 1 MM IN A FOR 30-FOOT					MEAN Δλ	WT.	MEAN DEV.	ROWLAND *minus* MT. W	75-FOOT	REMARKS
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
3910.984 1.135	4 3	(0.158)	0.153	0.153	0.152	0.150	0.152	14	0.002	−0.001	0.154	Poor quality
3956.476 .603	4 4	(.116)	.121	.119	.120	.120	.120	17	.001	+ .007	.118	
3979.664 .783	4 4	(.114)	.117	.119	.116	.118	.117	14	.002	+ .002	.119	
3948.818 .925	4 4	....	.099	.100	.103	.104	.102	17	.002	+ .005	(.095)	
3918.464 .563	4 4	....	(.091)	.098	.098	.100	.099	11	.001	.000	(.091)	
3989.912 90.011	4 3	....	(.085)	.094	.094	.094	.094	14	.001	+ .005	(.088)	
3919.208 .309	3 3	....	(0.082)	.091	.094	.094	.093	11	.002	+ .008	(0.081)	
3926.086 .165	4 3	....	....	(0.069)	0.075	0.074	0.075	8	0.001	+0.004	....	

As is well known, the solar lines decrease in width and the spectrograms improve in definition in passing from the red to the violet. This change is striking when one attempts measurements near the limit of resolution in the two regions in immediate sequence. In Part II of Table III are the data for 8 pairs to the violet of λ 4000, mean separation 0.107 A, for which the mean error in Rowland is +0.004 A, approximately half that for the pairs to the red of λ 4000 with the same mean separation. A difference of this order is consistent with the appearance of pairs in the two regions, for, in the violet, owing to the better resolution, the contrast between the intensities of the background on the two sides of a component is much less marked for pairs of like intensity and separation.

As the Mount Wilson measures, made under the most diverse conditions, differ systematically from the Rowland values, the assumption of corresponding errors in the Rowland tables seems a well-grounded conclusion. The establishment beyond doubt of the errors in a few cases may, however, strengthen the evidence that the discrepancies in general represent actual errors in the Rowland tables. The interpretation of the registered curves for the pairs at λ 5167 and at λ 5169 given in an earlier paragraph (p. 8) does not appear open to serious criticism, as no recognized peculiarity in the action of the registering microphotometer has the effect of displacing the axes of curves for lines so nearly resolved. An example still more free from possible criticism is supplied by the solar pair at λ 5107. These are lines of the best quality, and are completely resolved upon the Mount Wilson plates. A spectrum of the fourth order is reproduced in Plate X (*b*) and the curve in Fig. 4. The Mount Wilson micrometer measures give a separation of 0.192 A for the mean of 33 measures on the six series of spectrograms with an average deviation of 0.002 A. The separation given by the registered curve is 0.190 A, while that from the Rowland wavelengths of the constituent lines is 0.204 A, a discrepancy of +0.013 A.

Another example that appeals to the eye, even without measurement, is found in the pairs at λ 3918 and λ 3919. A spectrum of the fifth order is shown in Plate X (*c*), and the corresponding registered curves in Fig. 5. The eye decides at once which is the wider separation, and both the micrometer measurements and the registered curves confirm it. The separations deduced from the Rowland wave-lengths are 0.099 A and 0.101 A, but the separations upon the Mount Wilson plates are 0.099 A and 0.093 A, respectively.

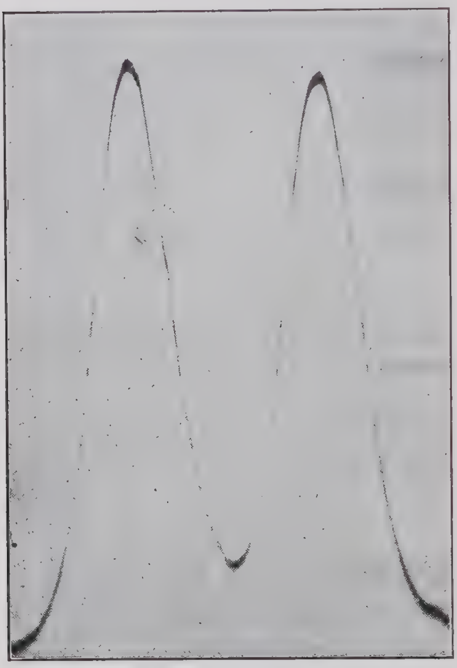

FIG. 4.—Microphotometer curve of Fe pair λ 5107.

6. THE ROWLAND PLATES

Through the kindness of Professor Ames some of the original plates taken by Rowland for the solar spectrum map were loaned to the Observatory. Two plates of the third order are especially good, showing very little grain. In the introduction to the *Preliminary Table of Solar Spectrum Wave-Lengths* Rowland says:

> In some cases when a double line is particularly difficult to separate, measurements are given on the two components and also on the line unresolved. This last measurement is placed in parenthesis between the other two measurements. Thus:
>
> | W.-L. | INTENSITY |
> |---|---|
> | 3738.454 | 3 |
> | (3738.466) | 6 *d* |
> | 3738.505 | 2 |
>
> means that there is a line at *w.-l.* 3738.466 with the intensity 6; and that with good definition this line may be resolved into two components having intensities 3 and 2 and the given wave-lengths.

Such a complex appearing in the table at λ 3926 is well resolved on the Rowland plates, more plainly resolved, in fact, than other doubles on the plates, which, from the absence of any indication to the

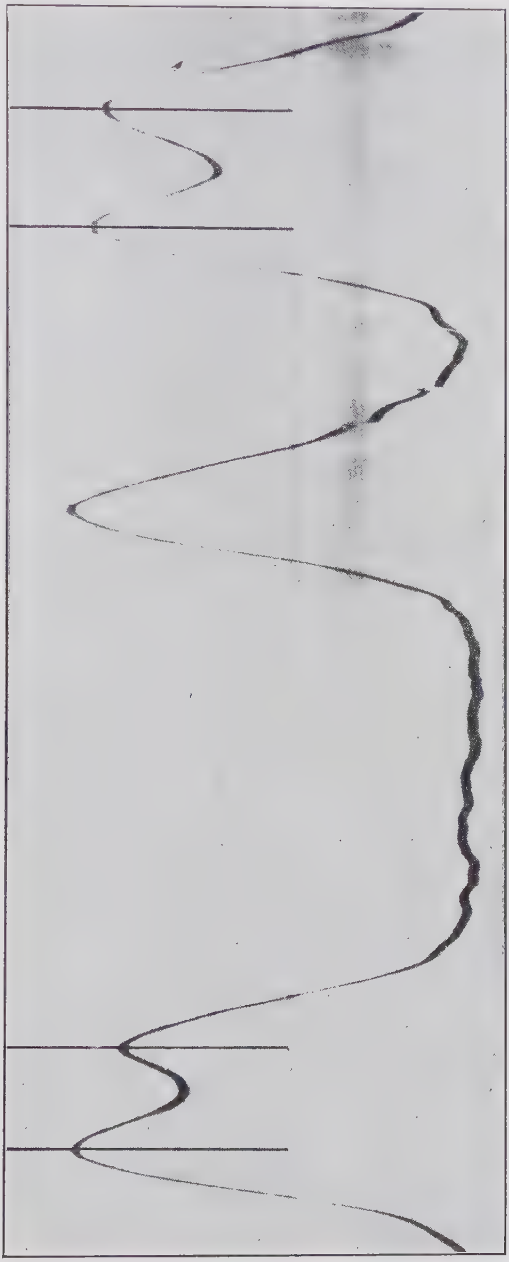

FIG. 5.—Microphotometer curve of Fe pair λ 3918 and Fe-Cr pair λ 3919

contrary, were measured without difficulty. It is thought, therefore, that the quality must have been considered very satisfactory by Rowland and Jewell. Though the Rowland plates excel the Mount Wilson plates (Seed Process) in fineness of grain, the slightly larger scale of the latter compensates for this, so that, in fact, they are somewhat easier to measure. For comparison ten pairs were measured upon both the Rowland and Mount Wilson plates. The data are shown in Table IV. In the third and fourth columns is given an indication of the resolution. For four pairs $s=c$; the

TABLE IV

COMPARISON BETWEEN MOUNT WILSON AND ROWLAND THIRD-ORDER PLATES

WAVE-LENGTH	INT.	RESOLUTION		MT. WILSON MEASURES		ROWLAND TABLE $\Delta\lambda$	ROWLAND TABLE *minus* MT. W PLATE
		Rowland 1 mm=0.64 A	Mt. Wilson 1 mm=0.57 A	Rowland Plate $\Delta\lambda$	Mt. Wilson Plate $\Delta\lambda$		
3910.984 1.135	4 3	$s=c$	$s=c$	0.151	0.153	0.151	−0.002
3914.426 .477	3 2	$s<<<<<c$	$s<<<<<c$	.046	.044	.051	+ .007
3918.464 .563	4 4	$s<<c$	$s<c$	.101	.098	.099	+ .001
3919.208 .309	3 3	$s<<<c$	$s<<c$	.094	.091	.101	+ .010
3926.086 (.123) .165	4 3	$s<<<<c$	$s<<<c$	.075	.069	.079	+ .010
3948.818 .925	4 4	$s<<c$	$s<c$	.106	.100	.107	+ .007
3956.476 .603	4 4	$s=c$	$s=c$	.124	.119	.127	+ .008
3979.664 .783	4 3	$s=c$	$s=c$	.120	.119	.119	.000
3989.912 90.011	4 3	$s<<<c$	$s<<c$	.094	.094	.099	+ .005
4024.726 .881	3 4	$s=c$	$s=c$	0.154	0.156	0.155	−0.001
Mean . . .							+0.0045

intensity in the space between the components equals that of the continuous spectrum background at their outer edges, i.e., the resolution is complete. The pair at λ 3914 shows only the merest trace of resolution. For the other five pairs the resolution upon the Mount Wilson plates, though far from complete, is a degree better than upon the Rowland plates.

(*a*) The Rowland values exceed the Mount Wilson measures on the Rowland plates by +0.0023 A; this suggests that the personal equation of the observer plays an important rôle. (*b*) The Mount Wilson measures on the Rowland plates exceed the measures on the Mount Wilson plates by +0.0022 A, i.e., owing to the better resolution the influence of an adjacent line is less on the Mount Wilson plates with a correspondingly lessened tendency toward overspacing. The two effects (*a*) and (*b*) account for the Rowland *minus* Mount Wilson mean of +0.0045 A. The discrepancies between the Mount Wilson and the Rowland separations measured upon the same plates are most manifest for pairs near the limit of resolution, thus:

Fairly Resolved		Near Limit of Resolution	
	R−Mt. W		R−Mt. W
λ 3910	0.000	λ 3914	+0.005
3918	−0.002	3919	+0.007
3948	+0.001	3926	+0.004
3956	+0.003	3989	+0.005
3979	−0.001		
4024	+0.001		

7. DISCUSSION

In view of the diversity in the methods of measurement employed at Mount Wilson and the wide range in the scale and resolution of the spectrographic material, it seems probable that the effects due to personal equation have been greatly reduced, if not eliminated, and that the differences between the Rowland and Mount Wilson measures represent systematic errors in the former. A tendency toward over-separation of the components of close pairs appears to accompany departure from the conditions for finest definition. As the Rowland plates were taken for the average run of lines, it seems probable that the errors for closely adjacent lines would be systematically positive. That the evidence from the Mount Wilson observations is cumulative is seen from the data in Table V, where the results from various lines of investigation are assembled. The magnitude of the error depends upon several factors, the proximity, the intensity, the character, and the wave-length of the components. For lines of like intensity and character in a definite spectral region the error increases with decrease of separation; for lines of the same separation, character, and region it increases with the intensity; for lines of the same separation, character, and intensity it is distinctly greater for longer wave-lengths.

TABLE V

RÉSUMÉ OF DATA

Source	Table I	Table II			Table III		Table IV		Pairs 5167–5919
Wave-length.	4465	4685	4680	4570	4640	3940	3960	3940	5168
Intensity....	2.5	3.5	3.5	3.6	3.5	3.8	3.8	3.1	6.5
Separation...	0.087	0.073	0.274	0.145	0.075	0.107	0.125	0.080	0.149
R−Mt. W...	+0.006	0.012	+0.003	+0.008	+0.013	+0.004	0.000	+0.005	+0.017

The difficulties of measurement and the systematic character of the errors encountered in this investigation occur in the determination of the absolute wave-lengths of lines with closely adjacent companions. Differential measures upon similar spectra such as those of the two limbs of the sun are not subject to such systematic errors, provided the spectra are of equal intensity, a condition that gains in importance when the measurements are upon lines not completely isolated.

A résumé of the circumstances that accompany the systematic differences between the Rowland and the Mount Wilson measurements suggests as a cause a personal equation whose influence appears in estimating or neglecting the effect of contrast. Early in this investigation a tendency toward an increase in the measured separations accompanying a lessened intensity of the photographic plate became evident. For example, six Mount Wilson observers, including the four with the widest experience in solar work, measured the separation of the pair at λ 5455 upon two spectra identical except for a slight difference in intensity. The measurements agree in showing a separation upon the weaker exposure greater by 0.013 A; an appeal to the microphotometer curves supports the smaller value.

An indication of wider import is that a falling off in definition due to any cause is a condition productive of the errors in question. It is immaterial whether the decrease in the sharpness of the lines with the consequent loss in definition is caused by a slight change in focus or a widening of the slit. Two third-order spectrograms, No. 56 and No. 199, throw light on the question. Upon Plate No. 56, considered at the time to be of excellent quality, seven close pairs were measured by three experienced observers with fairly concordant results. Later, spectrogram No. 199 was taken in the same region with a two-normal slit and a backed plate. The plates were taken with the same focal setting and are practically of the same intensity, but No. 199 is very superior in definition. The five exposures were measured by two of the previous observers with very concordant results, which appear in Table VI, together with those from a fifth-order spectrogram. The separations given by No. 56 are consistently larger than those from No. 199, the average difference being +0.008 A. The differences, moreover, are greater for the pairs nearest to the limit of resolution. For four such pairs it is +0.012, while for the three of better quality it is +0.004 A. The appearance of pairs very near the limit of resolution is extremely sensitive to instrumental conditions, 1 mm change of focus in the 30-foot spectrograph—one part in nine thousand—being recognizable in the spectrum of close pairs in which both lines are of good quality. The close agreement between Plate No. 199 and the fifth-order plate furnishes a criterion for deciding between the two third-order plates and for the rejection of the measurements made upon No. 56.

An explanation of these discrepancies based upon the psychological effects of contrast appears to be in harmony with the observations. On line 5167, Plate X, represented by the curve in Fig. 1, the intensity at the symmetrical points *B* and *C* is the same, but *B* is seen against the relatively strong continuous background at *A*, while *C* is seen against the weaker background at *D*.

Under the influence of the difference in contrast, *B* appears of greater intensity than *C* and is of greater influence in estimating the position of the maximum, which is consequently moved outward. A similar effect operates in fixing the maximum of the adjacent line; the combined result is therefore a separation more or less in error in the positive direction, the amount depending upon the personal equation of the observer. The observations show that the increase in this contrast difference, which occurs whenever the intensity in the region of overlap is decreased relatively to that of the continuous spectrum, is accompanied by increase in the measured separation. The influence in question appears in the case of the pairs at λ 5167 and at λ 5169, where the micrometer measures always exceed those made by the registering microphotometer (p. 8); also in Table II, where the micrometer results for the maximum average larger by 0.005A than those found by analysis, and in Table VI, where the spectrogram taken with a wider slit gives separations greater by 0.008 A than Plate No. 199.

TABLE VI

INCREASE IN SEPARATION ACCOMPANYING DECREASE IN DEFINITION

λ	Int.	Plate 56	Plate 199	Plate 56—Plate 109	Order V
4455.980 .064	2 3	0.086	0.074	+0.012	0.073
4457.600 .712	2 2	.112	1.08	+ .004	.106
4459.199 .301	2 3	.104	.091	+ .013	.095
4464.844 .938	2 1	.093	.091	+ .002	.088
4472.884 .967	1 0	.082	.076	+ .006	.077
4476.185 .253	4 3	.082	.070	+ .012	.066
4482.338 .438	5 3	0.104	0.094	+0.010	0.092
Mean......	....	0.095	0.086	+0.008	0.085

When the spectrograms differ only in intensity, the matter is complicated by photographic effect. Whether or not increase of exposure lessens the difference in contrast on the two sides of a line in a close pair depends upon where on the characteristic curve of the plate the density of the continuous spectrum and that of the region of overlap, respectively, fall—a question not yet investigated. The effect studied by Eberhard[1] may also be of influence, though from an investigation by Koch[2] it appears to be negligible.

The wave-lengths of 54 solar lines separated 0.25 to 0.50 A from adjacent lines have been measured upon plates of high dispersion, the neighboring free-standing lines being used as standards. The mean variation from Rowland is ±0.003 A. As over 250 lines were used for reference, it appears that the accidental errors in the relative wave-lengths of the *Preliminary Table of Solar Wave-Lengths* for lines practically isolated are much less than ±0.01 A, a value frequently assumed. It is probable, as Frost and Adams[3] remark, that errors of this magnitude occur but rarely, and mainly then for lines whose measurement is inherently difficult, such as the very weak or strong lines. The proximity and character of neighboring lines introduced disturbing elements that have apparently affected the results for closely adjacent lines and given the errors a systematic character. Information upon which to base a discriminating judgment has not been available, and the user of the table, who has not solar spectrograms of high dispersion at command, could hardly do otherwise than ascribe equal precision to the data for lines that appear in the table without distinguishing signs, though the errors for certain classes of lines are relatively large and systematic.

SUMMARY

1. The measurement of solar lines near the limit of spectrographic resolution is a matter of extreme difficulty and liable to be systematically in error.

2. The Mount Wilson separations have been determined, as far as possible, upon each of five series of spectrograms with dispersions varying from 1 mm = 1.8 A to 1 mm = 0.23 A.

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[1] *Phys. Zeit.*, **13**, 288, 1912.

[2] *Annalen der Physik*, **42**, 1, 1913.

[3] *Publications of the Yerkes Observatory*, **2**, 155.

3. Separations obtained from curves produced by the registering microphotometer are, for incompletely separated components, smaller than those found by filar-micrometer settings.

4. For pairs near the limit of resolution analysis based upon microphotometer curves, settings upon the edges of the doublet and of lines similar to its components, and ordinary filar-micrometer measurements have been employed. The first appears most reliable and yields smaller values than the third.

5. Separations equal to the theoretical spectrographic resolution, though serving to detect duplicity, are not sufficient for filar-micrometer measurements of wave-lengths to the third decimal place in angstrom units.

6. Spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution.

7. Filar-micrometer measurements of the separation between the components of close doublets varies with the width of the slit, the precision of the focal settings, and the density of the spectrograms.

8. Whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation. This seems to be an effect of contrast, the observer locating the maximum nearer that edge of the line for which the contrast is greatest, i.e., nearer the free edge.

9. The differences between the Rowland and Mount Wilson determinations of the separation of the components of close pairs of solar lines are systematic. For pairs to the red of λ 4000, component intensities 3 and 4, with mean separations of 0.274 A, 0.145 A, and 0.075 A, the Rowland values exceed the Mount Wilson values by +0.003 A, +0.008 A, and +0.013 A, respectively.

10. That these differences are errors in the Rowland values is made probable by the agreement between the diverse methods used, by the concordance between the spectrograms of different orders, and by the tendency toward over-separation with any departure from best conditions.

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THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF CALCIUM AND LITHIUM

By JANET T. HOWELL

INTRODUCTION

After the discovery of the Zeeman effect the analogous decomposition of spectral lines in an electric field was looked for by many investigators. In 1913 the effect was discovered in the diffuse series of hydrogen by J. Stark[1] and A. Lo Surdo[2] under entirely different experimental conditions. Since then it has been studied in hydrogen and helium by both methods and Stark has investigated the transverse effect for lithium, mercury, and a number of other elements.

Although a very large number of data have been accumulated since the discovery of the new effect, the work in this important field is still in its infancy. The results obtained by Stark and Lo Surdo differ markedly for the elements investigated, and the number as yet uninvestigated is large. The results have a most important bearing on atomic theory,[3] but, so far, they seem to have led to new complications rather than to new solutions. The contradictions between the results given by Stark and Lo Surdo suggest the possibility that the electric effect may vary under different conditions, but no definite conclusion on this subject has been reached. To make a comprehensive theory of the effect possible, much more investigation is necessary. Both the methods employed thus far have serious limitations, hence new ways of attacking the problem are important. It is especially important, from the point of view of solar work, to find a method adapted to the investigation of the heavy elements of the reversing layer. Although no new method has been perfected as yet, a preliminary survey of a large number of elements under low dispersion has brought out some new and interesting results in calcium and lithium which are reported here.

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[1] *Berichte der K. Preuss. Akad. der Wiss.*, **47**, 932, 1913.

[2] *Rendiconti d. Lincei*, **22**, 2d sem., 664, 1913.

[3] A. Garbasso, *ibid.*, **22**, 2d sem., 635, 1913; *Il Nuovo Cimento*, **7**, 354, 1914; *ibid.*, **9**, 376, 1915. W. Wien, *Berichte der K. Preuss. Akad. der Wiss.*, **48**, 70, 1914; W. Voigt, *Annalen der Physik*, **4**, 197, 1901; *Göttingen Nachr. Ges. Wiss.*, 1914; E. Gehrcke, *Verh. der deutschen phys. Ges.*, **16**, 431, 1914; N. Bohr, *Phil. Mag.*, **27**, 506, 1914.

The results obtained in previous work can be summarized briefly as follows:

STARK'S METHOD AND RESULTS

The great number of data contributed by Stark and his coworkers have been collected and discussed in Stark's *Elektrische Spektralanalyse chemischer Atome.*[1] They used the luminous canal rays behind the cathode in a discharge tube as the source of light, and submitted them to an auxiliary electric field of 28,500–74,000 volts per centimeter. The source of potential for this auxiliary field was a dynamo of 4500 volts and a storage battery of 3800 volts. In his earlier work Stark used a small concave grating of 1.5 m radius giving a dispersion in the first order of 1 mm = 9 A. Later, in photographing the fine division of the hydrogen lines, he decreased the dispersion somewhat, but more than doubled the electric field. He used two methods for obtaining the spectra of elements in the electric field: (1) a luminous gas, (2) the bombardment of the salts of alkali metals by the canal rays. He also mentioned the possibility of producing metallic lines by the bombardment of metal electrodes by canal rays.

Stark investigated the transverse effect for H, He, Li, Hg, Al, C, Ca, Mg, Na, and Th, and the longitudinal effect for H and He.[2] In both cases he observed in a direction perpendicular to the direction of motion of the canal rays in order to avoid complications due to the Doppler effect. The general conclusions derived from his results have been summarized by Fulcher[3] and may be put briefly as follows:

1. The diffuse series of H, He, and Li show a separation directly proportional to the field-intensity and of an order of magnitude of 3–18 A for a field of 28,500 volts per cm.

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[1] Leipzig: S. Hirzel, 1914.

[2] *Annalen der Physik*, **43**, 965, 1914; J. Stark and G. Wendt, *ibid.*, **43**, 983, 1914; J. Stark and H. Kirschbaum, *ibid.*, 43, 991, 1017, 1914; J. Stark, *ibid.*, **48**, 193, 1915.

[3] *Astrophysical Journal*, **41**, 359, 1915.

2. The hydrogen components are symmetrical as to the displacement from the original line and probably as to intensity in sources at rest. The red components are more intense when the field and the motion of the luminous particles are in the same direction, and the violet components when they are in the opposite direction.

3. Helium and lithium show asymmetry both as to intensity and as to displacement.

4. Unlike the magnetic effect, the electric effect varies from line to line of the same series. The number of components and the maximum displacement increase with the term-number.

5. The diffuse series shows large effects, while the separation in the sharp main and subordinate series of He and Li is less than 1 A except for the He lines λ 3613.8 and λ 4169.1.

6. Lines of Al, C, Ca, Hg, Mg, Na, and Th show practically no effect. The displacement in the diffuse series of Hg is the largest, being of the order of magnitude of 0.4 A.

7. In the transverse effect the components are polarized, the outer lines parallel to the field, the inner perpendicular.

8. In the longitudinal effect of hydrogen and helium the lines are unpolarized and agree in number and position with the perpendicular components of the transverse effect.

LO SURDO'S METHOD AND RESULTS

Lo Surdo observed the region immediately in front of the cathode in a discharge tube, where the luminosity of the negative glow and the sudden fall of potential fulfilled the conditions for electric decomposition. Investigating the conditions in the dark space,[1] he showed that the increase in the cathode fall at a certain pressure depends solely on the current-density. He therefore used very narrow tubes. He found, experimentally, that when the plain electrode completely fills the tube, the length of the dark space is independent of the diameter of the tube. It is essential, in any case, to have the electrode completely fill the tube so that the lines of force may remain parallel.

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[1] *Il Nuovo Cimento*, **9**, 368, 1915.

In his earlier work Lo Surdo used a tube of 4 mm internal diameter and 20 cm long, but later decreased the diameter to 1.5 mm. The pressure was regulated in general to a 2 mm dark space and the tube was excited by storage batteries giving a potential difference of 5000–8000 volts. The image of the region in front of the cathode was polarized by a nicol and focused on the slit of a 4-prism spectroscope. In the transverse effect the varying electric field gives the lines the shape of a Y, making it easy to identify the components.

The hydrogen results given by Lo Surdo[1] and Puccianti[2] differ somewhat from those of Stark. Lo Surdo finds two parallel components in all the hydrogen lines investigated (α, β, γ, δ) and perpendicular components agreeing in number with the term-number of the series. Stark, even in his early papers, found more components than this, and his latest work shows the hydrogen lines to be very complex. Lo Surdo's examination of the longitudinal effect in $H\gamma$ agrees with Stark's result.

Brunetti[3] has recently published two papers on the electrical decomposition of helium lines. The work shows the presence of very interesting satellites which follow a different field law from the regular components. The components are, in general, somewhat different from Stark's, especially as regards polarization.

APPARATUS

The apparatus used in this work was essentially of the Lo Surdo form. Professor Stark was kind enough to prepare for the Mount Wilson Observatory a hydrogen tube, of the ingenious form devised by him. But as the life of such tubes is short, and as no glass-blower having sufficient skill to make others was available, we were compelled to have recourse to the very simple form of tube used by Lo Surdo. Moreover, Stark has made a survey of most of the more promising elements with his apparatus, but the Lo Surdo tube has been applied only to hydrogen and helium. The discordant results of the two schools of investigators seemed to indicate that the nature of the Stark effect is dependent on the mode of excitation. So the application of the Lo Surdo method to all the available elements was a hopeful point of departure. A diagram of the apparatus is shown in Fig. 1.

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[1] *Rendiconti d. Lincei*, **23**, 1st sem., 82, 143, 252, 326, 1914.

[2] *Ibid.*, pp. 329, 331, 1914.

[3] *Il Nuovo Cimento*, **10**, 34, 41, 1915.

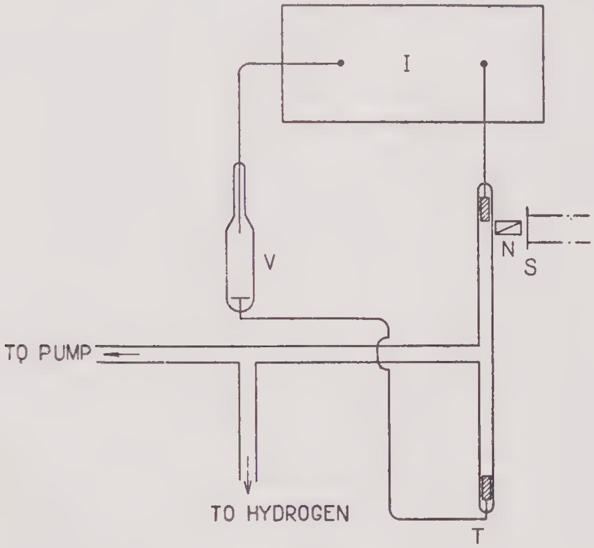
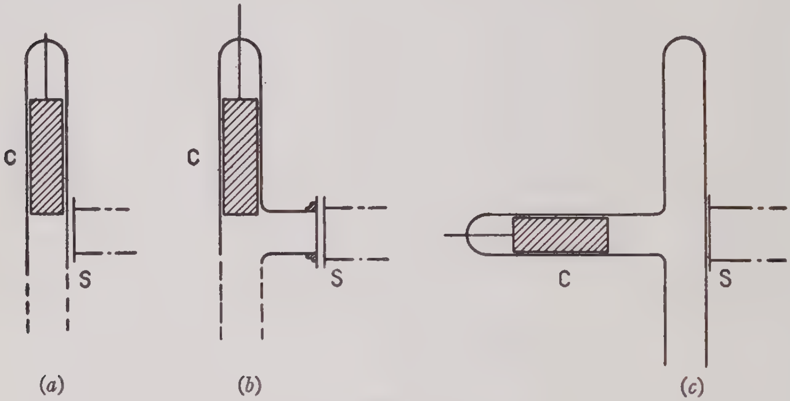


FIG. 1

T = Tube
V = Valve tube for rectifying the discharge
I = Induction coil
N = Nicol
S = Slit of 3-prism spectroscope

The spectroscope contained three prisms, two of ultra-violet glass, one of quartz (compound right- and left-handed), and gave a dispersion of 1 mm = 18 A at Hγ; 1 mm = 12 A at H and K. The source of light was placed directly in front of the slit with a small nicol interposed. This arrangement introduced some astigmatism, but, as only the maximum displacement was measured, the integrated effect along the line did not interfere. As the collimator was entirely filled, the arrangement was economical of light and also of space. An oil pump was used which held the pressure steadily at about 6 mm dark space. No value can be given for the current through the tube as no suitable instrument was available for measuring it. The current in the primary of the coil was 11 amperes, the voltage 110, and a Wehnelt interrupter was used for which the maximum spark-gap of the coil was 17 cm. A valve tube in the circuit rectified the discharge.

As the tube cracked on overheating, it was run intermittently and not more than three minutes at a time. The interval covered by the exposure varied from 30 minutes to 6 hours, the actual exposure being about one-half this time. Seed "27" plates were used. The tube was a simple T-tube of heavy-walled glass tubing with electrodes sealed in at the ends. The internal diameter was 6 mm and the length 20 cm. The electrodes were solid cylinders of metal nearly filling the tube and were sealed in with fine platinum wires. Three types of tubes were used whose cathodes are shown in Fig. 2.



(*a*) (*b*) (*c*)

FIG. 2

S = Slit of spectroscope
C = Cathode

Types (*a*) and (*b*) were used in the transverse effect. When the electrodes were of aluminium, which did not sputter to any extent during the life of the tube, the simple form (*a*) was used. When the cathode was made of a metal that sputtered easily, or when a salt was used on the cathode that discolored the sides of the tube, type (*b*) was found to be very satisfactory indeed. Type (*c*) was used in the longitudinal effect and worked well except for its extreme fragility. Types (*a*) and (*b*) lasted for several days, if used with care and caution, but in (*c*) the impact of the rays melted the glass directly opposite the cathode and the life of the tube was only from 30 minutes to an hour. A new tube could be made and set up in 20 minutes, but the fragility seriously limited the length of exposure possible. If any further work is done with tubes of this type it would be well to make them of quartz.

Photographs of the transverse effect showed electrical components in the Y-shape described by Lo Surdo. In the longitudinal effect the electrode did not entirely fill the tube, with the result that the field fell off at the edges and the components tapered inward and were easily identified.

METHODS OF OBTAINING SPECTRA

The discharge tube was always filled with hydrogen so as to have standard lines for field determination. The metallic cathode, bombarded by anode rays, gave the spectrum of the metal close to its surface, in just the proper region for observing the electric effect. This was true of all the metals investigated. Stark mentioned this method of obtaining spectra in a vacuum tube. Goldstein[1] investigated the phenomenon and found that the spectra appeared only when the gas in the tube was nitrogen, and were much strengthened at a liquid air temperature. Robinson[2] recently published a paper on the cathode spectra of metals. He was able to obtain spectra in H, CO, and, with special brilliance, in O, but found it necessary to have thin cathodes. No evidence of metallic lines was obtained with electrodes 1–2 mm thick. Since I had no difficulty in producing brilliant metallic spectra near the cathode with cylindrical electrodes of 5 mm diameter and from 1.5 to 2 cm long, it follows that the thin electrode is certainly not necessary. The spectra were obtained with about equal brilliance in air, oxygen, and hydrogen. Comparative photographs were taken of the cathode spectra of aluminium and iron at different pressures and there seems to be no particular connection between the cathode spectra and the sputtering of the metal. Although iron sputters easily, and aluminium almost none at all, the aluminium spectrum was obtained over a larger range of pressure than the iron spectrum. The aluminium lines λ 3962 and λ 3944 appeared before the pressure was low enough for the Crookes dark space to appear and at 3–4 mm dark space they extended 3 cm from the electrode. The iron spectrum does not appear until the pressure is reduced to 3 mm dark space. The spectra of Fe, Ni, Al, Mg, Zn, and Ca were obtained by this method and, as found by Robinson, they were in general like the spark. The metallic lines are easily identified, as they are more intense near the cathode, and it seems likely that further study of them will bring out many interesting differences from arc and spark spectra. So far the only one that has been studied in detail is the copper[1] spectrum. In the cathode spectrum of magnesium there were three lines which clearly originate in the electrode and which do not agree with any known magnesium lines. The wave-lengths are 4155, 4113, and 4103. The last is probably the same as λ 4106.8 given by Fowler,[2] but the other two seem to be new.

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[1] *Physikalische Zeitschrift*, **6**, 14, 1905.

[2] *Astrophysical Journal*, **42**, 473, 1915.

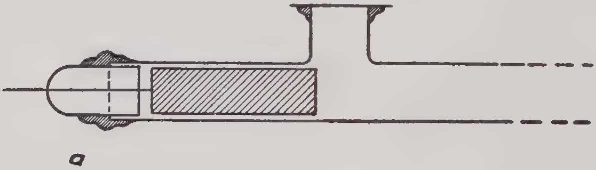

FIG. 3

When the cathode was covered with a thin coat of the chloride of a metal, carefully dried, the bombardment of the salt by the anode rays gave a very brilliant spectrum. Li, Ca, and Sr were used in this way. When using salts, the (*b*) or (*c*) type of tube was found best because the walls became discolored very fast. In order to renew the supply of salt between exposures the electrodes were made demountable as shown in Fig. 3. The electrode was sealed in at *a* with sealing wax.

METHOD OF MEASURING PLATES

The hydrogen lines Hβ and Hγ were used to determine the field-strength for each plate. Hβ gave a very clear separation when polarized parallel, but was too indistinct for measurement when polarized perpendicular to the field. Hence for these plates only Hγ could be used. The red component of Hγ was usually confused by an air line. On some plates this line was absent, but whenever it was present only the violet component was measured. The field-strength was determined from Stark's measurements for the separation at 28,500 volts per centimeter. His later work showed the hydrogen separation to be symmetrical, and he claimed much greater accuracy for the total separation than for the displacement of the components from the center given in his earlier paper. It seemed best, therefore, to multiply the displacement of the violet component by two and use this value for the total separation when the red component could not be measured. A certain amount of inaccuracy is introduced here because the intensity of the components is slightly unsymmetrical for moving sources and this might shift the center of the middle component to one side. As no shift could be detected in the middle component, the error is apparently less than the error of measurement. There is, however, a large error in this method of determining the field-strength. Stark thought that his calculation of the field-strength from the potential difference and the distance between the electrodes was probably too large; hence the resultant separations for a given field-strength were probably too small. Also there is great likelihood that the magnitude of the separation may vary somewhat in going from the Stark to the Lo Surdo method where the conditions are different. This seems especially likely, since the field-strength often differed by 15 or 20 per cent in determinations from Hβ and Hγ on the same plate, while the determinations from Hγ on different plates, taken under the same circumstances, rarely varied more than 5 per cent. The general character of the components agreed with Stark's results, except that a single center component was always found in Hγ when polarized parallel, instead of the two faint ones given by Stark. This is probably due to the fact that the discharge was not absolutely unidirectional, and that a faint unseparated line is superimposed on the components. This indirect process of determining field-strengths undoubtedly introduces an error that may be as great as 10 per cent, but no better method was available.

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[1] *Astrophysical Journal*, **42**, 473, 1915.

[2] *Proc. R. S.*, **71**, 419, 1903.

Another uncertainty lies in the possibility of a complication from the Doppler effect when photographs are taken along the field. It is quite possible that this may have affected the measurements on Hγ. The center line of Hγ in the longitudinal effect is composed mainly of light from in front of the dark space and appears very sharp; consequently there is certainly no Doppler effect present there. The components, however, are composed almost entirely of light from the negative glow on the surface of the cathode where the Doppler effect would be at a maximum. However, Stark always found an undisplaced line as well as a displaced line in his photographs of the Doppler effect in canal rays, and the displaced line varied in displacement and intensity with the velocity of the rays. At the pressure used in these experiments it seems safe to assume that the displaced line, if present, would be very faint and very close to the stationary one. As the components of Hγ are themselves faint, only the stationary line would show. Lo Surdo photographed the longitudinal effect for Hγ and found symmetrical unpolarized components agreeing in position and number with those observed by Stark. Apparently, therefore, there was no Doppler effect noticeable in his case. H and K and Li λ 4602 give components strong enough to be confused by a Doppler effect, but the components do not show two maxima of intensity. At any rate, these lines have not thus far been examined in canal rays for a Doppler displacement; nothing definite, therefore, can be said about them in this respect.

The maximum separation of components was selected for measurement and their displacement from the center was determined by reference to a neighboring unaffected line and to the unseparated line outside the field.

The accuracy of the results varies with the dispersion, and the character of the lines. Some idea of the degree of accuracy will be given under the results on each line.

RESULTS

Entirely negative results were found for Fe, Ni, Al, Mg, Zn, and Sr.

The aluminium lines λ 3962 and λ 3944 were very greatly strengthened toward the cathode—more than the increase in intensity at the cathode could account for. Photographs of these lines with an echelon showed wings to the red, but they did not vary with the field-strength. The aluminium spectrum was examined from λ 4860 to λ 2350 with a single quartz prism spectroscope, but no electric effect was found. In Mg, Fe, Ni, Zn, and Sr the spectra were photographed from λ 4900 to λ 3500, and no electric effect large enough to be detected with the low dispersion used was observed.

TABLE I

LITHIUM

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)

λ in A	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+1.00	8	+0.48	8	
	−2.48	6	−2.00	6	
4132.93.......	+2.26	2	+1.78	2	
	−0.18	5	−0.18	5	Unpolarized
	−3.10	1	−2.24	1	
TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (STARK)					
4602.37.......	+0.65	8	+0.56	8	
	−0.28	1	−0.28	1	Doubtful
	−1.81	6	−1.64	6	
4132.93.......	+1.87	3	+1.57	3	
	−0.37	4	−0.33	4	
	−2.62	2	−2.44	2	
LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)					
4602.37.......	+0.57	8	+0.34	6	
	−2.01	6	−1.53	3	
4132.93.......	+1.16	1	+0.77	1	
	−0.26	5	−0.26	5	Unpolarized
	−1.99	0	−1.50	0	

Results for lithium.—The results for lithium are given in Table I, together with Stark's results reduced to the same field-strength. The numbers marked + indicate components to the red, those marked −, components to the violet. The field close to the cathode was in the neighborhood of 25,000 volts per centimeter. In the longitudinal effect, the displacement of the red component in λ 4602 was doubled because the photographed line was a blend of the red component and the unaffected line outside the field. For the same reason the displacement of the center component to the violet in λ 4132 (longitudinal) was doubled. As the discharge was not entirely unidirectional there is always a faint unseparated line superimposed on the electrical components, and this makes the displacement of the center component of λ 4132 too small in the transverse effect. But, as it was uncertain what percentage of the total intensity was due to this unseparated line, the measured displacement is given. Owing to the uncertain nature of the middle component of λ 4132 the total separation of the outer components is more accurate than their displacements from the center. The results for the transverse effect are in general larger than those found by Stark. As the error of the field-strength determinations is uncertain, the only idea of the precision of the results that can be given is the average deviation from the mean of the results on different plates. This deviation is given in Table II. The components of Hγ are more distinct when polarized perpendicular than when polarized parallel; consequently the error is greater for the parallel components. The deviation of λ 4602 refers to the total separation, that of λ 4132 to the separate components. Photographs of the lithium lines are given in Plate XI.

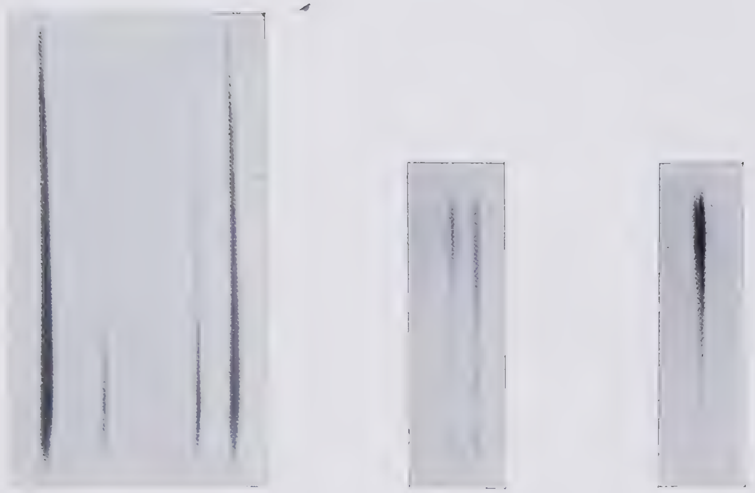
TABLE II

AVERAGE DEVIATION IN A FOR LITHIUM RESULTS

λ in A	Transverse Effect		Longitudinal Effect	
	Parallel Comp.	Perpend. Comp.	Parallel Comp.	Perpend. Comp.
4602.37......	0.32	0.04	0.14	0.04
4132.93......	0.28	0.02	0.12	0.12

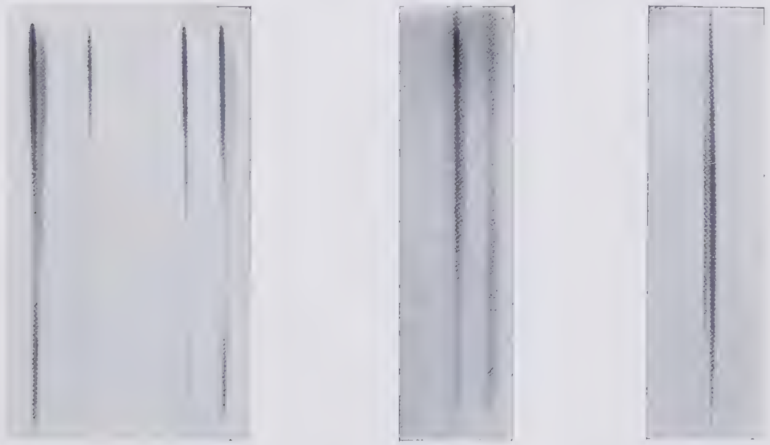
Results for calcium.—The results for the H and K lines of calcium are given in Table III. In the earlier photographs a cathode of metallic calcium was used, but it had a decided tendency to rectify the discharge in the opposite direction and acted as an

PLATE XI



λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

THE TRANSVERSE EFFECT IN CALCIUM AND LITHIUM



λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

THE LONGITUDINAL EFFECT IN CALCIUM AND LITHIUM

anode such a large part of the time that the photographs were very poor. Later, calcium chloride on an aluminium cathode was used and proved very satisfactory. The longitudinal effect gave very clear components, but in the transverse effect the lines were both broad and hazy, although the Y-shape proved the effect to be due to the electric field. The best photographs showed the lines to be composed of two very diffuse components, an intense one slightly displaced to the red, and a fainter one more displaced to the violet.

Table III contains measurements on the separation of these very indistinct components in the transverse effect. The actual separation of the components is small, but the width of the lines, owing to their diffuse character, is large. As the red component is more intense and more diffuse than the violet, the width is almost symmetrical with respect to the unseparated line. The total width of H is 2.38 A parallel and 2.16 A perpendicular; that of K is 2.64 A parallel and 2.42 A perpendicular.

TABLE III

CALCIUM H AND K

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ in A	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63…….	+0.22	6	+0.16	6	Unpolarized
	−0.86	2	−0.74	2	
3933.83…….	+0.22	9	+0.22	9	Unpolarized
	−0.92	3	−0.74	3	

LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ in A	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63…….	+1.27	3	+1.23	3	Unpolarized
	+0.01	8	−0.02	8	Unpolarized
	−1.17	0	−1.11	0	Unpolarized
3933.83…….	+1.42	4	+1.38	4	Unpolarized
	+0.06	9	−0.02	9	Unpolarized
	−1.30	1	−1.26	1	Unpolarized

The results of the transverse effect indicate pretty clearly that H and K are both composed of a strong unpolarized component 0.2 A to the red and of a fainter polarized component to the violet. The longitudinal components are certainly unpolarized. The middle line shows evidence of polarization on the edges, but the polarized edges are such a small part of the total intensity of the line that the effect is scarcely measurable. The H and K lines were investigated for circular polarization, but no trace of it was found.

Photographs of H and K are shown in Plate XI. The separation of H and K into components in the transverse photograph does not show in the reproduction, but the progressive widening with increasing field-strength proves the electrical nature of the effect. The violet component of H in the longitudinal photograph does not reproduce well, but is analogous to that of K which shows plainly. The Y-shape in the longitudinal components means that the discharge was passing from only a small section of the electrode. This happened frequently when calcium chloride was used. When the whole surface of the electrode was acting, the components extended across the field, tapering in at the edges.

The average deviation of the results is given in Table IV. It is calculated for the total separation in the transverse effect and for the separation of the components in the longitudinal.

TABLE IV

DEVIATION IN A FOR H AND K COMPONENTS

λ IN A	TRANSVERSE EFFECT		LONGITUDINAL EFFECT	
	Parallel Comp.	Perpend. Comp.	Red Comp.	Violet Comp.
3968.63......	0.12	0.05	0.04	0.03
3933.83......	0.10	0.09	0.05	0.01

DISCUSSION OF RESULTS

Instead of contributing new law and regularity to the electric decomposition of spectral lines, the results for lithium and calcium seem to contradict the laws already formulated. In former investigations the longitudinal effect has given only unpolarized components, but the lithium lines λ 4602 and λ 4132 are here shown to have very clearly polarized components along the field.

Previously only the diffuse series of different elements have shown any appreciable effect, but H and K belong to a principal-pair series of calcium, while the lines of the diffuse series at λλ 4457, 4435, and 4425 showed no effect at all. The lines at λ 3737 and λ 3706, which are analogous to H and K, were too faint on my plates for any examination, but I hope to investigate them in the near future.

It is still somewhat uncertain whether the electric effect is the same under different conditions or not. The difficulty in determining consistent field-strengths from the hydrogen lines is no very strong evidence on either side, because the difficulty of making accurate measurements on the components of $H\gamma$ was so great. The values found here for lithium in the transverse effect are larger than Stark's values, but the difference is not great enough to show any real dissimilarity. The general character of the components is the same. The chief reason for thinking that there may be a real difference between electrical components under different conditions is the fact that Stark[1] found no effect in the H and K lines of calcium (transverse). It was doubtful at first whether the production of spectra through the bombardment of metallic electrodes was favorable to an electrical effect. The fact that using metallic calcium as an electrode gave the same result as covering the electrode with calcium chloride proves this to be an entirely favorable method. The photographs with metallic calcium were unsatisfactory, owing to the tendency of calcium to act as an anode, but a few good ones obtained in this way showed the electric effect quite plainly.

The result most clearly shown by this work is the absolute necessity of increasing the intensity of the source so that high dispersion may be used. The effect in the majority of elements is so small that it will take high dispersion to bring it out.

GENERAL CONCLUSIONS

The spectra of H, Li, Ca, Fe, Ni, Mg, Al, Zn, and Sr were examined near the cathode in a discharge tube under low dispersion. Fe, Ni, Mg, Al, Zn, and Sr showed no electric effect. This negative result brings out the absolute necessity of increasing the dispersion in dealing with the heavier elements. The effect, if it exists in these elements, is small and can be brought out only under much stronger fields and with higher dispersion. Both Stark's method and that of Lo Surdo give such faint luminosity that the present problem is to find a method for producing very brilliant spectra in a strong electric field.

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[1] *Annalen der Physik*, **43**, 1017, 1914.

New results were found in lithium and calcium which show most interesting variations from those previously obtained.

The longitudinal effect in lithium gives clearly polarized components, whereas all the longitudinal components in hydrogen and helium have been unpolarized.

The H and K lines of calcium show electrical components with a separation, especially in the longitudinal effect, which is comparable in magnitude with the separations in H, He, and Li. This is surprising, owing to the relatively large atomic weight of calcium and to the fact that H and K do not belong to a diffuse series.

In conclusion, I wish to express to Dr. Hale and the members of the Observatory staff my very deep appreciation for the opportunity of a year's work at the Observatory. The problem of the electric effect was undertaken at Dr. Hale's suggestion, and has been guided by his advice and encouragement throughout.

Mount Wilson Solar Observatory
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THE STRUCTURE OF THE LITHIUM LINE λ 6708 AND ITS PROBABLE OCCURRENCE IN SUN-SPOT SPECTRA

By ARTHUR S. KING

In connection with an investigation of the electric furnace spectrum of calcium, it became highly desirable to obtain definite evidence as to the origin of the line λ 6708 which usually appears in calcium spectra. It has been uncertain whether calcium could under certain conditions give a line of this wave-length, or whether the line is in all cases the well-known red line of lithium, which may appear as an impurity in other spectra. The question is important also in connection with sun-spot spectra, in which Hale and Adams[1] found a strong line of the wave-length specified. They also observed it much intensified in the outer regions of calcium and other arcs. The writer[2] found the line very strong in the furnace spectrum of calcium. If in these cases it is produced by calcium, the fact is important chiefly as an indicator of reduced temperature in sun-spots, but if the line is always due to lithium, it furnishes the clearest evidence we have of the presence of lithium in the sun. This latter view was taken by Hemsalech and De Watteville,[3] who in their study of the calcium flame spectrum considered that λ 6708 is due to lithium impurity and that it is the line occurring in sun-spots. Holtz,[4] however, in measuring the calcium arc spectrum according to the international standards, ascribed λ 6708 to calcium.[5]

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[1] *Mt. Wilson Contr.*, No. 15; *Astrophysical Journal*, **25,** 75, 1907.

[2] *Mt. Wilson Contr.*, No. 35; *Astrophysical Journal*, **29,** 190, 1909.

[3] *Comptes Rendus*, **149,** 1369, 1909.

[4] *Zeitschrift für wissenschaftliche Photographie*, **12,** 101, 1913.

[5] After this manuscript was prepared, my attention was called to an article by H. G. Woodward (*Astrophysical Journal*, **41,** 169, 1915), in which the possibility of obtaining a calcium-arc spectrum free from the line λ 6708 is demonstrated in a very satisfactory manner. The portion of the present paper bearing on the origin of the line supplements that of Mr. Woodward with different evidence and reaches the same conclusion.

The relatively low dispersion used in the study of the calcium furnace spectrum showed λ 6708 as clearly double, though not resolved. It thus appeared possible that both calcium and lithium entered into the production of the line. Higher dispersion was then resorted to, the second order of the vertical plane grating spectrograph of 30-foot focus being employed, which gives a dispersion of approximately 1 mm = 0.89 A. A large number of spectrograms were then taken for a wide variety of conditions of the arc and furnace, the line being studied both with calcium and with lithium in the source. A satisfactory resolution of what proved to be a highly complex line was obtained in this way. Measurement of the components appearing under various conditions were referred to the calcium line λ 6717.940 (Rowland). When lithium was used in the arc or furnace, λ 6718 was put on the plate by means of the occulting device above the slit or by screening out the lithium line during the calcium exposure by a strip of paper passed in front of the plate.

RESULTS

When the source is such that λ 6708 might be due either to calcium or to lithium impurity, that is, with metallic calcium in the arc or furnace, or with clean carbons in the arc, a narrow doublet is usually obtained, of which the violet component is about twice as strong as the red. When lithium chloride was used in the source, a reversed line appeared which was photographed on numerous plates in juxtaposition to the narrow doublet; but it was evident that the reversed line could not have sharpened down to either component of the doublet without decided shift of its center. It thus appeared that the doublet occurring in the calcium spectrum is distinct from the lithium reversed line and might belong to calcium. However, a very similar doublet was obtained by Zeeman[1] in absorption when white light was passed through an exhausted vessel containing lithium vapor, the relative intensity of the components being the same in the two cases, while the interval of 0.144 A measured by Zeeman is in fair agreement with the mean value of 0.152 A obtained from six arc and furnace photographs. Kent[1] observed the same doublet in emission from a vacuum tube containing lithium, and measured the separation as 0.151 A. This close agreement, together with other observations to be described, indicates that the doublet is in no case due to calcium.

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[1] *Proceedings of the Amsterdam Academy*, **15** (II), 1130, 1913; **16** (I), 155, 1913.

A remarkable condition became evident when the quantity of lithium in the arc was such as to give a rather narrowly reversed line. A bright line then appeared inside of the reversal and slightly to the red of its center. Visual observations showed the same appearance as that recorded on the plates. As the quantity of lithium in the arc was increased, the bright line was gradually suppressed by the absorption of the vapor producing the reversal of the main line. Turning the spectrograph so as to bring the axis of the arc-image parallel to the slit showed the emission line weakening toward the upper pole, where the denser absorbing vapor appeared as a red hood beneath the arc terminal.

Further experiments were then made with a very small quantity of lithium in the arc, a drop of dilute solution of the chloride being placed on either carbon or copper poles. By projecting the image on a long slit, the spectrum line was made to register the condition in the arc from pole to pole. Near the upper terminal and extending to the middle of the arc, the line consisted of an unsymmetrical triplet. The bright line usually appearing within the reversal was present as the middle component, while the sides of the reversal had become two sharp lines, sometimes with incipient wings. The red component of the triplet had a close satellite on its red side. The triplet blurred in the middle of the arc, and from here to the lower (positive) pole appeared a gradual development of the two components of the close doublet which it had been thought might belong to calcium. Putting the calcium line λ 6718 on the plate by a second exposure to serve as a standard, it was found that the doublet in the lithium arc agreed closely as to wave-length, separation, and relative intensity of components with the doublet given by calcium in the arc or furnace. Unless it was added by separate exposure, the photographs showed no trace of the calcium line λ 6718, which is good evidence that the doublet was not due to calcium mixed with the lithium, since in the calcium arc λ 6718 is much stronger than λ 6708.

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[1] *Astrophysical Journal*, 40, 337, 1914.

The components of the lithium triplet were studied to best advantage in the furnace spectrum, especially with regard to the separation of the side components. In the vacuum furnace, with a small quantity of lithium vapor present, the components of the triplet were sharp, with quite the appearance of Zeeman components, the phenomenon being clearly distinct from the regular reversal effect.

Measurement of a number of photographs proved that the interval between the side components was decidedly variable, this seeming to be controlled by the amount of vapor present. Nine furnace photographs gave intervals ranging from 0.25 A to 0.36 A, a difference much greater than the error of measurement for components as sharp as these. As the separation became wider, the middle component became weaker and the satellite by the red component strengthened. With a larger quantity of vapor, the side components lost their sharpness and gave the regular reversed line with widely shaded wings. A description of a succession of furnace runs at about 2100° C. without fresh supply of lithium will serve to illustrate the changes and the transition to the doublet stage. Starting with a small supply of vapor, the components came closer together on successive plates with a strengthening of the central line until the three were no longer resolved. The next photograph showed the doublet with the usual separation of components. The interval of this doublet seems to be constant, as the measurements on good plates have never differed by more than 0.007 A.

The variation of the lithium components is thus shown to be the same in arc and furnace, and the controlling agency in each has seemed to be the amount of vapor in the source. The appearance of the two sets of components in the arc near the positive and negative poles respectively can scarcely be dependent upon the discharge conditions, since either set can be produced in the furnace without material temperature change. The whole effect agrees with the observations of Nutting,[1] who studied line structure for a number of elements by means of the echelon spectroscope. He observed only a broadening of the red lithium line, but in other spectra frequently recorded a "twinning" of sharp components which spread apart with increasing material in the arc and changed into the winged structure of regular reversal. In some cases a central component was present, as is the case for the lithium line. The variable interval of the components suggests electrical resolution, and Stark[1] has recently developed a theory of the connection between electrical resolution and the widening of spectrum lines. The production of the effects in the furnace shows that they are not due to an external electric field, but it would seem that there are strong possibilities in the action of interatomic fields, in the variation of which the vapor-density might well be the controlling agency.

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[1] *Astrophysical Journal*, **23**, 64, 1906.

An observation of the structure of λ 6708 by Jewell in an early paper by Rowland[2] is explained by the foregoing results. Jewell notes: "With but little material in the arc this is a difficult triplet. The violet component is very strong, the red component about half as strong, and between them but nearer the red component is a very narrow line much weaker than either of the others." This appearance would result from the use of a concave grating and the projection of most of the length of the arc on the slit. The strong violet component of the doublet would then blend with the violet component of the triplet, producing an unsymmetrical triplet of which the central and red components seen by Jewell correspond with those on my photographs.

In Table I, are given measurements of those members of the group at λ 6708 whose wave-lengths seem to be invariable. The mean wave-length of the variable side components of the triplet appears to be that of the center of the reversed line into which they gradually develop. The measurements were made in arc spectra from the sharp calcium line λ 6717.940 (Rowland), the wave-length of which in the international system is given by Holtz as 6717.70.

The mean wave-length of the close doublet in the international system is 6707.81, which agrees exactly with the value given by

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[1] *Jahrbuch der Radioaktivität und Elektronik*, **12**, 349, 1915.

[2] *Astronomy and Astrophysics*, **12**, 344, 1893.

Holtz, who measured it as an unresolved line ascribed to calcium. This mean wave-length of the doublet agrees closely with the wave-length of the reversed line; but the agreement would cease if the doublet were faint, as when produced by a small impurity, so that the measured wave-length would be that of the strong violet component. A discrepancy as large as 0.07 A might thus result. The possibility of variable wave-lengths due to complex structure has been recognized by other observers, and we may have here the key to the condition which Burns[1] notes: that "lines of impurities are not as yet recommended for standards," since a small impurity may give a distinct set of components.

TABLE I

WAVE-LENGTHS OF COMPONENTS OF λ 6708

	No. Plates	λ (Rowland)	λ (I. A.)
Violet component of doublet	7	6707.977	6707.74
Red component of doublet	3	6708.125	6707.88
Center of reversed line	8	6708.053	6707.81
Bright line inside of reversal	6	6708.072	6707.83

Various conditions of the lithium components are shown in Plate XII. The first three enlargements are of the narrow doublet and two states of the triplet in the furnace, the strength of the red satellite in No. 3 giving a quadruple structure. The next three are for the arc with increasing quantities of vapor, the side components being almost sharp in No. 4, while Nos. 5 and 6 show the regular reversal with the bright line inside. No. 6 was taken with the arc-image parallel to the slit, showing the extinction of the bright line near the upper terminal. No. 7 shows the simultaneous production of both sets of components, the one near the positive, the other near the negative pole.

COMPARISON WITH λ 6708 IN THE SUN-SPOT SPECTRUM

Sun-spot spectra showing λ 6708 have been photographed by Mr. Ellerman and Mr. Nicholson, using the first and second orders of the 75-foot spectrograph on Mount Wilson, and kindly placed at

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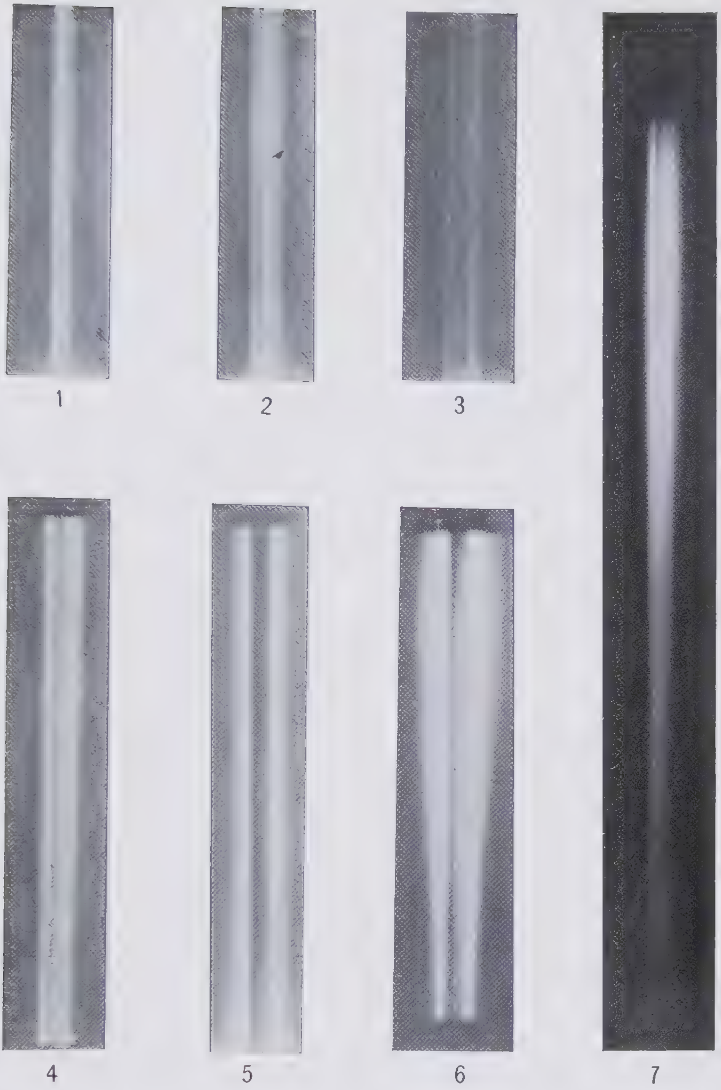
[1] *Bulletin of the Bureau of Standards*, **12**, 179, 1915.

PLATE XII



λ 6708 in Furnace and Arc Spectra

1. Doublet appearing in furnace or arc with trace of lithium vapor.
2. Triplet which takes the place of the doublet when slightly more vapor is present.
3. Appearance of line in furnace with more vapor than for No. 2.
4. Triplet in arc, with side components beginning to show the widening characteristic of reversal.
5. Reversed line in arc with inner component still visible.
6. Reversed line in arc showing fading of inner component near upper electrode where absorption is strongest.
7. Combination of triplet and doublet in arc when vapor density gradually increases from lower to upper terminal.

my disposal for comparison with the laboratory plates. The line seems to be absent from the regular solar spectrum, the faint line in Rowland at λ 6708.176 apparently being too far to the red. In the spot spectrum, the line is rather wide and of low density, so that the first-order plates, with a scale of 1 mm=0.713 A, proved much better for measurement. The mean value from four plates is λ 6708.065. Considering the character of the line, which, exclusive of wings, is about 0.2 A wide, this is in fair agreement with λ 6708.053 for the reversed lithium line and is very close to the mean value, λ 6708.062, of the center of this line and of the bright line appearing within the reversal. Such dissymmetry as is present in the sun-spot line is toward the red, so that the condition of the vapor is probably not that giving the close doublet with a strong violet component. Second-order plates taken with nicol prism and quarter-wave plate show a decided Zeeman effect for the sun-spot line. This, together with the strength of the line, makes it probable that λ 6708 is not one of the fluting lines which are plentiful in this region, though possibly it may be blended with such a line.

The material here presented appears to furnish a strong probability that the sun-spot line is due to lithium, and we have the remarkable condition that an element of very wide distribution in terrestrial substances is apparently unable to show its spectrum in the solar atmosphere except over sun-spots.

The relative intensities of λ 6708 and the strong orange line λ 6104 were compared in a set of furnace spectra of lithium at various temperatures. At 1650° C., λ 6708 was very strong and reversed, while λ 6104 was barely visible. Above 2000°, the latter line gained in relative intensity, though still much weaker than λ 6708. This behavior of λ 6708 justifies its use as a criterion of low temperature. λ 6104, if present in the spot spectrum, is extremely faint. Other lithium lines will be searched for when spot photographs of the more refrangible region are available, but if they are not found their absence may be fairly ascribed to a low-temperature condition, since λ 6708 is by far the most sensitive to a weak excitation in laboratory sources.

SUMMARY

1. The complex lithium line λ 6708 may appear under different conditions with two distinct sets of components, either as an unsymmetrical doublet or as a triplet of variable separation.

2. In a third stage, the side components of the triplet change into the ordinary reversal, within which the central component still can be seen.

3. All three conditions of the line may be produced in either arc or furnace, the doublet and triplet sometimes appearing at the same time in different parts of the arc.

4. The observations show that the line at λ 6708, often found in calcium and other spectra, has the same structure and wave-length of components as the lithium line when produced by a source containing but little of the substance. The line is probably always due to lithium, and if low-temperature conditions are present in the source, may attain a very high relative intensity.

5. Measurements of the strong sun-spot line λ 6708, which is probably absent from the photosphere, agree so closely with the wave-length of the arc and furnace line as to leave little question regarding the presence of lithium in the solar atmosphere; while the high intensity of the line at low furnace temperatures is evidence of the correspondingly low temperature of sun-spots.

Mount Wilson Solar Observatory
July 7, 1916

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OBSERVATIONAL EVIDENCE THAT THE RELATIVE POSITIONS OF FRAUNHOFER LINES ARE NOT SYSTEMATICALLY AFFECTED BY ANOMALOUS DISPERSION

By CHARLES E. ST. JOHN

I. INTRODUCTION

The suggested mutual influence of Fraunhofer lines, a quasi-repulsion between closely adjacent lines, offers a ready means of testing the theory of anomalous dispersion in the solar atmosphere. Professor Julius[1] developed the deduction and applied it to the displacements between the center and limb edges of the penumbrae of sun-spots given in my paper on "Radial Motion in Sun-Spots."[2] He found what he considered positive evidence of the effect. I rediscussed[3] the data from this point of view, including a large number of omitted lines fulfilling the conditions of selection, and calling attention to errors involved in the method used by Julius which introduced systematic differences of the sign required by the hypothesis. Of this discussion Julius says:

> A few months later St. John published the elaborate article in which he criticized my treatment of his observations very severely, in several respects justly. I had not overcome the difficulties of handling rightly the Mount Wilson data, nor had I entirely avoided bias. St. John made certain alterations in the method of grouping and comparing the measured displacements, added a number of omitted and of new cases, and thus reached the conclusion that there was no indication at all of a mutual influence.
>
> Although I am satisfied by St. John's improved discussion of the data that, in the Evershed effect, mutual influence is not so conspicuous as my defective treatment of those measurements had made it appear, I still believe that future research will bring it to light.[4]

The magnitude of the effect in this case Julius now says can scarcely be expected, under the most favorable conditions, to exceed

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[1] *Astrophysical Journal*, **40**, 1, 1914.

[2] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 327, 1913.

[3] *Mt. Wilson Contr.*, No. 93; *Astrophysical Journal*, **41**, 28, 1915.

[4] *Astrophysical Journal*, **43**, 49–50, 1916.

0.002 A, a quantity so near the limit of precision for such observations that its definitive establishment must prove a matter of extreme difficulty, a quantity so small that the question becomes one of theoretical rather than practical interest to the astrophysicist.

Albrecht[1] found by comparing the Rowland and International wave-lengths of iron lines that the violet and red components of close solar pairs showed, relative to the general displacement, specific displacements to the violet and red of 0.007 and 0.005 A, respectively, for a mean separation of 0.22 A from the companion line. These results he interpreted as effects of anomalous refraction, an interpretation approved by Julius, who accepts the data as definitely establishing the existence of mutual influence between the lines in the general solar spectrum.[2]

II. SUN-ARC DISPLACEMENTS

Quantities of the order given by Albrecht's investigations are well within the range of present instruments and methods. The displacements of the solar lines in sun-arc comparisons furnish a direct and suitable means of investigating the question, as a difference of 0.012 A between the displacements of lines with violet and red companions is greater than the average value of the sun-arc displacements, quantities determinable with high accuracy. Such data, center of sun *minus* center of arc, have been accumulating at this Observatory for three or four years. The early observations were made with the 75-foot spectrograph. The length of the plates (90 cm) and the scale (1 mm = 0.7 A) are such that a range of spectrum of 600 A is covered by a single exposure. This has insured such interlocking that the results may be considered free from serious relative errors. Recently a series of plates was begun under still more refined conditions with the remodeled 30-foot spectrograph of the 60-foot tower telescope. Measurements now in progress upon these plates are confirming the absolute values previously obtained for the groups of stable arc lines.

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[1] *Astrophysical Journal*, **41**, 333, 1915.

[2] *Ibid.*, **43**, 53, 1916.

In the Mount Wilson classification of iron lines,[1] those whose wave-lengths are independent of arc conditions at constant pressure are distributed among groups *a*, *b*, and *c*4; to groups *c*5, *d*, and *e* are assigned the lines subject to pole-effect,[2] whose wave-lengths therefore vary with the current-strength and the part of the arc used.

A. *Groups a, b, and c*4.—In the present situation as to solar observations, conclusions based upon the relative behavior, in the sun and arc, of lines belonging to groups *a*, *b*, and *c*4 carry the maximum weight. Of these groups 211 lines, sufficiently separated from neighboring lines for measurement of precision and free from known blends, have been examined for sun-arc displacements.

An investigation entitled "The Accuracy Obtainable in the Measured Separation of Close Solar Lines; Systematic Errors in the Rowland Table for Such Lines,"[3] has recently been concluded. This has furnished the means for determining the minimum separation between lines in the solar spectrum, consistent with their accurate measurement upon spectrograms taken with the instrument employed in obtaining these sun-arc displacements.

The data relative to all lines of these groups having close companions are given in Tables I and II. To identify the lines common to Albrecht's list and mine, his values are shown in the last column. The first section of each table contains measurements free from known sources of error. The measurements in the second section are of little or no weight, the lines themselves being blends or the adjacent lines too near.

The 56 lines having companions to the violet, mean separation 0.275 A, and the 29 having companions to the red, mean separation 0.320 A, show displacements of +0.0036 and +0.0038 A, respectively, in agreement with the mean of 0.0038 A from the 211 lines, instead of approximately +0.009 A and −0.003 A, values required to harmonize with the observations of Albrecht, if the deviations found by him represent actual shifts of 0.005 and 0.007 to the red and violet relative to the general displacement of +0.004 A. The behavior of these 85 lines with closely adjacent companions, lines of high quality in both solar and arc spectra, fails to show the large differences found by Albrecht, but, on the other hand, furnishes strong observational evidence that the sun-arc shifts for such lines are the same as for isolated lines and are therefore not measurably affected by mutual influence.

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[1] *Transactions of the International Union for Co-operation in Solar Research*, 4, 74, 1913.

[2] St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, 42, 231, 1915.

[3] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

TABLE I

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUPS *a*, *b*, AND *c*4 COMPANIONS TOWARD THE VIOLET IN SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
3964.663...	2: 3	.247	+0.001	....	4443.365..	1:3	.369	+0.003	....
3969.413...	6:10	.527	+ .013	−0.004	4531.327..	2:5	.204	+ .004	−0.002
3977.009...	3: 2	.170	+ .003	....	4556.306..	3:4	.243	+ .004	+ .002
4000.611...	2: 2	.208	+ .002	....	4592.840..	2:4	.133	+ .004	− .010
4001.814...	1: 3	.219	+ .007	....	4741.718..	1:3	.456	+ .001	....
4067.429...	5: 3	.290	+ .007	....	4787.003..	3:2	.276	+ .003	− .012
4087.252...	1: 3	.391	+ .004	....	4789.849..	2:3	.321	+ .002	− .012
4123.907...	1: 5	.243	+ .006	....	4939.868..	2:4	.452	+ .004	− .002
4133.062...	1: 4	.199	+ .005	....	5041.255..	3:4	.186	.000	− .002
4134.840...	3: 5	.251	+ .004	+ .006	5041.936..	2:4	.141	+ .012	+ .001
4137.156...	4: 6	.478	+ .002	....	5051.825..	1:4	.142	+ .006	....
4146.225...	1: 3	.311	.000	....	5079.409..	3:4	.251	+ .003	− .006
4147.836...	2: 4	.334	.000	+ .002	5098.885..	1:3	.134	+ .002	− .005
4149.533...	2: 4	.173	− .001	....	5107.823..	4:4	.204	+ .006	− .004
4152.343...	2: 3	.235	− .006	....	5143.111..	2:3	.153	+ .004	....
4156.970...	1: 3	.139	+ .001	....	5227.362..	3:5	.319	+ .002	− .003
4172.923...	2: 4	.120	− .003	....	5250.817..	2:3	.432	+ .003	....
4174.095...	3: 3	.385	+ .008	....	5328.696..	2:2	.181	+ .006	....
4191.843...	6: 3	.248	+ .006	+ .003	5333.089..	1:4	.247	+ .006	+ .005
4202.198...	1: 8	.329	+ .006	....	5365.596..	5:3	.527	+ .002	+ .003
4220.509...	1: 3	.297	− .002	....	5405.989..	1:6	.435	+ .006	....
4242.897...	2: 2	.131	− .004	....	5429.911..	1:6	.194	+ .007	....
4282.565...	2: 5	.438	+ .004	....	5447.130..	2:6	.333	+ .004	+ .003
4290.542...	2: 1	.165	+ .002	....	6137.210..	8:3	.381	+ .006	....
4291.630...	1: 2	.255	+ .002	+ .007	6191.779..	6:9	.386	+ .005	−0.004
4315.262...	3: 4	.124	+ .006	− .007	6462.965..	5:3	.181	+0.008	....
4369.941...	1: 4	.373	+ .003	....					
4427.482...	2: 5	.216	+ .004	.000	Mean, 56 lines........		.275	+0.0036	....
4430.785...	1: 3	.429	+ .001	....					
4435.321...	5: 2	.192	+0.005	....	Mean, 23 lines, Albrecht.............		.283	−0.0018	....

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4294.301...	2: 5	.097	−0.013	−0.005	5167.678..	15:5	.181	+0.019	−0.016
4407.871...	2: 4	.061	− .027	+0.011	6254.456..	1:5	.074	−0.014	+0.017
4547.192...	1: 2	.091	+ .003	....					
4552.725...	2: 1	.093	+ .006	....					
4633.100...	1: 4	.109	+0.001	....	Mean, 7 lines........		.101	−0.0036	....

B. *Groups c*5, *d, and e.*—Measurements upon iron lines of groups *c*5, *d*, and *e* are so easily affected by conditions in the arc, that consistent results can be expected only when these conditions are constant. In obtaining the sun-arc displacements at Mount Wilson, the Pfund arc used had a length of 6 mm and was fed by a current of 6 amperes from a 110-volt storage battery. The slit was normal to the axis and in the equatorial zone of a two-and-one-half-fold enlarged image produced by an achromatic lens. Under the constant conditions, the displacements for lines of groups *c*5, *d*, and *e*, relative to each other, are probably free from serious error, and comparison between the behavior of lines with companions, and that of isolated lines, can yield results of weight.

TABLE II

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUPS *a*, *b*, AND *c*4 COMPANIONS TOWARD THE RED IN THE SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4000.403...	2:2	.218	0.000	....	4454.552..	5:3	.401	+0.002	+0.003
4030.339...	5:2	.307	− .001	....	4461.818..	3:4	.347	+ .004	+ .008
4067.139...	3:5	.290	+ .011	....	4466.727..	1:5	.375	+ .008	....
4082.264...	3:2	.325	− .003	....	4489.911..	3:4	.342	+ .006	+ .004
4096.129...	2:3	.133	+ .003	....	4619.468..	1:3	.243	+ .004	....
4098.335...	4:5	.354	.000	....	5079.921..	1:4	.223	+ .003	....
4136.678...	6:4	.478	+ .005	....	5107.619..	4:4	.204	+ .004	+ .008
4140.089...	3:6	.469	+ .006	....	5131.642..	1:2	.300	+ .003	....
4142.025...	2:4	.305	+ .001	....	5195.113..	2:4	.534	+ .004	+ .006
4171.068...	4:4	.145	+ .006	....	5328.236..	2:8	.279	+ .011	− .007
4175.082...	1:4	.210	− .001	....	5341.213..	1:7	.124	+ .010	....
4224.337...	3:4	.336	+ .005	....	5397.344..	1:7	.478	+ .008	....
4226.584...	20:2	.320	+ .006	....	6136.829..	3:8	.381	+0.002	+0.004
4267.122...	2:3	.421	− .006	....					
4337.216...	3:5	.509	+ .004	+0.002	Mean, 29 lines.......		.320	+0.0038	....
4351.711...	5:2	.219	+0.004	....	Mean, 8 lines, Albrecht.............		.375	....	+0.0035

SECTION B. MEASUREMENTS WITHOUT WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
3997.547...	2:4	.091	+0.022	+0.001	4476.185	3:4	.068	+0.025	+0.008
4204.101...	4:3	.062	− .011	+ .048	5012.252..	1:4	.083	+ .017	− .002
4210.494...	3:4	.067	+ .002	+0.027	5169.069..	4:3	.151	+ .015	....
4245.422...	2:4	.098	+ .006	....	5476.500..	3:1	.278	0.000	+0.011
4367.749...	2:5	.090	+ .005	....					
4391.123...	1:2	.069	+0.003	....	Mean, 10 lines.......		.105	+0.0084	....

The Mount Wilson data include 125 lines of the related groups *c*5 and *d*, and 34 lines of group *e*. The mean sun-arc displacement for the 125 lines of groups *c*5 and *d* is −0.0063 A. As the displacements for these lines show an increase for lines in the red, the mean based upon the lines in the region covered by the lines to be tested is taken as the standard or normal displacement. The mean displacement for the 34 lines of group *e* is +0.0142 A. Lines of these groups were included by Albrecht in reaching the conclusion that mutual influence displaces the violet components 0.007 A less and the red components 0.005 A more than the average. It is evident that a comparison, to have definite meaning, must be between lines of the same or similar groups. For lines of groups *c*5 and *d* the displacements of the violet and red components of close pairs should then be of the order of −0.012 A and 0, respectively; for lines of group *e* +0.007 and +0.019 A. An inspection of Tables III, IV, and V shows no indication of such systematic differences, but rather a remarkable agreement between lines in the open and lines with close companions, an agreement, however, in large measure fortuitous for arc lines of this character.

TABLE III

Relative Displacements of the Solar and Arc Lines of Iron, Groups *c*5 and *d* Companions toward the Violet in Solar Spectrum

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4024.881...	3:4	.155	−0.007	....
4083.917...	4:1	.134	− .003	....
4154.976...	4:4	.309	− .002	....
4187.204...	2:6	.426	− .004	....
4227.606...	20:4	.702	− .004	−0.004
4233.772...	4:6	.444	− .004	+ .002
4247.591...	1:4	.127	− .007	....
4250.287...	2:8	.490	− .003	....
4469.545...	1:4	.229	.000	....
4607.831...	1:4	.321	− .003	....
4872.332...	1:4	.220	− .003	− .006
4885.620...	2:3	.356	−0.003	+0.003
4983.433..	2:3	.439	−0.021	....
4985.730..	3:3	.298	− .006	....
5006.306..	4:5	.410	− .005	....
5137.558..	3:3	.308	+ .002	....
5208.776..	5:2	.180	− .005	−0.006
5615.877..	2:6	.357	.000	− .004
5641.667..	1:2	.461	− .016	....
5655.715..	1:2	.320	− .008	....
5659.052..	1:4	.299	−0.008	−0.003
Mean, 21 lines.......		.333	−0.0052	....
Standard displacement.		......	−0.0050	....

SECTION B. MEASUREMENTS WITHOUT WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4668.331...	2:4	.088	−0.011	−0.010
4957.785*..	4:4	.305	+0.018	−0.004
5139.644*.	4:4	.217	+0.003	+0.002
Mean, 3 lines........		......	+0.003	....

* Blend.

A comprehensive view of the results of the discussion based upon the behavior of 372 iron lines for which the measurements of the relative sun-arc displacements are of sufficient weight to furnish conclusions of value, and of which 142 have close companions, may be had from Table VI, where the deviations from the mean are accounted favorable to the anomalous-dispersion hypothesis when they show that the displacements of the red and violet components of close solar pairs are respectively greater and less than the mean for all lines of the group, as should be the case if mutual influence

TABLE IV

RELATIVE DISPLACEMENTS OF THE SOLAR AND ARC LINES OF IRON, GROUPS *c*5 AND *d* COMPANIONS TOWARD THE RED IN THE SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4101.421...	3:2	.419	+0.002	....	5125.300..	1:3	.123	−0.011	−0.002
4109.215...	1:3	.394	− .003	....	5139.427..	4:4	.217	− .002	+ .006
4158.959...	5:5	.394	− .007	....	5273.339..	2:3	.219	− .004	+ .004
4191.595...	3:6	.248	− .003	....	5283.802..	1:6	.479	− .002	....
4196.372...	2:4	.327	− .006	....	5476.778..	5:3	.345	− .004	+ .015
4225.619...	1:3	.255	− .005	....	5573.075..	1:6	.253	− .003	....
4236.112...	1:8	.242	− .003	....	5662.744..	1:4	.411	− .009	....
4859.928...	30:4	.599	− .004	....	5709.601..	5:5	.174	− .007	....
4910.198...	2:3	.307	− .005	....	5952.943..	1:4	.443	− .016	....
4938.997...	2:4	.419	− .005	+0.005	6246.535..	2:8	.239	− .008	....
4957.480...	8:5	.305	− .008	+ .003	6400.217..	2:8	.321	−0.007	+0.001
4982.682...	2:4	.312	− .023	+ .013					
4985.432...	3:3	.298	− .007	+ .009	Mean, 25 lines.......		.326	−0.0062	....
5005.896...	5:4	.410	−0.005	+0.002	Standard displacement.		......	−0.0063	....

SECTION B. MEASUREMENTS WITHOUT WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4125.776...	1:3	.074	0.000	....	4707.457..	2:5	.215	−0.009	+0.006*
4210.494...	3:4	.067	+0.002	+0.027	4727.582..	2:3	.094	+0.014	+0.003
					Mean, 4 lines........		......	+0.002	....

* Blend.

TABLE V

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUP *e*

COMPANIONS TOWARD VIOLET					COMPANIONS TOWARD RED				
λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
5195.647...	4:2	.534	+0.007	−0.001	5365.069..	3:5	.527	+0.018	0.000
5370.166...	1:6	.384	+ .013	....	5411.124..	1:4	.304	+ .016	....
5404.357...	2:5	.329	+ .015	− .012	5463.174..	3:3	.320	+ .014	+ .009
5463.494...	3:3	.320	+ .018	− .002	5598.524..	4:4	.187	+ .016	+ .008
5930.406...	2:6	.508	+ .009	....	6078.710..	2:5	.517	+0.014	−0.002
6020.401...	2:4	.169	+0.004	−0.008					
Mean...............		.374	+0.0110	....	Mean...............		.371	+0.0156	....
Standard displacement...............................								+0.0142 A	

were superposing upon the general displacement a measurable shift of the red and violet components of close pairs to the red and violet, respectively. In sign the mean result is unfavorable to the anomalous-dispersion hypothesis, but in magnitude it is practically zero, being 0.0003 A. On the assumption that the Albrecht deviations are real shifts, the displacements of lines with violet companions should exceed those of lines with red companions by 0.010–0.012 A. They are, however, within the limits of error, identical.

TABLE VI

RÉSUMÉ OF BEARING OF SUN-ARC DISPLACEMENTS ON MUTUAL INFLUENCE

No. of Lines	Group	Companion	Sun *minus* Arc	Normal	Summation of Deviations	
					Favorable	Unfavorable
56	*a*, *b*, *c*4	To Violet	+0.0036	+0.0038	....	−0.0112
29	*a*, *b*, *c*4	Red	+ .0038	+ .0038	0.0000	....
21	*c*5, *d*	Violet	− .0052	− .0050	....	− .0042
25	*c*5, *d*	Red	− .0062	− .0063	....	− .0025
6	*e*	Violet	+ .0110	+ .0142	....	− .0192
5	*e*	Red	+0.0156	+0.0142	....	−0.0070
142						

Mean deviation unfavorable to mutual influence. . . . . −0.0003A

The data for the lines in the second sections of Tables I, II, III, and IV are given in order to make the presentation of the Mount Wilson observations complete. Because of the recognized sources of possible error, they have not been considered in the discussion; their inclusion, however, would tend to increase the foregoing unfavorable deviation from the mean. Of these rejected lines the 17 used by Albrecht are more fully considered in the fourth division of this contribution.

The evidence of equality between the displacements of lines with companions to the violet and red, respectively, given by these 142 lines for which the conditions are favorable to the evoking of mutual influence, is clear and positive. It seems, therefore, a justifiable conclusion from this equality that mutual repulsion does not occur to a measurable amount, and that some other explanation applies to the Albrecht deviations from the mean.

III. RELATIVE SEPARATION IN SOLAR AND ARC SPECTRA

Mutual influence appearing as a quasi-repulsion requires that the difference in wave-length between a line and a closely adjacent companion be greater in the solar spectrum than when determined in terrestrial sources. If the interpretation adopted by Albrecht for his observations be the correct one, the separation in the solar spectrum, when the mean distance between the components is 0.22 A, should exceed that in terrestrial sources by 0.012 A, a quantity easily within the range of measurement. The measurements considered in this section were made upon separate plates for the sun and arc. The solar measurements, being more difficult than the corresponding ones for the arc, have been made upon plates of higher dispersion, with a scale sufficient to yield reliable results for the particular combination of intensity and spacing.[1] The conditions for the sun-arc measurements and for the determinations of separation considered in this section are so different, that it seems hardly probable that personal equation enters in the same manner or to the same degree in both, or that different observers would obtain concordant results, as they do, if personal equation introduced appreciable systematic errors. Four measurers have worked upon the sun-arc plates, and two of these have made the major part of the determinations recorded in Table VII, which form the subject-matter of this section.

For the 45 pairs the mean difference between the separations in the solar and arc spectra is zero within the limit of precision. According to the theory of mutual influence, the repulsion should be the more marked, the closer the adjacent lines. Of the pairs in Table VII, 13 have separations under 0.2 A, the mean being 0.148 A. For these pairs the mean separation in the solar spectrum is less by 0.0015 A than in terrestrial sources; for the remaining 32 pairs with a mean separation of 0.349 A, it is greater by 0.0003 A. In both cases the mean separations in sun and arc are practically identical.

The mean difference in separation without regard to sign, 0.003 A, and individual differences are larger, however, than one would expect from accidental errors of measurement. The following considerations show that these large differences are systematic in sign and probably real. Three things appear distinctly in the Mount Wilson sun-arc observations:[1] (1) in groups *a* and *b* strong lines are displaced more than weak; (2) lines of groups *c*5 and *d* are displaced to the violet; (3) lines of group *e* show the maximum displacement to the red. If, therefore, the components of a pair differ greatly in intensity or belong to different groups, whether the separation in the solar spectrum is greater or less than in the arc depends upon the configuration of the pair, provided the behavior of lines with closely adjacent companions is similar to that of free-standing lines. For example, the pair at λ 4939, with components *d* and *a*, should be wider

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[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

TABLE VII

COMPARATIVE SEPARATION IN SOLAR AND ARC SPECTRA

λ Rowland	Intensity, Element, or Group		Separation		Sun *minus* Arc	λ Rowland	Intensity, Element, or Group		Separation		Sun *minus* Arc
	V	R	Sun	Arc			V	R	Sun	Arc	
3647.561...	4	12	.422	.415	+0.007	4489.911..	Fe	Mn	.340	.340	0.000
3707.959...	5	5	.105	.102	+ .003	4654.672..	*a*	*d*	.120	.132	− .012
3711.364...	4	3	.184	.184	.000	4938.997..	4	2	.423	.426	− .003
3722.071...	3	2	.102	.100	+ .002	4939.416..	*d*	*a*	.449	.442	+ .007
3735.014...	40	4	.459	.460	− .001	4985.432..	3	3	.296	.294	+ .002
3743.508...	Fe	Ti	.108	.106	+ .002	5005.896	4	5	.406	.405	+ .001
3745.717...	8	6	.337	.340	− .003	5079.158..	*e*	*b*	.248	.245	+ .003
3748.408...	10	1	.227	.228	− .001	5107.619..	4	4	.191	.193	− .002
4000.403...	2	2	.204	.200	+ .004	5139.427..	4	4	.211	.212	− .001
4058.915...	Fe	Mn	.170	.173	− .003	5148.222..	*e*	*d*	.185	.190	− .005
4063.436...	4	20	.314	.310	+ .004	5169.069..	3	4	.129	.129	.000
4067.139...	5	3	.291	.296	− .005	5208.596..	Cr	Fe	.169	.170	− .001
4079.996...	3	3	.370	.370	.000	5227.043..	3	5	.322	.323	− .001
4106.420...	2	2	.171	.168	+ .003	5273.339..	3	2	.215	.214	+ .001
4136.678...	4	6	.475	.474	+ .001	5365.069..	*e*	*a*	.527	.533	− .006
4143.572...	4	15	.452	.450	+ .002	5404.028..	*a*	*e*	.319	.303	+ .016
4154.667...	4	4	.308	.310	− .002	5446.797..	Ti	Fe	.332	.334	− .002
4191.595...	6	3	.240	.240	.000	5463.174..	3	3	.319	.316	+ .003
4226.904...	Ca	4	.701	.710	− .009	5455.671..	*e*	*a*	.156	.162	− .006
4315.138...	Ti	Fe	.115	.115	.000	5476.500..	*a*	*d*	.280	.284	− .004
4427.266...	Ti	Fe	.213	.215	− .002	6136.829..	8	3	.373	.373	.000
4430.356...	1	3	.425	.422	+ .003	6400.217	8	2	.310	.311	−0.001
4454.552...	Fe	Ca	.399	.399	0.000						

45 lines, mean separation.............................. 0.291 A
17 lines, sum of positive differences.................... + .064
20 lines, sum of negative differences.................... − .070
8 lines, zero difference.............................. .000
Mean Δλ Sun −Δλ Arc.............................. −0.0002 A

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[1] *Mt. Wilson Contr.*, No. 93, p. 35; *Astrophysical Journal*, **41**, 63, 1915.

apart in the sun, while the pair at λ 4654, with components *a* and *d*, should be closer in the sun than in the arc, as the observations show. Taking as a lower limit a 50 per cent difference in intensity and considering the characteristic behavior of the groups, one can predict the relative separation in solar and arc spectra for 23 pairs as shown in Table VIII. The homogeneity of the two classes seems conclusive evidence that lines with close companions conform to the behavior of isolated lines of the same group and that their relative position in the solar spectrum is, therefore, not determined by mutual influence.

TABLE VIII

Classification of Solar Separations Based upon the Normal Behavior of Isolated Lines

Predicted Separation, Solar > Arc					Predicted Separation, Solar < Arc				
λ Rowland	Sep.	Intensity or Group		Sep. Sun *minus* Sep. Arc	λ Rowland	Sep.	Intensity or Group		Sep. Sun *minus* Sep. Arc
		V	R				V	R	
3647.561......	.422	4	12	+0.007	3735.014......	.459	40	4	−0.001
4063.436......	.314	4	20	+ .004	3745.717......	.337	8	6	− .003
4136.678......	.475	4	6	+ .001	3748.408......	.227	10	1	− .001
4143.572......	.452	4	15	+ .002	4067.139......	.291	5	3	− .005
4430.356......	.425	1	3	+ .003	4191.595......	.240	6	3	.000
4939.416......	.449	*d*	*a*	+ .007	4226.904......	.701	20	4	− .009
5227.043......	.322	3	5	− .001	4654.672......	.120	*a*	*d*	− .012
5404.028......	.319	*a*	*e*	+0.016	4938.997......	.423	4	2	− .003
					5079.158......	.248	*e*	*b*	+ .003
Mean.........	.397	....		+0.005 A	5148.222......	.185	*e*	*d*	− .005
					5365.069......	.527	*e*	*a*	− .006
					5455.671......	.156	*e*	*a*	− .006
					5476.500......	.280	*a*	*d*	− .004
					6136.829......	.373	8	3	.000
					6400.217......	.310	8	2	−0.001
					Mean.........	.325	....		−0.0035A

IV. THE SYSTEMATIC DEVIATIONS OF ALBRECHT

Dr. Albrecht plotted the differences between the Rowland and the "corrected" International wave-lengths of the iron lines against wave-lengths as abscissae,[1] and found that the values for lines with violet and red companions were, respectively, above and below the curve, requiring, when the average distance between the lines and their companions was 0.22 A, corrections of −0.005 and +0.007 A to reduce them to the mean. He saw no reason for considering these systematic deviations to be due to the rôle played by personal equation in the measurement of close pairs. He regarded them, therefore, as evidence of anomalous dispersion in the sun.

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[1] *Astrophysical Journal*, **44**, 14, 1916.

As the extensive Mount Wilson data for sun-arc displacements and for the comparative separations of the components of close pairs in solar and terrestrial spectra indicate, within the limits of precision, a total absence of mutual influence, it seems necessary to conclude either that the Mount Wilson data, though depending upon concordant results from several observers and methods, are affected by a personal equation introducing systematic errors just balancing the effects of mutual influence, or that the Rowland wave-lengths for lines in close pairs, depending upon a single observer, are systematically in error. A slight over-spacing of such pairs, displacing the components in opposite directions, would introduce an effect of the sign required by the theory. An investigation of the accuracy obtainable in the measurement of close solar pairs and of possible errors in the Rowland table has lately been carried out at this Observatory.[1] For the 30 pairs composed of lines of intensities 3 and 4 which formed the basis of the investigation, the Rowland separations are larger than those found at Mount Wilson; the sign of the Rowland errors is such as would explain the Albrecht deviations as errors in the Rowland tables. For separations of 0.274, 0.145, and 0.075 A the errors are +0.003, +0.005, and +0.013 A, respectively, a march of magnitude with proximity of components similar to that observed by Albrecht.

The investigation showed that spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution, that whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation.

Professor Aitken finds in measures upon close double stars with 12- and 36-inch telescopes the same tendency toward over-separation with increased overlap of the stellar images, as appears in measures of incompletely resolved solar pairs. I am under obligation to him for the following data:

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[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

SEPARATIONS OF DOUBLE STARS

Under 1″.0 12-Inch *minus* 36-Inch		Between 1″.0 and 2″.0 12-Inch *minus* 36-Inch	
33 plus residuals	+2″.47	10 plus residuals	+0″.85
7 minus "	−0.45	18 minus "	−1.46
2 zero "	0.00	4 zero "	0.00
42 "	+2.02	32 "	−0.61
Mean "	+0.05	Mean "	−0.02

Concerning these data, Professor Aitken says:

> The theoretical resolving power of the 12-inch is 0″.39, hence, taking distorted images into account, due to bad seeing, pairs with angular separations less than 1″.0 would have little space between the two images and the closer pairs would usually be in contact, or would overlap. For pairs over 1″.0 and certainly for pairs over 1″.25, there would always be clear space between the images on any night I would use for work. I think the tabulation shows that there is a systematic tendency to overmeasure small distances with the 12-inch telescope—assuming that the 36-inch measures are exact—a tendency which disappears with distances of 1″ or more. The small negative residual (12-inch−36-inch) for the larger distances is obviously of a more accidental character than the positive residual for the smaller distances, since we have 18 minus and 10 plus signs in the one case against 33 plus and 7 minus signs in the other. This result is in harmony with many other similar comparisons I have made both of my own measures and of measures by different observers at about the same epoch with telescopes of different apertures.

These results point to an explanation of the systematic deviations observed by Albrecht, based upon errors in the Rowland wave-lengths for lines in close pairs. An extensive investigation of all the cases occurring in Albrecht's tables was undertaken. From experience gained in obtaining the data in *Contribution* No. 120, it has been possible to take advantage of the best available conditions for observations upon such difficult objects as close solar pairs. The wave-lengths of the solar lines used by Albrecht have been referred to those of the neighboring free-standing lines. All measurements have been made by two observers upon spectrograms of a scale and dispersion that experience has shown to be the most reliable in each particular case. In every instance the data depend upon three or more concordant plates.

From the characteristic behavior of the Fe lines in the arc, it is evident that conclusions drawn from the differences between the Rowland and International wave-lengths must carry the maximum weight when based upon the stable lines of groups *a*, *b*, and *c*4. In the discussion of the Albrecht deviations, these lines, therefore, are considered separately from the unstable lines of groups *c*5, *d*, and *e*, for which the errors in the International wave-lengths are known to be systematic in sign and relatively large. According to the theory the effect of mutual influence increases with decreasing separation of the reacting solar lines; at the same time, however, the difficulties of measurement become increasingly great. As the evidence in such cases is relatively of great importance, pairs for which the separation from the companion is less than 0.1 A are given separate consideration. The discussion of the two series of data follows under heads, A, B, C, and D.

A. *Groups a, b, and c*4.—The general result from Table IX is that for 88 lines the sum of the Albrecht deviations is 0.377 A, and that of the counterbalancing Rowland errors is 0.351 A, a mean unbalanced deviation of 0.0003 A. The data, however, vary in reliability. For the 54 lines of greater weight, groups *a*, *b*, and *c*4 in Part I, the sum of Albrecht deviations is 0.226 A, that of the Rowland errors, 0.255 A. That the Albrecht deviations and the Rowland errors are mutually explanatory appears clearly from the parallelism between them. In Table X the largest Albrecht deviations are in the first column and the smallest in the third, the Rowland errors for the same lines being in the second and fourth columns of the two parts of the table. For the 27 lines showing the smallest Albrecht deviations the mean mutual influence is zero, and the Rowland error correspondingly small. If the larger deviations are due to mutual influence, a lack of parallelism should be conspicuous for them, but here the correspondence is particularly close. Among the 27 lines with the largest deviations, discrepancies are pronounced for λ 5195 and λ 5476 indicated by "?" in the table. These lines have companions to the red for which, on the hypothesis of mutual influence, large displacements are characteristic. Their

TABLE IX

The Albrecht Systematic Deviations and Errors in the Rowland Table

PART I. LINES OF GROUPS *a*, *b*, AND *c*4. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
3647.988.....	4:12	0.427	.986	−0.002	+0.002	
3680.069.....	2: 9	.248	.062	− .007	− .007	
3737.281.....	5:30	.222	.280	− .001	+ .003	
3746.058.....	8: 6	.341	.054	− .004	− .008	
3748.650.....	10: 1	.242	.646	− .004	− .008	
3887.196.....	3: 7	.254	.198	+ .002	.000	
3888.671.....	2: 5	.111	.663	− .008	− .008	
3895.803.....	3: 7	.220	.805	+ .002	.000	
3969.413.....	6:10	.527	.410	− .003	− .004	Comp. in shade of λ 3968
4132.235.....	2:10	.135	.222	− .013	− .012	
4134.840.....	3: 5	.251	.836	− .004	+ .006	
4144.038.....	4:15	.466	.029	− .009	− .005	
4147.836.....	2: 4	.334	.831	− .005	+ .002	
4191.843.....	6: 3	.248	.839	− .004	+ .003	
4291.630.....	2: 2	.354	.625	− .005	+ .007	
4308.081.....	3: 6	.174	.065	− .016	− .006	
4315.262.....	3: 4	.124	.252	− .010	− .007	
4427.482.....	2: 5	.216	.476	− .006	.000	
4531.327.....	2: 5	.204	.326	− .001	− .002	
4556.306.....	3: 4	.243	.309	+ .003	+ .002	
4592.840.....	2: 4	.133	.831	− .009	− .010	
4787.003.....	3: 2	.276	.994	− .009	− .012	
4789.849.....	2: 3	.321	.840	− .009	− .012	
4939.868.....	2: 3	.452	.867	− .001	− .002	
5028.308.....	1: 2	.369	.310	+ .002	.000	
5041.255.....	3: 4	.186	.250	− .005	− .002	
5041.936.....	2: 4	.141	.943	+ .007	+ .001	
5079.409.....	3: 4	.251	.402	− .007	− .006	
5098.885.....	1: 3	.134	.876	− .009	− .005	
5107.823.....	4: 4	.204	.820	− .003	− .004	Quality excellent in sun and arc
5167.678.....	15: 5	.181	.669	− .009	− .016	Very difficult*
5227.362.....	3: 5	.319	.355	− .007	− .003	
5270.558.....	3: 4	.120	.541	− .017	− .026	Very difficult
5273.558.....	3: 2	.219	.555	− .003	− .014	
5333.089.....	1: 4	.240	.088	− .001	+ .005	
5365.596.....	5: 3	.527	.593	− .003	+ .003	
5447.130.....	2: 6	.333	.125	− .005	+ .003	
5455.834.....	2: 4	.163	.810	− .024	− .007	
6191.779.....	6: 9	0.386	.775	−0.004	−0.004	
Mean, 39 lines..........................				−0.0054	−0.0039	

SECTION B. COMPANION TO RED

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
3705.708.....	2: 9	0.141	.714	+0.006	+0.007	
3735.014.....	4:40	.471	.013	− .001	+ .004	
3743.508.....	2: 6	.118	.511	+ .007	+ .006	
3834.364.....	4:10	.142	.377	+ .013	+ .009	
4337.216.....	3: 5	.509	.216	.000	+ .002	
4454.552.....	5: 3	.401	.553	+ .001	+ .003	
4461.818.....	3: 4	.347	.824	+ .006	+ .008	
4489.911.....	3: 4	.342	.910	− .001	+ .004	
4679.027.....	2: 6	.382	.029	+ .002	+ .005	
5107.619.....	4: 4	.204	.627	+ .008	+ .008	Quality excellent in sun and arc*
5195.113.....	2: 4	.534	.113	.000	+ .006	Arc λ depends on unstable standards
5208.596.....	2: 5	.180	.598	+ .002	+ .003	
5328.236.....	2: 8	.279	.235	− .001	− .007	
5476.500.....	3: 1	.278	.496	− .004	+ .011	Weak line 0.11 to violet vitiates measures
6136.829	3: 8	0.381	.835	+0.006	+0.004	
Mean, 15 lines..........................				+0.0029	+0.0049	

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* *Astrophysical Journal*, 44, 21 and 30, 1916.

TABLE IX—*Continued*

PART II. LINES OF GROUPS *c*5 AND *d*. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
4227.606.....	20:4	0.702	.596	−0.010	−0.004	
4233.772.....	4:6	.444	.769	− .003	+ .002	
4872.332.....	1:4	.220	.329	− .003	− .006	
4957.785.....	5:8	.305	.785	.000	− .004	
4985.730.....	3:3	.298	.729	− .001	+ .003	
5006.306.....	4:5	.410	.303	− .003	− .002	
5139.644.....	4:4	.217	.642	− .002	+ .002	
5208.776.....	5:2	.180	.766	− .010	− .006	
5603.186.....	3:4	.103	.178	− .008	− .011	
5615.877.....	2:6	.357	.870	− .007	− .004	
5659.052.....	1:4	0.299	.042	−0.010	−0.003	
Mean, 11 lines............................				−0.0052	−0.0030	
SECTION B. COMPANION TO RED						
4191.595.....	3:6	0.248	.598	+0.003	+0.011	
4637.685.....	4:5	.508	.685	.000	+ .004	
4707.457.....	2:5	.215	.459	+ .002	+ .006	
4938.997.....	2:4	.419	.996	− .001	+ .005	
4957.480.....	8:5	.305	.481	+ .001	+ .003	
4982.682.....	2:4	.312	.682	.000	+ .013	
4985.432.....	3:3	.298	.433	+ .001	+ .009	
5005.896.....	5:4	.410	.898	+ .002	+ .002	
5125.300.....	1:3	.123	.296	− .004	− .002	
5139.427.....	4:4	.217	.431	+ .004	+ .006	
5273.339.....	2:3	.219	.343	+ .004	+ .004	
5476.778.....	5:3	.345	.775	− .003	+ .015	
6400.217.....	2:8	0.321	.221	+0.004	+0.001	
Mean, 13 lines............................				+0.0010	+0.0059	
PART III. LINES OF GROUP *e*. SECTION A. COMPANION TO VIOLET						
5195.647.....	4:2	0.534	.644	−0.003	−0.001	
5404.357.....	2:5	.329	.344	− .013	− .012	
5463.494.....	3:3	.320	.488	− .006	− .002	
5594.884.....	4:1	.193	.882	− .002	+ .009	
6020.401.....	2:4	0.169	.391	−0.010	−0.008	
Mean, 5 lines............................				−0.0068	−0.0028	
SECTION B. COMPANION TO RED						
5365.069.....	3:5	0.527	.068	−0.001	0.000	
5463.174.....	3:3	.320	.169	− .005	+ .009	
5598.524.....	4:1	.187	.525	+ .001	+ .008	
6008.186.....	6:4	.599	.186	.000	+ .012	
6078.710.....	2:5	0.517	.707	−0.003	−0.002	
Mean, 5 lines............................				−0.0016	+0.0054	

significance in this case is, however, open to question, for λ 5195, intensity 4, is 0.534 A from its companion of intensity 2, and λ 5476, intensity 1, stands between two lines, a weaker one, 0.113 A to the violet, a stronger one, 0.278 A to the red. The arc wavelength of λ 5195 is derived from unstable standards of group *d*, and is subject to the errors attaching to them. Of the arc wave-length of λ 5476 St. John and Ware remark, "Near diffuse line λ 5476.587. Separation difficult."[1] Notes upon the observing records for its solar wave-length indicate that the measurements are influenced by the weak violet companion which, even on plates of high dispersion, is contiguous to the violet edge and tends to produce a fictitious shift to the violet.

TABLE X

Correspondence between Albrecht Deviations and Rowland Errors Based upon Lines of Groups *a*, *b*, and *c*4 in Table IX

Companion to Violet				Companion to Red			
20 Largest Deviations		19 Smallest Deviations		7 Largest Deviations		8 Smallest Deviations	
Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.
−0.007	−0.007	+0.002	−0.002	+0.007	+0.006	+0.004	−0.001
− .008	− .004	+ .003	− .001	+ .006	+ .007	+ .002	.000
− .008	− .004	.000	+ .002	+ .009	+ .013	+ .003	+ .001
− .008	− .008	.000	+ .002	+ .008	+ .006	+ .004	− .001
− .004	− .003	+ .006	− .004	+ .008	+ .008	+ .005	+ .002
− .012	− .013	+ .002	− .005	+ .006?	.000?	+ .003	+ .002
− .005	− .009	+ .003	− .004	+0.011?	−0.004?	− .007	− .001
− .006	− .016	+ .007	− .005	....	....	+0.004	+0.006
− .007	− .010	.000	− .006				
− .010	− .009	− .002	− .001				
− .012	− .009	+ .002	+ .003				
− .012	− .009	− .002	− .001				
− .006	− .007	.000	+ .002				
− .005	− .009	− .002	− .005				
− .004	− .003	+ .001	+ .007				
− .016	− .009	+ .003	− .007				
− .026	− .017	+ .005	− .001				
− .014	− .003	− .003	− .003				
− .007	− .024	+0.003	−0.005				
−0.004	−0.004	....	....				
SUMS AND MEANS							
−0.181	−0.177	+0.028	−0.034	+0.055	+0.036	+0.018	+0.008
−0.0090	−0.0088	+0.0015	−0.0018	+0.0079 (7)	+0.0051 (7)	+0.0022	+0.0010
				+0.0076 (5)	+0.0080 (5)		

B. *Groups c5, d, and e.*—As to the 34 lines of the unstable groups *c*5, *d*, and *e*, the situation is so complicated that an attempt to follow its intricacies would be profitless. Four considerations are involved: (1) The systematic errors in the Rowland values tending to account for the deviations found by Albrecht. The average error is 0.003 A, somewhat smaller than for lines of groups

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[1] *Mt. Wilson Contr.*, No. 61, p. 28; *Astrophysical Journal*, 36, 41, 1912.

a, *b*, and *c*4. Such a difference is consistent with the relative appearance of the lines in arc and solar spectra, for the lines of groups *c*5, *d*, and *e*, diffuse, unsymmetrical, and difficult to measure in the arc, are, in general, the sharpest and most accurately measurable in the solar spectrum. (2) The undetermined systematic errors in the International wave-lengths of these lines, positive for groups *c*5, and *d*, negative for group *e*. In recognition of these errors standard conditions for the arc were adopted at the Bonn meeting of the International Solar Union and redeterminations of the iron standards have been undertaken in various laboratories. Preliminary results at this Observatory indicate that the errors are systematic and of the sign expected. (3) The systematic errors in the data for pressure-shift. At the time of publication of the pressure data, the sensitiveness of these lines to varying arc conditions had not been recognized and the needed precautions against pole-effect were not taken. Recent observations under standard arc conditions give 0.008 A per atmosphere for groups *c*5 and *d* instead of the 0.022 A previously published. For the lines of group *e* the later result is +0.002 A per atmosphere instead of the negative value −0.016 A.[1] (4) Albrecht's effort to make the International and Rowland wave-lengths homogeneous by "correcting" the International values to a pressure of half an atmosphere. That this operation still left systematic differences appears in Table XI based upon data from Albrecht's Table I. For reasons given later and because the points fall so far from the curve that they have had little if any influence on its course, λ 4204, λ 4210, and λ 5371 have not been taken into account.

A reference-curve based upon $\Delta\lambda'$ for all lines is on the average too high for the lines of groups *c*5 and *d*, whose proportional representation is small. This tends to increase their deviations to the violet and to decrease those to the red. Such a tendency shows in Part II of Table IX, where the mean deviation to the violet is 0.006 A, that to the red 0.003 A, while the Rowland errors are 0.001 and 0.005 A, respectively. A lowering of the curve by 0.004 A would bring the deviations and errors into practical agreement. As a systematic effect of this order is indicated by the negative residuals in Table XI, curves have been drawn for the lines of groups $c5$ and d alone, using $\Delta\lambda$ and $\Delta\lambda'$ from Albrecht's Table I as ordinates. The mean displacement for the same lines of these groups with companions to the red, deduced from either curve, is 0.002 A instead of 0.006 A, a result in close agreement with the corresponding Rowland error of 0.001, while the mean for the lines with companions to the violet remains practically unchanged. This systematic effect, introduced by referring the lines of groups $c5$ and d to a curve based largely upon lines of other groups, is apparently of a sign and magnitude to account for the inequality of the displacements for lines with red and violet companions, which Albrecht considered the principal objection to an explanation based upon personality in the Rowland measures.[1]

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[1] *Mt. Wilson Contr.*, No. 106, p. 16; *Astrophysical Journal*, **42**, 231, 1915.

TABLE XI

SYSTEMATIC VARIATION FROM HOMOGENEITY IN $\Delta\lambda'$

Region	Group	$\Delta\lambda$	$\Delta\lambda'$	$\Delta\lambda'$ Group $c5$, d *minus* $\Delta\lambda'$ Group a, b
4200–4300	$c5$, d	.159	.165	+0.001
	a, b	.163	.164	
5000–5100	$c5$, d	.165	.173	−0.007
	a	.178	.180	
5100–5200	d	.155	.168	−0.004
	a	.170	.172	
5200–5300	d	.164	.175	−0.002
	a	.175	.177	
5300–5400	d	.177	.187	−0.009
	a	.194	.196	
6300–6500	d	.195	.212	−0.004
	b	.212	.216	

C. *Lines less than 0.1 A from companion.*—Of the 104 lines considered by Albrecht, the remaining 16 are separated from the companion line by less than 0.1 A. The data relative to these very close pairs are given in Table XII. The wide range in the deviations, +0.048 to −0.002 A for the 6 lines with companions

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[1] *Astrophysical Journal*, **41**, 355, 1915.

TABLE XII

Lines Less than 0.1 A from Companion

PART I. COMPANION TO VIOLET

λ Rowland	Int.	Element and Class	ΔλR	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
3722.639	3	Ni					
(.692)	10		0.090				
.729	6	Ti–Fe, *a*		.714	−0.015	−0.014	Resolved only with best definition. Fe line a blend with titanium
4294.204	2	Ti					
....			0.097				
.301	5	Fe, *b*		.287	−0.014	−0.005	
4407.810	2	V					
....			0.061				
.871	4	Fe, *c*4		.878	+0.007	+0.011	
4668.243	2						
....			0.088				
.331	4	Fe, *d*		.322	−0.009	−0.010	
4727.582	3	Fe					
....			0.094				
.676	2	Mn		.662	−0.014	−0.029	In arc, Mn λ 4727.476 Kilby, .478 Janicki, .464 Burns; Fe .408 Janicki, .410 Burns; blend .438 Kayser, .434 Goos, 435 "unreliable" St. John and Ware; weight 0, Albrecht
4878.313	3	Ca					
....			0.094				
.407	4	Fe, *c*5		.406	−0.001	−0.008	
5202.439	2	Fe?					
....			0.077				
.516	4	Fe, *b*		.510	−0.006	−0.006	
5371.656	4	Cr?					
....			0.078	.692			
.734	3	Fe, *a*			−0.042	−0.042	Misidentification by Rowland. Not a pair in solar spectrum but a single line due to Fe
6147.950	2						
....			0.090				
8.040	3	Fe, *d*		.038	−0.002	−0.005	
6254.382	1						
....			0.074				
.456	5	Fe, *b*		.479	+0.023	+0.017	Doubtful, Albrecht

PART II. COMPANION TO RED

λ Rowland	Int.	Element and Class	ΔλR	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
3997.547	4	Fe, *b*		.549	+0.002	+0.001	
....			0.091				
.638	2						
4204.101	3	Fe, *b*		.106	+0.005	+0.048	Only incipiently separated from red companion by the highest resolving power used. Blend in solar spectrum. High dispersion shows weak line contiguous to violet edge. Burns gives two lines in arc 4203.953, intensity 1, and .985, intensity 3, of which the stronger alone was used by Albrecht. Both things contribute to the abnormal displacement to violet
....			0.062				
.163	4	La					

TABLE XII—*Continued*

PART II (*Continued*). COMPANION TO RED

λ Rowland	Int.	Element and Class	ΔλR	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
4210.494	4...	Fe, *c*5		.499	+0.005	+0.027	Resolution incipient. Blend in solar spectrum. With high dispersion a line contiguous to the violet edge is conspicuous. The measurements, being upon the blend of the two, give the apparent shift to the violet
....			0.067				
.561	3...						
4476.185	4...	Fe, *b*		.182	−0.003	+0.008	Resolution incipient. In the arc, "Unreliable. Hazy on the red edge," St. John and Ware; "Ur," Goos; "Not suitable for standard," Janicki; "Close double in vacuum," Babcock
....			0.008				
.253	3...	Ag					
4727.582	3...	Fe, *d*		.584	+0.002	+0.003	
....			0.094				
.676	2...	Mn					
5012.252	4...	Fe, *a*		.250	−0.002	−0.002	
....			0.083				
.335	1...						

to the red, and −0.041 to +0.017 A for the 10 lines with companions to the violet, indicates at once the low weight to be assigned to the data as evidence of mutual influence. Three pairs of lines, companions to the red, λ 4204, λ 4210, and λ 4476, are at the limit of spectrographic resolution for lines of these intensities. The probable errors are therefore large.[1] The record of observations in the last column furnishes sufficient grounds for omitting them from a definitive discussion. The three other pairs with companions to the red are measurable with high precision and the conditions are apparently favorable to the appearance of mutual influence; the mean displacement to the violet, however, is minute, 0.0007 A, and balanced by the Rowland errors.

Of the 10 lines with companions to the violet, 4 are of doubtful or no weight in a definitive discussion; λ 3722 itself is a blend and is in a complex very difficult of resolution; the arc wave-length of λ 4727 is uncertain; the indicated pair at λ 5371 is a single line; λ 6254 is not separable from its companion upon a fourth-order spectrogram, upon which the head of the oxygen band at λ 6276 is completely resolved, a resolution not reached by the Rowland plates for that region. The deviations for the remaining 6 lines are balanced by Rowland errors.

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[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

The most favorable condition for the appearance of mutual influence is in the case of lines with companions at the closest distance consistent with precision of measurement, as the probability is then greatest that the affected lines are situated upon a steep portion of the "anomaly-curve" due to the adjacent line.[1] Observations upon lines that, aside from closeness to a companion line, are free from known sources of error are consequently of crucial importance in their bearing upon the hypothesis of anomalous dispersion in the solar atmosphere, and failure here would indicate that, within the present attainable precision in measurement, anomalous dispersion does not systematically alter the relative positions of the Fraunhofer lines. Data for the 9 lines fulfilling these conditions are given in Table XIII. Their average separation is 0.082 A; ratio of companion to line, 3.2:2.7; weight, according to Albrecht, 1.7. The displacement to the violet for the lines with companions to the red should be larger than the displacement to the red for lines with companions to the violet. The deviation to the violet is, however, vanishingly small, much less than that to the red, and both are balanced by the corresponding Rowland errors.

TABLE XIII

CORRESPONDENCE BETWEEN ALBRECHT DEVIATIONS AND ROWLAND ERRORS FOR THE SMALLEST MEASURABLE SEPARATIONS, LINES FREE FROM EXTRANEOUS SOURCES OF ERROR

COMPANION TO RED				COMPANION TO VIOLET			
λ Rowland	Δλ	λMt.W. *minus* λR.	Albrecht	λ Rowland	Δλ	λMt.W. *minus* λR.	Albrecht
3997.547...	0.085	+0.002	+0.001	4294.301..	0.078	−0.014	−0.005
4727.582...	0.084	+0.002	+0.003	4407.871..	.068	+ .007	+ .011
5012.252...	0.082	−0.002	−0.002	4668.331..	.077	− .009	− .010
				4878.407..	.092	− .001	− .008
Means.....	0.083	+0.0007	+0.0007 A	5202.516..	.077	− .006	− .006
				6148.040..	0.099	−0.002	−0.005
				Means....	0.082	−0.0042	−0.0038 A

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[1] *Astrophysical Journal*, **43**, 53, note, 1916.

D. *Correlation.*—The correspondence between the Albrecht deviations and the Rowland errors appears distinctly in Tables X and XIII. A line-to-line comparison for these 63 lines, to which a maximum of weight may be ascribed, shows a striking parallelism. A direct correspondence between the 104 Albrecht deviations and the Rowland errors for the same lines is shown to be a practical certainty by the large correlation coefficient +0.56 and its small probable error ±0.05.

The facts observed by Albrecht and the corresponding results for this investigation are strikingly complementary.

ALBRECHT	ST. JOHN
Fraunhofer lines as given in Rowland's table are displaced when they have close companions.	Rowland wave-lengths of lines with close companions are systematically in error.
a) The displacement is to the violet when the adjacent line is to the red.	*a′*) The sign of the error is negative when the adjacent line is to the red.
b) The displacement is to the red when the adjacent line is to the violet.	*b′*) The sign of the error is positive when the adjacent line is to the violet.
c) The displacement in (*a*) is greater than in (*b*).	*c′*) A systematic excess in the Albrecht displacement to the violet for lines of groups *c*5 and *d* tends to make (*a′*) greater than (*b′*).
d) The displacement increases as the separation between the lines diminishes.	*d′*) The Rowland error increases as the separation of the lines diminishes.
e) The displacement is inappreciable for separation somewhat under 0.7 A.	*e′*) The Rowland errors cease to be systematic for separation somewhat under 0.5 A.

There seems to be no explanation of the *pari passu* march of the Albrecht observations and those recorded in Section IV of this paper other than that they are two phases of the same phenomenon.

V. ADJACENT LINES DUE TO DIFFERENT ELEMENTS

Under the title, "Mutual Repulsion of Contiguous Spectrum Lines," Sir Joseph Larmor says:

> Thus *very close spectrum lines ought to repel each other*, to a degree that experience alone can reveal.

But this conclusion implies that the adjacent lines represent independent vibrations. If they are two components of a single compound vibration of the molecule, the argument is not applicable.

If an iron line in the solar spectrum has a very close adjacent line *due to another substance,* while in the arc spectrum that substance is not present, then a displacement of the solar line relative to the arc line may be looked for.[1]

If such a mutual repulsion is operative to a measurable degree in the solar atmosphere, the sun-arc displacements of the Fe lines should be greater for the red and less for the violet components of a pair when the adjacent line is due wholly or in part to another substance than when the influencing line is iron or weak and unknown.[2] The most dependable results should be given by lines of groups *a*, *b*, and *c*4, both because of their stability in the arc and because of their greater number. For these 85 lines a residual of 0.002 A would have considerable weight. For the lines of groups *c*5, *d*, and *e*, the errors due to their sensitiveness to arc conditions are relatively large, and definite indications of characteristic behavior require correspondingly larger residuals for a like number of lines. The comparison between the displacements of 72 Fe lines, Table XIV, under the influence of adjacent lines in which other substances are concerned with the 70 for which mutual influence is supposed to be smaller shows no differences greater than the limit of error.

Among the 45 pairs in Table VII for which the separations were measured in solar and arc spectra are 8 close pairs of lines that fulfil the Larmor conditions, as the arc wave-lengths for the different elements were determined in separate arcs. For these 8 pairs, mean separation 0.23 A (Table XV), there is no systematic difference between the separations in the sun and in independent terrestrial sources, and the absence of mutual repulsion is still shown by the 4 closer pairs, mean separation 0.14 A. It is evident that the effect is not within the present means of observation.

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[1] *Observatory,* **497,** 103, 1916.

[2] NOTE.—According to Sir Joseph Larmor an increase of refractive power is accompanied by a lowering of the aethereal elasticity which results in an increase of the free period and consequently a real displacement of the red component of a close solar pair toward longer wave-length. Similar considerations indicate a displacement of the violet component toward shorter wave-length. Such changes in vibration-frequency occur only when the anomalous refractive power is due to an adjacent line originating in an independent vibrating system, that is, in general, to a line of another element. The view of Professor Julius is that, owing to a mutual influence of two close Fraunhofer lines, the refractive power of the medium for the spectral region between them is decreased, less R- and V-light is anomalously refracted or scattered away from the observer on their opposed edges, and the centers of gravity of the dispersion bands are consequently farther apart. As the mutual influence consists simply in the superposition of the anomalous refractive powers due to the two absorption lines, the displacements should be independent of the elements involved. Julius adds the proviso that the elements coexist in the mixture at the same levels.

TABLE XIV

SUN-ARC DISPLACEMENTS IN RELATION TO ORIGIN OF THE ADJACENT LINE

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Companion	Group	No. of Lines	Sun-Arc	No. of Lines	Sun-Arc	Remarks
To violet.....	*a*, *b*, *c*4	30	+0.0040	26	+0.0031	Favorable by 0.001 A
To red.......	*a*, *b*, *c*4	19	+ .0041	10	+ .0031	Unfavorable by 0.001 A
To violet.....	*c*5, *d*	10	− .0031	11	− .0072	Three abnormal values included
		9	− .0037	9	− .0047	Three abnormal values omitted
To red.......	*c*5, *d*	10	− .0066	15	− .0060	Equal within limit of error
To violet.....	*e*	1	+ .013	5	+ .011	Equal within limit of error
To red.......	*e*	2	+0.016	3	+0.015	Equal within limit of error

(4) Adjacent line wholly or in part not iron.
(6) Adjacent line iron or weak and unknown.

TABLE XV

SEPARATIONS IN THE SOLAR SPECTRUM COMPARED WITH SEPARATIONS DETERMINED FROM INDEPENDENT ARCS WHEN THE ADJACENT LINE IS DUE TO ANOTHER ELEMENT

λ Rowland	Elements	Separation		Sep. Sun *minus* Sep. Arc
		Sun	Arc	
3743.508........	Fe, Ti	.108	.106	+0.002
4058.915........	Fe, Mn	.170	.173	− .003
4315.138........	Ti, Fe	.115	.115	.000
4427.266........	Ti, Fe	.213	.215	− .002
4454.552........	Fe, Ca	.399	.399	.000
4489.911........	Fe, Mn	.340	.340	.000
5208.596........	Cr, Fe	.169	.170	− .001
5446.797........	Ti, Fe	.332	.334	−0.002
Means..........	....	.231	.232	−0.0008

In a recent number of the *Astrophysical Journal*[1] Albrecht deduces the following unweighted results from the systematic deviations between the Rowland and the "corrected" International wave-lengths for Fe lines with close companions:

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[1] *Astrophysical Journal*, 44, 1, 1916.

Companion not Fe	Sum of Deviations
20 lines, comp. to R. 0.008	0.160
41 " " " V. 0.0047	0.193
61 " total sum	0.353
Mean	0.0055

Companion Fe	Sum of Deviations
18 lines, comp. to R. 0.0055	0.099
23 " " " V. 0.003	0.069
41 " total sum	0.168
Mean	0.0041

and concludes that "this material reduction of the displacement for pairs in which both components are due to iron is distinctly in line with Larmor's theory."

The data show an apparent Larmor effect of 0.0014 A. There are included, however, 3 lines whose evidence in a definitive discussion is open to question, namely, λ 4204, λ 4210, and λ 5371. The Albrecht deviations for them are 0.048, 0.027, and 0.042 A. These average 8 times the mean for the 102 lines. Aside from their extraordinary magnitudes, other grounds for assigning them a very low weight have been given in a previous section and are here briefly restated. λ 5371 is not in a solar pair with Cr but is a single line due to iron; λ 4204 and λ 4210 are only partially resolved from their red companions even on excellent fifth-order spectrograms; each has, moreover, a hitherto unnoted weak line contiguous to its violet edge; λ 4204 is double in the arc, the red component only was used in the comparison, and λ 4210 is subject to pole-effect. These conditions all combine to produce abnormal violet displacements. If these three lines are omitted, the indication of a Larmor effect disappears, as the following tabulation shows:

Companion not Fe	Sum of Deviations
20−2=18 lines comp. to R.	0.085
41−1=40 " " " V.	0.151
58 " total sum	0.236
Mean	0.0041

Companion Fe	Sum of Deviations
18 lines comp. to R.	0.099
23 " " " V.	0.069
41 " total sum	0.168
Mean	0.0041

Other lines giving extraordinary values, namely, λ 3722, λ 5167, λ 5270, and λ 6254, are practically in the same category as the three mentioned above. Their omission does not change the evidence against the Larmor effect. The inconsistency between

0.004 A given by the 99 lines and 0.039 A given by 3 lines is so pronounced that one seems compelled to choose between them: The straight means only are here considered, as the system of weighting used by Albrecht unfortunately assigns the greater weight to observations presenting the greater difficulties of execution. For example, a combined weight of 8 is assigned to these 3 questionable lines in obtaining his final result, the average weight of the 102 lines being 1.25, so that in the weighted mean their influence is more than doubled.

Albrecht remarks further:

> As the displacement has not entirely disappeared for pairs in which both lines are due to iron, we must conclude that the components of these pairs represent only in part actual physical connection in the molecule, and in part entirely independent vibrations.[1]

It is implied that if the components are due to one compound vibration, that is, to a single element, and originate at the same level, no displacement would occur. This seems to be a relinquishment of the Julius point of view upheld in the former paper and opposed to the assumption of Julius that mutual influence is operative only when the lines originate at the same level.[2]

VI. PRESSURE IN THE SOLAR ATMOSPHERE

The pressure of 0.5 atmosphere found by Albrecht for the solar atmosphere was based upon the pressure-shifts for the iron lines, published previously to the appearance of his paper. *Contribution* No. 106 from this Observatory,[3] in which attention was called to the errors in the published data for groups *c*5, *d*, and *e*, due to pole-effect, was issued later. Using the old pressure-shifts of +0.022 A for groups *c*5 and *d*, and −0.016 A for group *e*, Albrecht obtained results of which he says:

> . . . . The correction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has brought in toward the curve the lines which are strongly affected by pressure—namely, those of groups *c*, *d*, and *e*. Without the application of these corrections, the lines of groups *c* and *d* are decidedly

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[1] *Astrophysical Journal*, **44**, 8, 1916.

[2] *Ibid.*, **43**, 49, 1916.

[3] St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

below the curve, and those of group *e* are above the curve. This "drawing in" of the points toward the curve is gratifying, as it shows that the reduction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has made them practically homogeneous with Rowland's wave-lengths in the sun—except, of course, for the systematic differences between the two systems as represented by the curve.[1]

A homogeneity attained by applying corrections based upon pressure data in which the errors are comparable to the original lack of homogeneity can hardly be regarded, in the light of fuller knowledge, as a justification of the method. That the practical homogeneity is still affected by systematic deviations for the lines of groups *c*5 and *d* is apparent in Table XI, and the lack of homogeneity, when the later pressure-shifts are used, is shown in Table XVI. Even an assumed pressure of zero fails to make them homogeneous with the Rowland wave-lengths, but they become so at a pressure of 5 atmospheres, a pressure in the solar atmosphere inconsistent, however, with the long series of sun-arc observations with which comparison is made in Table XVII. The discrepancies between the observed and calculated displacements are so large that it is evident that a homogeneity depending upon a correction for pressure only does not rest upon a substantial basis. In fact, so many elements enter into the problems raised by the differences between solar and arc wave-lengths that it seems advisable to obtain a much wider range of sun-arc and other data before entering upon a definitive discussion of the question of pressure in the solar

TABLE XVI

NON-HOMOGENEITY OF $\Delta\lambda'$ AT 0.5 ATMOSPHERE

REGION	GROUP	$\Delta\lambda$	$\Delta\lambda'$ AT 0.5 ATM.		$\Delta\lambda'$ AT 5 ATM.
			Old Pressure Observations	Later Pressure Observations	
5202–5341	*a*.	0.182	0.184	0.184	0.198
	d.	0.166	0.177	0.170	0.198
5365–5476	*a*.	0.208	0.210	0.210	0.224
	e.	0.217	0.209	0.218	0.225

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[1] *Astrophysical Journal*, 41, 351, 1915.

atmosphere. Of four suggested explanations of the differences between solar and arc wave-lengths, (1) pressure, (2) motion in line of sight, (3) difference in the gravitational fields of the sun and earth, (4) anomalous dispersion, Albrecht considers only the first. The omission of the fourth, a fundamental deduction from the hypothesis according to Julius, is remarkable in a paper purporting to establish anomalous dispersion in the sun.

TABLE XVII

SUN-ARC DISPLACEMENTS INCONSISTENT WITH A PRESSURE OF 5 ATMOSPHERES IN THE SUN

Group	No. of Lines	Calculated at 5 Atm.	Observed	O−C
a and *b*........	211	+0.016	+0.004	−0.012
*c*5 and *d*........	125	+0.032	−0.006	−0.038
e..............	34	+0.007	+0.014	+0.007

The outstanding indications that appear to be established by solar observations are that no one cause furnishes a satisfactory explanation of the differences between the wave-lengths in solar and arc spectra, and that pressure, though intimately concerned, does not play the supremely predominant rôle formerly attributed to it. A conclusion that the pressure in the solar atmosphere where the Fe lines originate is 0.5 atmosphere appears questionable when no account has been taken of variation with the solar intensity of the lines and other suggested effects.

I wish to express my appreciation of the assistance in this long and exacting investigation rendered by Miss Ware, whose unflagging interest and conscientious work have made the investigation possible and greatly increased the weight of the observations.

SUMMARY AND CONCLUSIONS

1. The mean sun-arc displacements for 211 Fe lines of the stable groups *a*, *b*, and *c*4 is +0.0038 A; that for 56 lines with companions to the violet, mean separation 0.275 A, is +0.0036 A; that for 29 lines with companions to the red, mean separation 0.320 A, is +0.0038 A.

2. The mean sun-arc displacement for 125 lines of groups $c5$ and d is -0.0063 A; that for 25 lines in the same region with companions to the red is -0.0062 A. The mean displacement for 21 lines with companions to the violet is -0.0052 A; that for all $c5$ and d lines in the same region is -0.0050 A.

3. The mean sun-arc displacement for 34 lines of group e is $+0.0142$ A; that for 5 lines with companions to the red and 6 with companions to the violet is $+0.0156$ and $+0.0110$ A, respectively.

4. The mean separation in the solar spectrum for 45 pairs is, within the limits of error, identical with that in arc spectra.

5. The behavior of lines with companions is like that of similar isolated lines. Whether the separation in the solar spectrum is greater or less than in arc spectra depends upon the configuration of the pair. For 8 pairs of the 45 it should be larger, for 15 smaller; it is respectively 0.005 A greater and 0.0035 A less than in arc spectra.

6. For 54 lines of groups a, b, and $c4$, separation >0.1 A, the sum of the systematic displacements found by Albrecht is 0.226 A, that of the counterbalancing errors in the Rowland wave-lengths is 0.255 A.

7. For 34 lines of groups $c5$, d, and e, separation >0.1 A, the sum of the systematic displacements is 0.151 A; corrected for systematic error it is 0.099 A; that of the balancing errors is 0.096 A.

8. Of the 16 remaining lines, separation <0.1 A, 7 are without weight in a definitive discussion. The sum of the systematic displacements for the remaining 9 lines, separation from companions 0.082 A, is 0.025 A; that of the balancing Rowland errors is 0.027 A.

9. The coefficient of correlation between the 104 displacements attributed by Albrecht to mutual influence and the 104 errors in Rowland wave-lengths found in this investigation is $+0.56\pm0.05$.

10. The sun-arc displacements for iron lines are independent of the origin of closely adjacent lines.

11. The separations of iron lines from those due to another substance are the same in solar and arc spectra.

12. From 102 lines Albrecht finds a small Larmor effect which disappears when 3 lines giving inconsistently large values are omitted.

13. The later recognized systematic errors in the published data on pressure-shift invalidate conclusions based upon them.

14. The correspondence between the errors in the Rowland wave-lengths of lines with close companions and the displacements noted by Albrecht is so complete that it appears a practical certainty that these displacements are another measurement of the errors.

15. They therefore furnish no valid evidence that the relative positions of the Fraunhofer lines are systematically displaced by mutual influence. On the other hand, the sun-arc displacements (370 lines) and the relative separations of the components of 45 close pairs in solar and arc spectra indicate that, within the limits of error, evidence of mutual influence is absent from the general solar spectrum and in so far as mutual influence is a necessary corollary of anomalous dispersion evidence for the existence of the latter is also absent.

Mount Wilson Solar Observatory
August 1916

ERRATA

Owing to a printer's error and to the loss in transit of page-proofs sent by Professor Störmer from Christiana, the following corrections are to be made in his article "Researches on Solar Vortices," *Mt. Wilson Contribution*, No. 109.

Page 2, second equation, *for* ϕ *read* ϕ_0.
6, *delete* 4th and 5th lines from bottom.
17, *interchange* legends of the two parts of Fig. 9.
55, second line, the quantities between the inequality signs are two parts of a single expression. The comma should be replaced by a multiplication sign.
55, *delete* last sentence of the second paragraph of Section XIV.

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